USE OF THE INSTRUCTOR'S MANUAL

The problem solution portion of this manual has been prepared for the instructor who wishes to occasionally refer to the authors' method of solution or who wishes to check the answer of his (her) solution with the result obtained by the authors. In the interest of space and the associated cost of educational materials, the solutions are very concise. Because the problem solution material is not intended for posting of solutions or classroom presentation, the authors request that it not be used for these purposes.

In the transparency master section there are approximately 40 solved problems selected to illustrate typical applications. These problems are different from and in addition to those in the textbook. Instructors who have adopted the textbook are granted permission to reproduce these masters for classroom use.

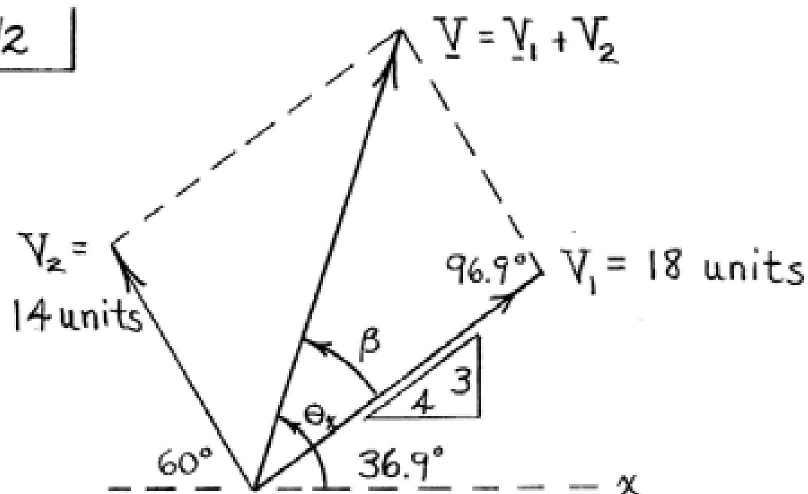
1/1

$$V = \sqrt{10^2 + 24^2} = 26$$

$$\cos\theta_x = \frac{-10}{26}, \quad \underline{\theta_x = 112.6°}$$

$$\underline{n} = \frac{\underline{V}}{V} = \frac{-10\underline{i} + 24\underline{j}}{26} = \underline{-0.385\underline{i} + 0.923\underline{j}}$$

1/2



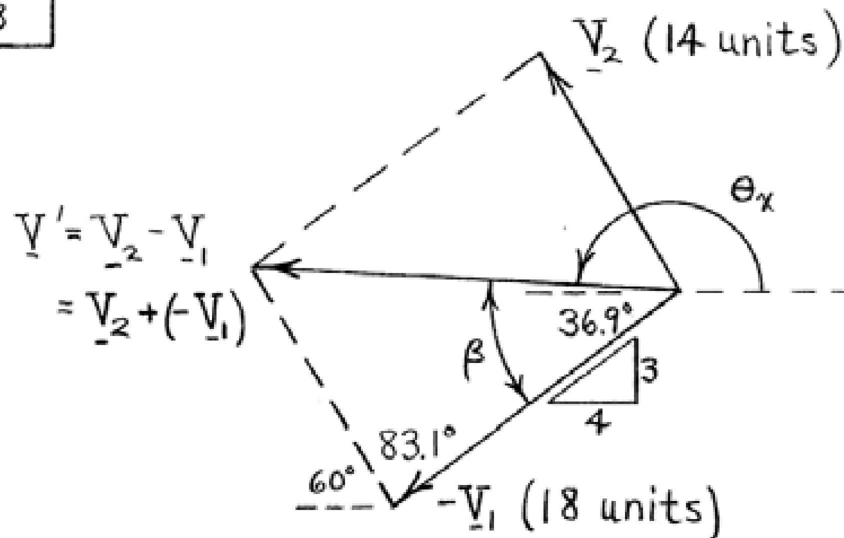
Graphically, $\underline{V = 24 \text{ units}}$, $\underline{\theta_x = 72°}$

Algebraically, $V^2 = 18^2 + 14^2 - 2(18)(14)\cos 96.9°$

$\underline{V = 24.1 \text{ units}}$

$\frac{\sin\beta}{14} = \frac{\sin 96.9°}{24.1}$, $\beta = 35.2°$

$\theta_x = \beta + 36.9° = \underline{72.1°}$



Graphically, $\underline{V' = 21 \text{ units}}$, $\underline{\theta_x = 176°}$

Algebraically, $V'^2 = 18^2 + 14^2 - 2(18)(14)\cos 83.1°$

$\underline{V' = 21.4 \text{ units}}$

$\frac{\sin\beta}{14} = \frac{\sin 83.1°}{21.4}$, $\beta = 40.4°$

$\theta_x + \beta = 217°$, $\theta_x = 217° - \beta = 217° - 40.4° = \underline{176.5°}$

1/4

$$F = \sqrt{80^2 + 40^2 + 60^2} = 107.7 \text{ lb}$$

$$\cos\theta_x = \frac{F_x}{F} = \frac{80}{107.7} = 0.743, \quad \underline{\theta_x = 42.0^\circ}$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{-40}{107.7} = -0.371, \quad \underline{\theta_y = 111.8^\circ}$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{60}{107.7} = 0.557, \quad \underline{\theta_z = 56.1^\circ}$$

1/5

$W = mg = 75(9.81) = \underline{736\text{ N}}$

$W = 736\text{ N}\left(\frac{1\text{ lb}}{4.4482\text{ N}}\right) = \underline{165.4\text{ lb}}$

1/6

$$F = W = \frac{G m_1 m_2}{r^2},$$

where $G = 6.673(10^{-11})\ m^3/(kg \cdot s^2)$

$m_1 = 80\ kg$

$m_2 = 5.976(10^{24})\ kg$

and $r = (6371 + 250)(10^3)\ m$

Substitute these numbers & obtain $W = 728\ N$

U.S. units : $W = 728\,N\left(\frac{1\ lb}{4.4482\ N}\right) = 163.6\ lb$

1/7

$$W = (130 \text{ lb})\left(\frac{4.4482 \text{ N}}{\text{lb}}\right) = \underline{578 \text{ N}}$$

$$m = \frac{W}{g} = \frac{130}{32.2} = \underline{4.04 \text{ slugs}}$$

$$m = \frac{W}{g} = \frac{578}{9.81} = \underline{58.9 \text{ kg}}$$

1/8 $A = 8.69$, $B = 1.427$

$(A+B) = 8.69 + 1.427 = \underline{10.12}$

$(A-B) = 8.69 - 1.427 = \underline{7.26}$

$(AB) = (8.69)(1.427) = \underline{12.40}$

$(A/B) = 8.69/1.427 = \underline{6.09}$

1/9

$$F = \frac{G m_e m_m}{d^2} = \frac{6.673(10^{-11})(5.976 \cdot 10^{24})^2(1)(0.0123)}{(384\,398 \cdot 10^3)^2}$$

$$= \underline{1.984(10^{20})\ N}$$

$$F = 1.984(10^{20})\ N \left(\frac{1\ lb}{4.4482\ N}\right) = \underline{4.46(10^{19})\ lb}$$

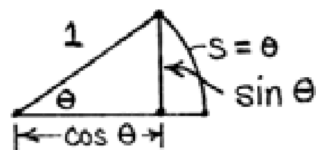
1/10 $20°\left(\frac{\pi \text{ rad}}{180°}\right) = 0.3491$ rad

$\sin 20° = 0.3420$

Percent error is $\frac{0.3420-0.3491}{0.3420}(100) = \underline{2.06\%}$

$\tan 20° = 0.3640$

Percent error is $\frac{0.3640-0.3491}{0.3640}(100) = \underline{4.09\%}$



The approximation $\sin\theta \cong \theta$ involves the approximation that the arclength $s = \theta$ is the vertical side of the triangle. The approximation that $\tan\theta \cong \theta$ involves, in addition, the approximation that 1 is the horizontal side of the triangle.

2/1

$$\begin{cases} F_x = 500 \cos 40° = \underline{383 \text{ N}} \\ F_y = -500 \sin 40° = \underline{-321 \text{ N}} \end{cases}$$

$$\underline{F} = 383\underline{i} - 321\underline{j} \text{ N}$$

2/2

$\underline{F} = 400(-\cos 30°\,\underline{i} + \sin 30°\,\underline{j})$

$= -346\underline{i} + 200\underline{j}$ lb

Scalar components: $\begin{cases} F_x = -346 \text{ lb} \\ F_y = 200 \text{ lb} \end{cases}$

Vector components: $\begin{cases} \underline{F}_x = -346\underline{i} \text{ lb} \\ \underline{F}_y = 200\underline{j} \text{ lb} \end{cases}$

2/3

$$\underline{F} = 5.2\left(-\frac{12}{13}\underline{i} - \frac{5}{13}\underline{j}\right)$$

$$= -4.8\underline{i} - 2\underline{j} \quad \text{kN}$$

(Note: Writing 2, rather than 2.00, indicates an exact result.)

2/4

$$\underline{F} = F\underline{n}_{AB} = 3000\left[\frac{15\underline{i} + 8\underline{j}}{\sqrt{15^2 + 8^2}}\right]$$

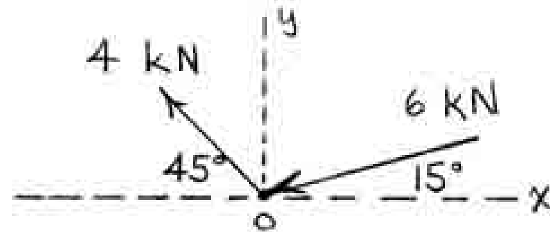
$$= 2650\underline{i} + 1412\underline{j} \text{ lb}$$

Scalar components : $\begin{cases} F_x = 2650 \text{ lb} \\ F_y = 1412 \text{ lb} \end{cases}$

2/5

$$\underline{F} = 1800\left(-\frac{3}{5}\underline{i} - \frac{4}{5}\underline{j}\right) = \underline{-1080\underline{i} - 1440\underline{j}\ \text{N}}$$

2/6



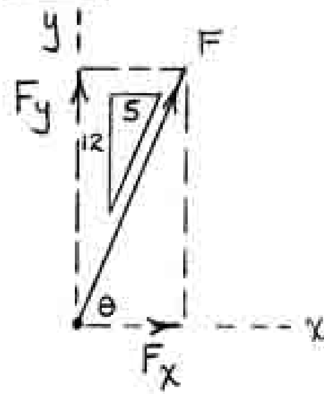
$$R_x = \Sigma F_x = -4\cos 45° - 6\cos 15° = -8.62 \text{ kN}$$

$$R_y = \Sigma F_y = 4\sin 45° - 6\sin 15° = 1.276 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \underline{8.72 \text{ kN}}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{1.276}{-8.62}\right) = \underline{171.6°}$$

2/7



$\cos\theta = \frac{5}{13}$, $\sin\theta = \frac{12}{13}$

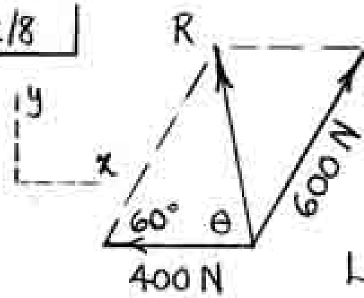
$F_y = F\sin\theta = F\frac{12}{13} = 70$

$\underline{F = 75.8 \text{ lb}}$

$F_x = F\cos\theta = 75.8\left(\frac{5}{13}\right)$

$= \underline{29.2 \text{ lb}}$

2/8



Law of cosines:

$$R^2 = 600^2 + 400^2 - 2(600)(400)\cos 60°$$

$\underline{R = 529 \text{ N}}$

Law of sines:

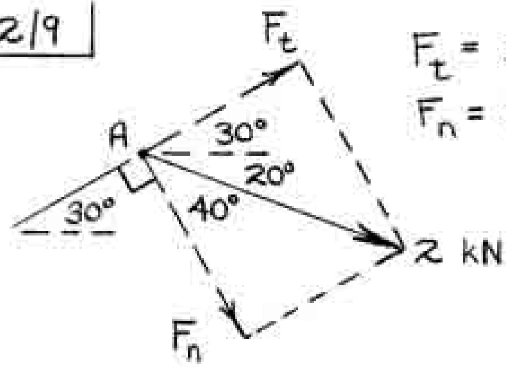
$$\frac{529}{\sin 60°} = \frac{600}{\sin\theta}, \quad \underline{\theta = 79.1°}$$

(b) $R_x = \Sigma F_x = 600\cos 60° - 400 = -100$ N

$R_y = \Sigma F_y = 600 \sin 60° + 0 = 520$ N

So $\underline{R} = -100\,\underline{i} + 520\,\underline{j}$ N

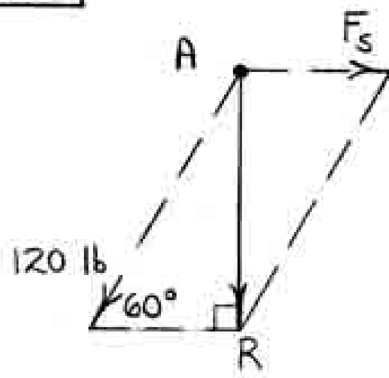
2/9



$F_t = 2 \cos 50° = \underline{1.286 \text{ kN}}$

$F_n = 2 \sin 50° = \underline{1.532 \text{ kN}}$

2/10



$$\cos 60° = \frac{F_s}{120}$$

$F_s = 60$ lb

$$\sin 60° = \frac{R}{120}$$

$R = 103.9$ lb

2/11

$$P_x = -260\left(\frac{12}{13}\right) = -240 \text{ N}$$

$$P_y = -260\left(\frac{5}{13}\right) = \underline{-100 \text{ N}}$$

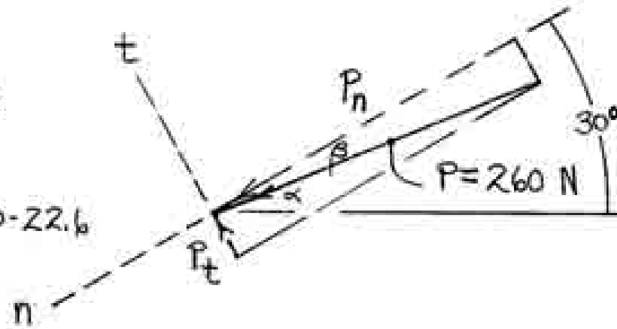
2/12

$\alpha = \tan^{-1} \frac{5}{12}$

$= 22.6°$

$\beta = 30 - \alpha = 30 - 22.6$

$= 7.38°$



$P_n = P\cos\beta = 260 \cos 7.38° = \underline{258 \text{ N}}$

$P_t = P\sin\beta = 260 \sin 7.38° = \underline{33.4 \text{ N}}$

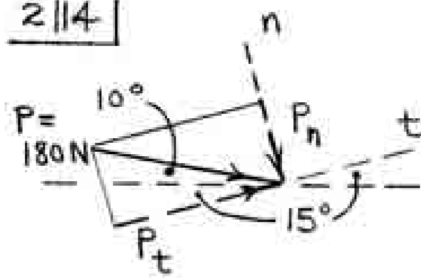
2/13

$R_x = \Sigma F_x = 400 + 400 \cos 60° = 600$ N

$R_y = \Sigma F_y = 400 \sin 60° = 346$ N

$\Rightarrow \underline{R} = 600\underline{i} + 346\underline{j}$ N

$R = \sqrt{600^2 + 346^2} = 693$ N



$P_t = 180 \cos 25° = \underline{163.1\ \text{N}}$

$P_n = -180 \sin 25° = \underline{-76.1\ \text{N}}$

2/15 (b) $\underline{R} = 400\,\underline{e}_t + 900\,\underline{e}_n$ lb

(a) $\underline{R} = [400\cos 15° - 900\sin 15°]\underline{i} + [400\sin 15° + 900\cos 15°]\underline{j}$

$= 153.4\,\underline{i} + 973\,\underline{j}$ lb

2/16 Using the coordinates of the problem figure:

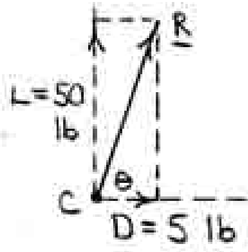
$$R_x = \Sigma F_x = 200 \cos 35° - 150 \sin 30°$$
$$= 88.8 \text{ N}$$

$$R_y = \Sigma F_y = 200 \sin 35° + 150 \cos 30°$$
$$= 245 \text{ N}$$

$$\therefore \underline{R} = 88.8\underline{i} + 245\underline{j} \text{ N}$$

2/17

$$\frac{L}{D} = \frac{50}{D} = 10 \; ; \quad D = 5 \text{ lb}$$



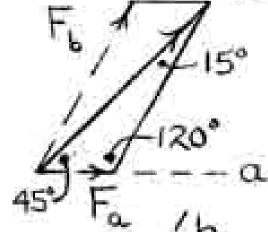
$$R = \sqrt{L^2 + D^2} = \sqrt{50^2 + 5^2}$$

$$= \underline{50.2 \text{ lb}}$$

$$\theta = \tan^{-1}\left(\frac{L}{D}\right) = \tan^{-1}\left(\frac{50}{5}\right)$$

$$= \underline{84.3°}$$

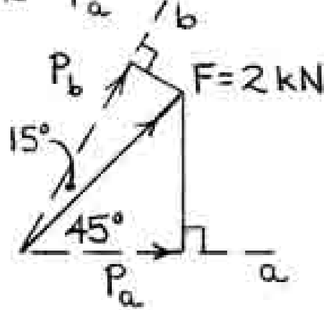
2/18



Components

$$\frac{\sin 120°}{2} = \frac{\sin 15°}{F_a}, \quad \underline{F_a = 0.598 \text{ kN}}$$

$$\frac{\sin 120°}{2} = \frac{\sin 45°}{F_b}, \quad \underline{F_b = 1.633 \text{ kN}}$$

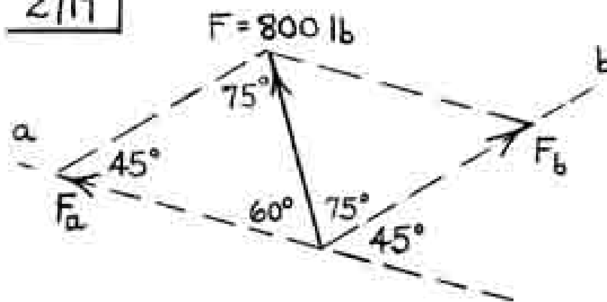


Projections

$P_a = 2 \cos 45° = \underline{1.414 \text{ kN}}$

$P_b = 2 \cos 15° = \underline{1.932 \text{ kN}}$

2/19

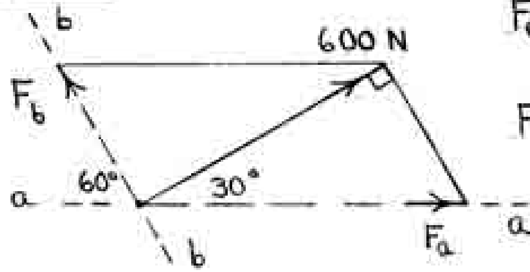


$$\frac{\sin 45°}{800} = \frac{\sin 75°}{F_a} = \frac{\sin 60°}{F_b}$$

Components: $\begin{cases} F_a = 1093 \text{ lb} \\ F_b = 980 \text{ lb} \end{cases}$

Projections: $\begin{cases} F_{p_a} = 800 \cos 60° = 400 \text{ lb} \\ F_{p_b} = 800 \cos 75° = 207 \text{ lb} \end{cases}$

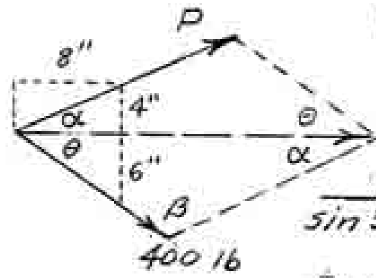
2/20



$$F_a = \frac{600}{\cos 30°} = \underline{693 \text{ N}}$$

$$F_b = 600 \tan 30° = \underline{346 \text{ N}}$$

2/21



$\alpha = \tan^{-1}\frac{4}{8} = 26.57°$

$\theta = \tan^{-1}\frac{6}{8} = 36.87°$

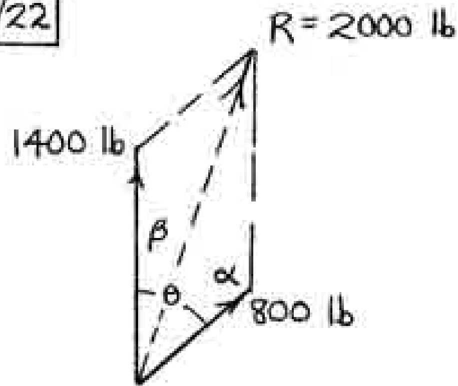
$\beta = 180 - (\alpha + \theta) = 116.57°$

$$\frac{P}{\sin 36.87°} = \frac{400}{\sin 26.57°}$$

$$P = 400\,\frac{0.6}{0.4472} = \underline{537\ \text{lb}}$$

$$\frac{T}{\sin 116.56°} = \frac{400}{\sin 26.57°}, \quad T = 400\,\frac{0.8944}{0.4472} = \underline{800\ \text{lb}}$$

2/22



Law of cosines: $2000^2 = 1400^2 + 800^2 - 2(1400)(800)\cos\alpha$

With $\alpha = 180 - \theta$ and $\cos(180 - \theta) = -\cos\theta$:

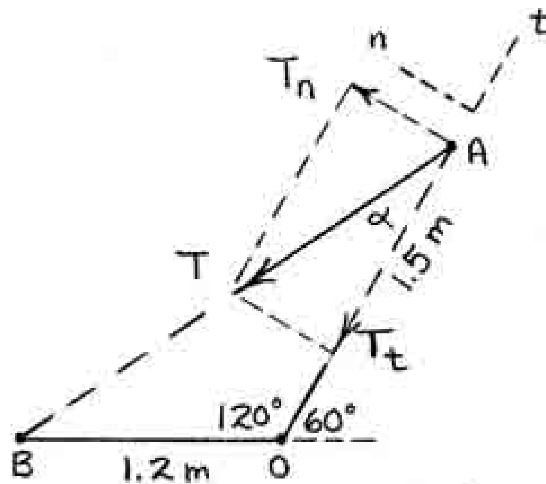
$2000^2 = 1400^2 + 800^2 + 2(1400)(800)\cos\theta$

$\underline{\theta = 51.3°}$

Law of sines: $\dfrac{800}{\sin\beta} = \dfrac{2000}{\sin(180° - 51.3°)}$

$\underline{\beta = 18.19°}$

2/23



$$\overline{AB}^2 = 1.2^2 + 1.5^2 - 2(1.2)(1.5)\cos 120°$$

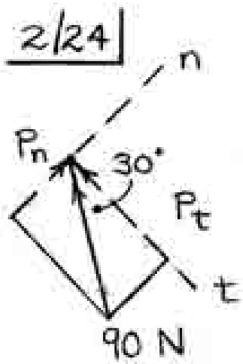
$$\overline{AB} = 2.34 \text{ m}$$

$$\frac{\sin\alpha}{1.2} = \frac{\sin 120°}{2.34}\ , \quad \alpha = 26.3°$$

$$T_n = T\sin\alpha = 750\sin 26.3° = \underline{333 \text{ N}}$$

$$T_t = -T\cos\alpha = -750\cos 26.3° = \underline{-672 \text{ N}}$$

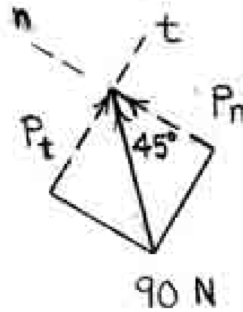
2/24



BC

$P_t = -90 \cos 30° = -77.9$ N

$P_n = 90 \sin 30° = 45.0$ N

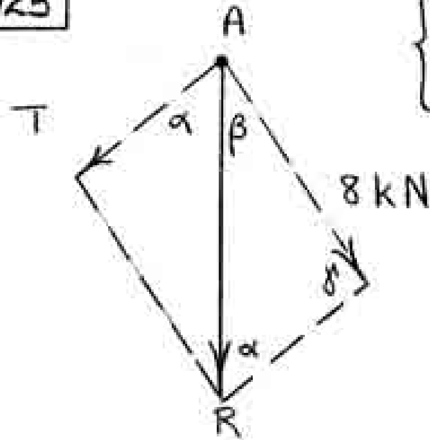


AB

$P_t = 90 \sin 45° = 63.6$ N

$P_n = 90 \cos 45° = 63.6$ N

2/25



$$\alpha = \tan^{-1}\frac{50}{40} = 51.3°$$
$$\beta = \tan^{-1}\frac{40}{60} = 33.7°$$

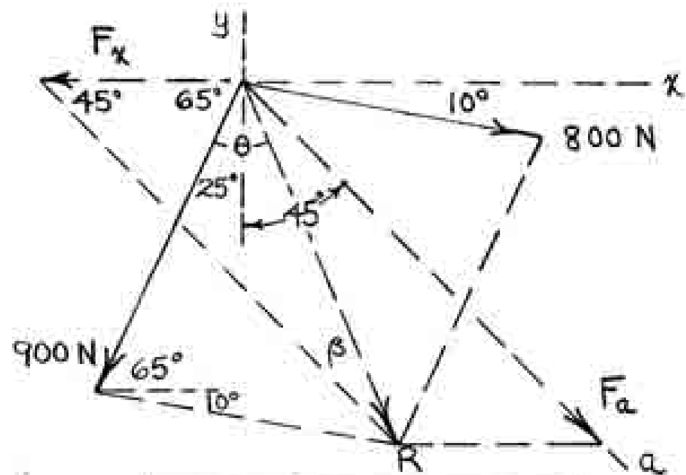
$$\gamma = 180 - \alpha - \beta = 95.0°$$

$$\frac{\sin\beta}{T} = \frac{\sin\alpha}{8}$$

$$\underline{T = 5.68 \text{ kN}}$$

$$\frac{\sin\gamma}{R} = \frac{\sin\alpha}{8}, \qquad \underline{R = 10.21 \text{ kN}}$$

2/26



Law of cosines: $R = \sqrt{900^2 + 800^2 - 2(900)(800)\cos 75^\circ} = 1038$ N

Law of sines: $\frac{1038}{\sin 75^\circ} = \frac{800}{\sin\theta}$, $\theta = 48.1^\circ$

$\beta = 180^\circ - 45^\circ - (65^\circ + 48.1^\circ) = 21.9^\circ$

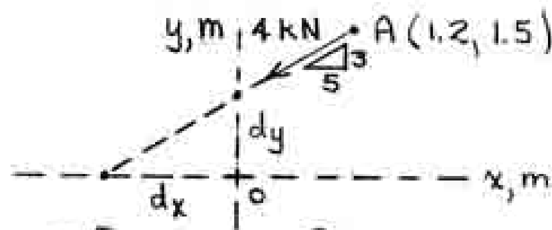
$\frac{1038}{\sin 45^\circ} = \frac{F_x}{\sin 21.9^\circ}$

$F_x = 547$ N

$\frac{F_a}{\sin(65^\circ + 48.1^\circ)} = \frac{}{\sin 45^\circ}$

$F_a = 1350$ N

2/27



$$\curvearrowleft + \quad M_O = 4\left[\frac{5}{\sqrt{34}}(1.5) - \frac{3}{\sqrt{34}}(1.2)\right] = \underline{2.68 \text{ kN}\cdot\text{m}}$$

As a vector, $\underline{\underline{M}_O = 2.68\underline{k} \text{ kN}\cdot\text{m}}$

$$\frac{1.5}{d_x + 1.2} = \frac{3}{5}, \quad d_x = 1.3 \text{ m}$$

$$\frac{d_y}{1.3} = \frac{3}{5}, \quad d_y = 0.78 \text{ m}$$

Coordinates of intercepts (in m): $\underline{(-1.3, 0), \ (0, 0.78)}$

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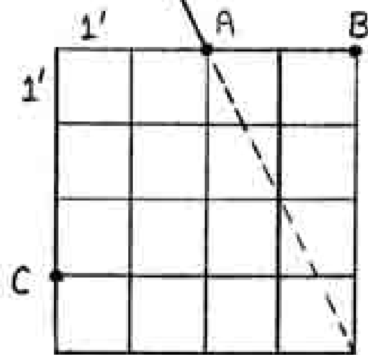
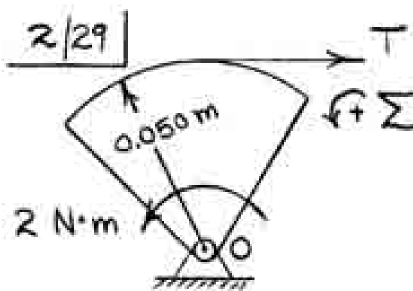


$\curvearrowright + \; M_B = 75\left[\frac{4}{\sqrt{20}}\,(2)\right]$

$= \underline{134.2 \text{ lb-ft}}$ (CW)

$\curvearrowleft + \; M_C = 75\left[\frac{4}{\sqrt{20}}(2) + \frac{2}{\sqrt{20}}(3)\right]$

$= \underline{235 \text{ lb-ft}}$ (CCW)



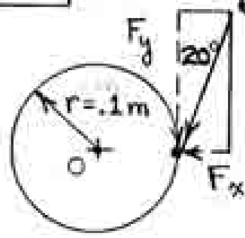
$$\circlearrowleft + \sum M_O = 2 - T(0.050) = 0$$

$$\underline{T = 40 \text{ N}}$$

2/30

$$+\circlearrowleft M_O = 180(9.81)(2.4) = \underline{4240\ \text{N}\cdot\text{m}}$$

2/31



$+\circlearrowleft M_O = r F_y$

$= (0.1)(60 \cos 20°)$

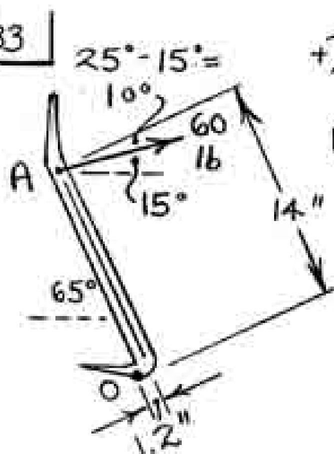
$= \underline{5.64 \text{ N}\cdot\text{m}}$

2/32

$\circlearrowleft + \; M_O = 250 \cos 15° (0.200) - 250 \sin 15° (0.030)$

$= 48.30 - 1.941 = \underline{46.4 \text{ N·m}}$

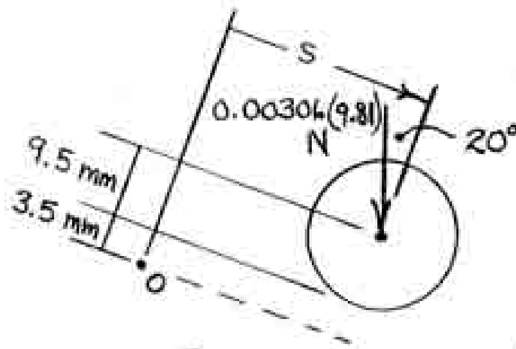
2/33



$+\circlearrowright M_O = (60 \cos 10°)(14) + (60 \sin 10°)(1.2) = 840 \text{ lb-in.}$

or $\underline{M_O = 70.0 \text{ lb-ft CW}}$

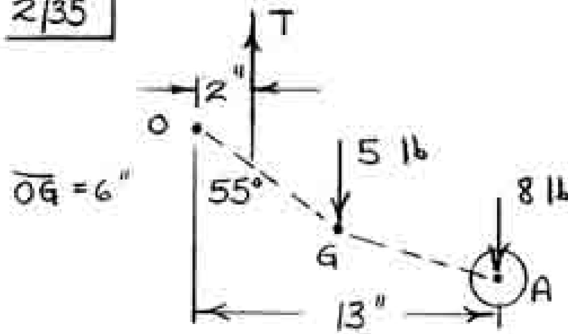
2/34



$$+\circlearrowleft M_O = 0.00306(9.81)\left[s\cos 20^\circ + (9.5+3.5)\sin 20^\circ\right]$$

$$= 0.1335 + 0.0282s \text{ N}\cdot\text{mm} \quad (s \text{ in mm})$$

2/35



The combined moment about O of the 5-lb and 8-lb weights is

$$\curvearrowright + \; M_O = 5(6 \sin 55°) + 8(13) = 128.6 \text{ lb-in.} \; \text{(CW)}$$

$$\curvearrowright + \; \Sigma M_O = 0: \; -T(2) + 128.6 = 0$$

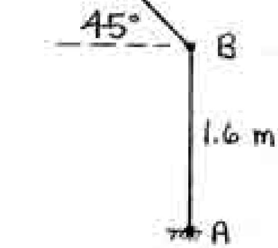
$$T = 64.3 \text{ lb}$$

2/36

P = 30 N

C

1.6 m

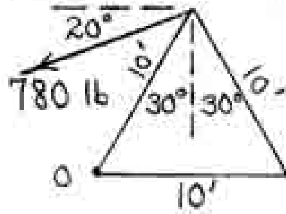


$$+\circlearrowleft M_B = 30(1.6) = 48 \text{ N}\cdot\text{m}$$

$$+\circlearrowleft M_A = 30\cos 45°(1.6 + 1.6\sin 45°)$$

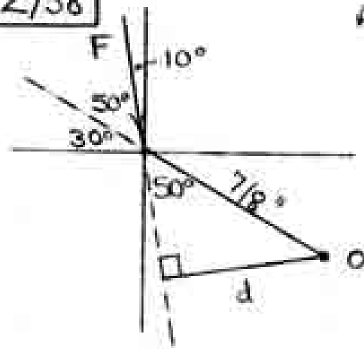
$$+ 30\sin 45°(1.6\cos 45°) = \underline{81.9 \text{ N}\cdot\text{m}}$$

2/37



$\circlearrowleft +$ $M_O = 780 \cos 20° \, (10 \cos 30°)$

$- 780 \sin 20° \, (5) = \underline{5010 \text{ lb-ft}}$



$$\stackrel{\curvearrowleft}{+} M_O = Fd$$

$$= 0.4\left(\frac{7}{8}\sin 50°\right)$$

$$= \underline{0.268 \text{ lb-in.}}$$

2/39

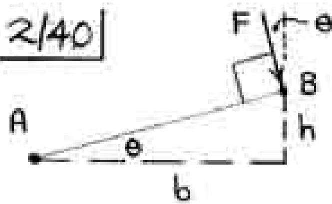
$d = 450 - 62.5 \cos 20°$

$= 391$ mm

$\circlearrowright + \quad M = Fd = 200(0.391)$

$= \underline{78.3 \text{ N·m}}$

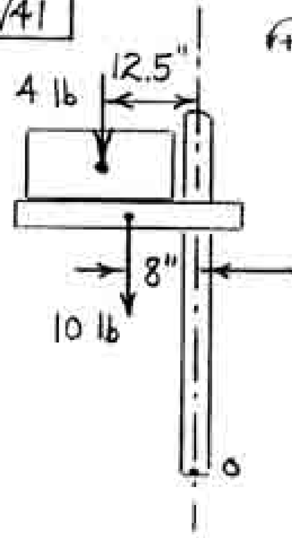
2/40



M_A is maximum when $\underline{F}$ is perpendicular to AB.

Thus $\theta = \tan^{-1}(h/b)$

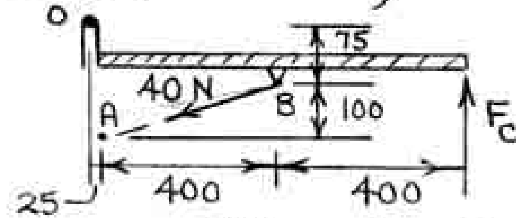
2/41



$$\curvearrowleft + \quad M_O = 4(12.5) + 10(8)$$

$$= \underline{130 \text{ lb-in.}}$$

2/42 (Dim. in mm)



$AB = \sqrt{400^2 + 100^2}$

$= 412 \text{ mm}$

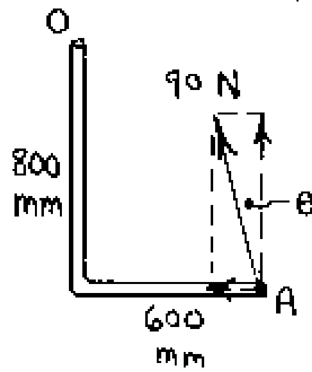
$$\circlearrowleft + \; M_O = \left(\frac{400}{412} \cdot 40\right)(75) + \left(\frac{100}{412} \cdot 40\right)(425)$$

$$= 7030 \text{ N}\cdot\text{mm} \quad \text{or} \quad \underline{7.03 \text{ N}\cdot\text{m}}$$

$$\circlearrowleft + \; \Sigma M_O = 0: -F_C(825) + 7030 = 0$$

$$\underline{F_C = 8.53 \text{ N}}$$

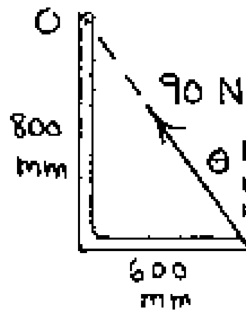
2/43



(a) $\theta = 15°$

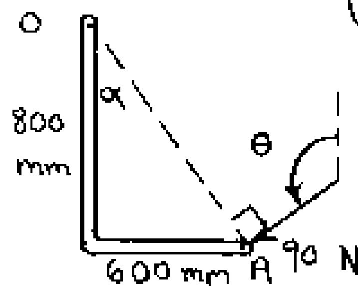
$\curvearrowleft M_O = 90\cos 15°(0.6) - 90\sin 15°(0.8)$

$= \underline{33.5\ \text{N}\cdot\text{m}}$



(b) $\theta = \tan^{-1}\left(\frac{600}{800}\right) = \underline{36.9°}$

$\underline{(\text{or } 217°)}$

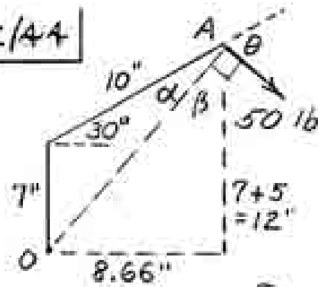


(c) $F \perp OA$, so

$\theta = 90° + \alpha$, $\alpha = \tan^{-1}\frac{600}{800} = 36.9°$

So $\theta = \underline{126.9°}$ (or $\underline{307°}$)

2/44



$\overline{AO} = \sqrt{12^2 + 8.66^2} = 14.80$ in.

$\frac{7}{\sin\alpha} = \frac{14.80}{\sin 120°}$, $\sin\alpha = \frac{7}{14.80}\,0.866$

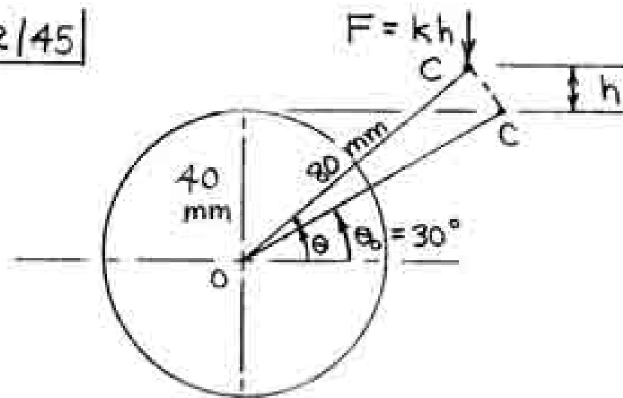
$= 0.410$

$\alpha = 24.18°$

$\theta = 180 - (24.18 + 90) = \underline{65.8°}$

$\overset{+}{\circlearrowleft} M_O = 50(14.80) = \underline{740 \text{ lb-in.}}$

2/45



$$M_o = F(80\cos\theta) = kh(80\cos\theta)$$

$$= k(80\sin\theta - 40)(80\cos\theta)$$

$$= 3200k(2\sin\theta\cos\theta - \cos\theta) = 3200k(\sin 2\theta - \cos\theta)$$

For maximum M_o, $\frac{dM_o}{d\theta} = 0$:

$$3200k(2\cos 2\theta + \sin\theta) = 0$$

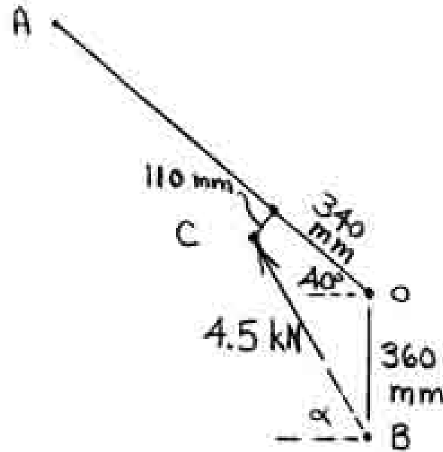
$$\Rightarrow 2(1 - 2\sin^2\theta) + \sin\theta = 0$$

$$4\sin^2\theta - \sin\theta - 2 = 0$$

$$\sin\theta = \frac{1 \pm \sqrt{1+32}}{8} = 0.843 \text{ or } -0.593$$

$\underline{\theta = 57.5°}$ (or $-36.4°$ – on dwell part of cam)

2/46

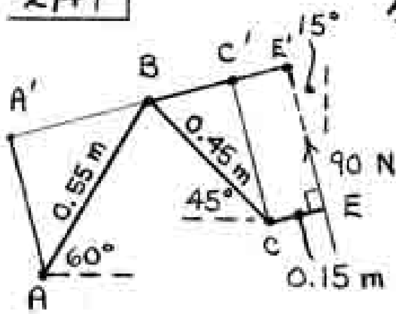


$$\alpha = \tan^{-1}\left[\frac{360 + 340\sin 40° - 110\sin 50°}{340\cos 40° + 110\cos 50°}\right]$$

$$= 56.2°$$

$$\curvearrowright + M_O = 4.5\,(0.360\cos 56.2°) = \underline{0.902\ \text{kN}\cdot\text{m CW}}$$

2/47



$$\circlearrowleft + \quad M_C = F(\overline{CE}) = 90(0.15) = \underline{13.50 \text{ Nm}}$$

$$M_B = F(\overline{BE'})$$

$$= 90(0.15 + 0.45 \sin 30°)$$

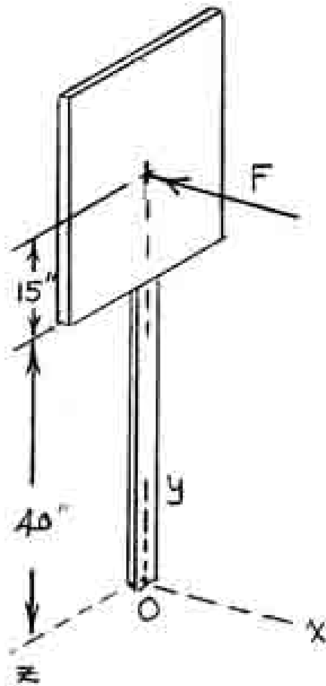
$$= \underline{33.8 \text{ N·m}}$$

$$M_A = F(\overline{A'E'})$$

$$= 90(0.15 + 0.45 \sin 30° + 0.55 \sin 45°)$$

$$= \underline{68.8 \text{ N·m}}$$

2/48



$$F = pA = 3.5\,\frac{(30)(24)}{144}$$

$$= 17.5 \text{ lb}$$

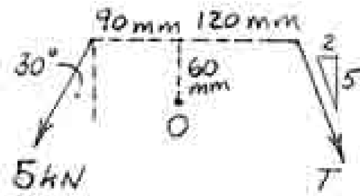
$$\underline{M}_O = \underline{r} \times \underline{F} = \frac{55}{12}\underline{j} \times (-17.5\underline{i})$$

$$= \underline{80.2\underline{k} \text{ lb-ft}}$$

2/49

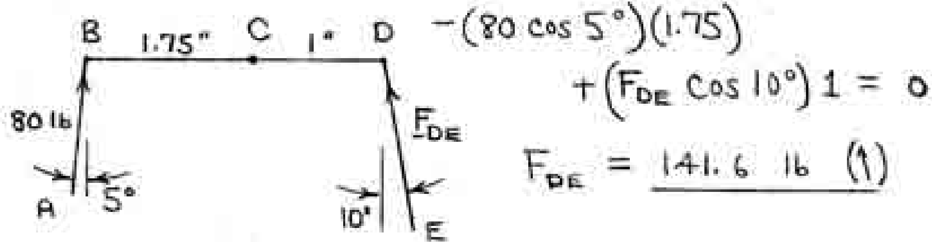
$M_O = 5[(\cos 30°)90 + (\sin 30°)60]$

$- T\left[\frac{5}{\sqrt{29}}(120) + \frac{2}{\sqrt{29}}(60)\right] = 0$



$539.7 - 133.7T = 0$, $\underline{T = 4.04 \text{ kN}}$

2/50

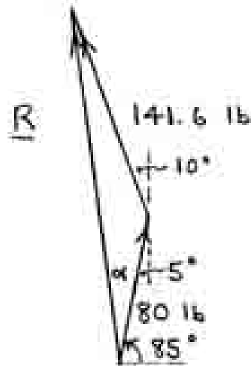


$\circlearrowleft + \Sigma M_C = 0:$

$-(80 \cos 5°)(1.75)$

$+ (F_{DE} \cos 10°)\,1 = 0$

$F_{DE} = \underline{141.6 \text{ lb} \ (\uparrow)}$



Law of cosines:

$R^2 = 80^2 + 141.6^2 - 2(80)(141.6)\cos 165°$

$R = \underline{220 \text{ lb}}$

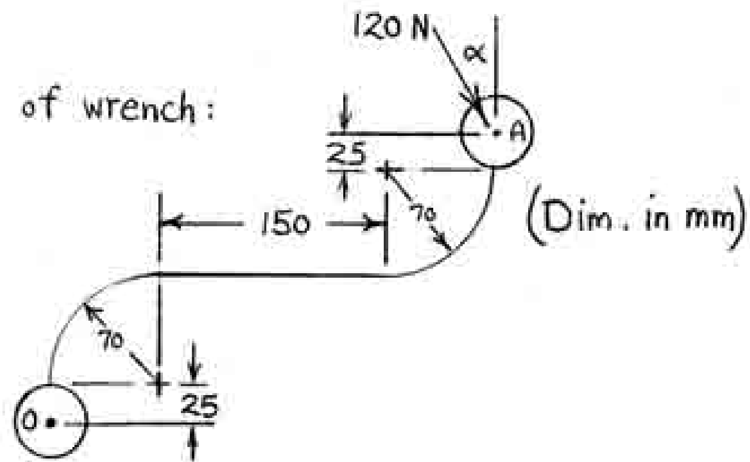
Law of sines:

$\dfrac{\sin\alpha}{141.6} = \dfrac{\sin 165°}{219.9}$

$\alpha = 9.60°, \quad \theta = 85° + \alpha = \underline{94.6°}$

2/51

Elements of wrench:



$\alpha = 30°$:

$$+\circlearrowleft M_O = 120 \cos 30° [70 + 150 + 70]$$
$$+ 120 \sin 30° [25 + 70 + 70 + 25] = 41\,500 \text{ N·mm}$$

or $\underline{M_O = 41.5 \text{ N·m CW}}$

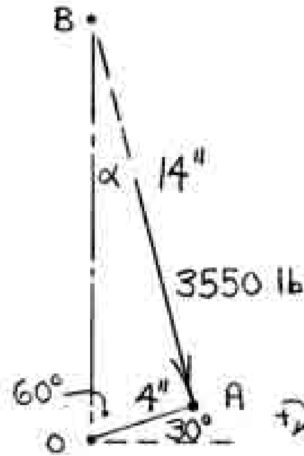
For maximum M_O:

$$\alpha = \tan^{-1}\left[\frac{25+70+25+70}{70+150+70}\right] = \underline{33.2°}$$

$$(M_O)_{max} = 120\sqrt{(25+70+25+70)^2 + (70+150+70)^2}$$

$$= 41\,600 \text{ N·mm or } \underline{41.6 \text{ N·m CW}}$$

2/52



Law of sines: $\frac{4}{\sin\alpha} = \frac{14}{\sin 60°}$

$\alpha = 14.33°$

$\overline{BO} = 14\cos 14.33° + 4\cos 60°$

$= 15.56$ in.

Consider 3550 lb acting at B:

$+\circlearrowleft M_O = (3550 \sin 14.33°)(15.56)$

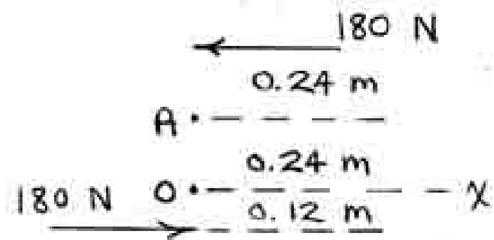
$= \underline{13{,}670 \text{ lb-in.}}$

(or $M_O = \underline{1139 \text{ lb-ft}}$)

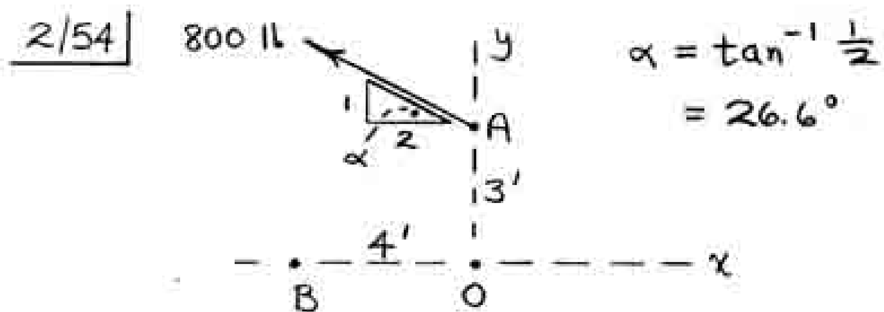
2/53 $\circlearrowleft + \; M = M_O = M_A = Fd$

$= 180(0.24 + 0.24 + 0.12)$

$= \underline{108 \text{ N·m CCW}}$



2/54



$\alpha = \tan^{-1}\frac{1}{2}$
$= 26.6°$

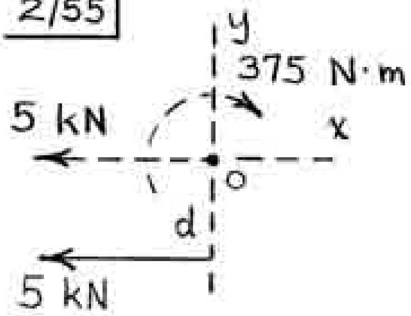
At O: $F = 800$ lb, 26.6°

$\circlearrowleft + M_O = 800\frac{2}{\sqrt{5}}(3) = 2150$ lb-ft CCW

At B: $F = 800$ lb, 26.6°

$\circlearrowleft + M_B = 800\frac{2}{\sqrt{5}}(3) + 800\frac{1}{\sqrt{5}}(4) = 3580$ lb-ft CCW

2/55

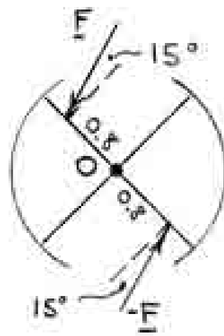


$+\circlearrowleft$ $M = Fd$: $375 = 5000d$

$d = 0.075$ m

$\therefore$ $\underline{y = -75 \text{ mm}}$

2/56

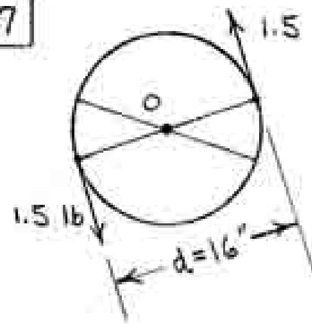


$$\curvearrowleft + \; M_O = \Sigma F d$$

$$25 = 2F(\cos 15°)(0.8)$$

$$F = \underline{16.18 \text{ N}}$$

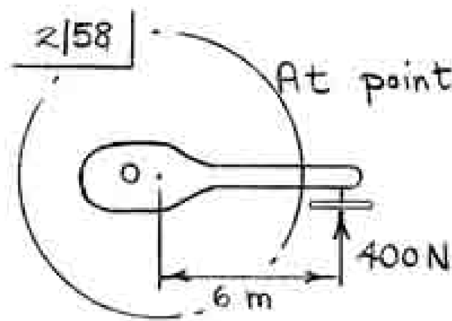
2/57



$\circlearrowleft M = Fd = (1.5)(16)$

$= \underline{24 \text{ lb-in.}}$

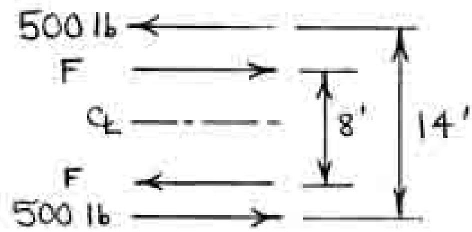
For constant forces: Increasing d increases M and increases circumferential hand movement. Decreasing d decreases M and decreases hand motion.



At point O: $R = 400\ \text{N}\ \uparrow$

$M_O = 400(6) = 2400\ \text{N}\cdot\text{m}$ CCW

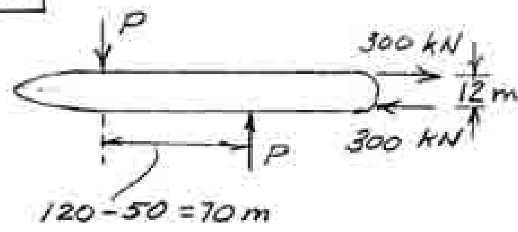
2/59



$$\circlearrowleft + \Sigma M = 500(14) - F(8) = 0$$

$$\underline{F = 875 \text{ lb}}$$

2/60

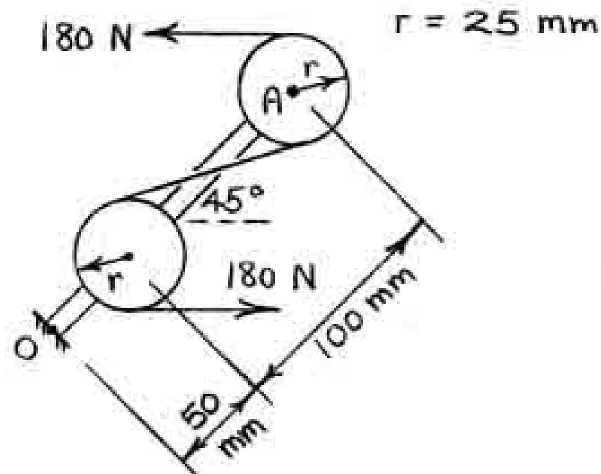


$70P = 300(12)$

$P = 51.4$ kN

2/61 $M = Fd$, $F = \frac{4000 \times 12}{4} = \underline{12{,}000 \text{ lb}}$

2/62

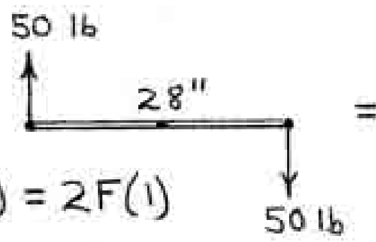


The system at O is a couple.

$\curvearrowleft + M = Fd = 180(100 \sin 45° + 25 + 25)$

$= 21\,700 \text{ N}\cdot\text{mm}$ or $\underline{21.7 \text{ N}\cdot\text{m CCW}}$

2/63



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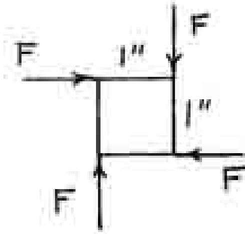
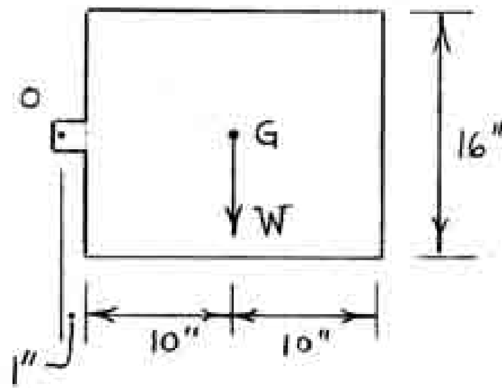


$M = 50(28) = 2F(1)$

$F = 700$ lb

2/64



The weight W of the door is

$$W = (\rho g)V = 489(20)(16)\left(\frac{1}{8}\right)\frac{1}{12^3} = 11.32 \text{ lb}$$

The equivalent force-couple system at O is

$$\begin{cases} \underline{R = 11.32 \text{ lb down}} \\ \underline{M_O = 11.32(11) = 124.5 \text{ lb-in. CW}} \end{cases}$$

Assumption: Neglect weight of small tab of steel near hinge.

2/65

(a) At A:

$F = 400 \text{ N}$ (20° from vertical)

$\Sigma M_A = (400 \cos 20°)(0.35)$

$= 131.6 \text{ N·m CW}$

(b) At O:

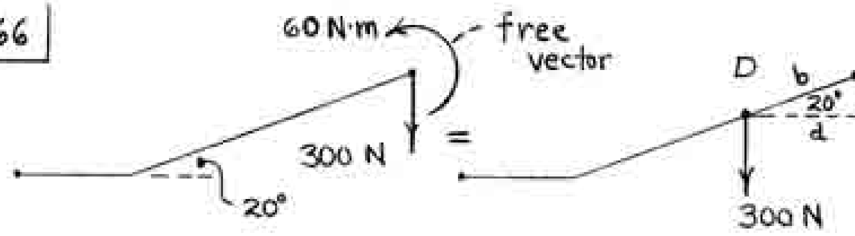
$F = 400 \text{ N}$ (20° from vertical)

$\Sigma M_O = 400 \cos 20° (0.35) + 400 \sin 20° (0.6)$

$= 214 \text{ N·m CW}$

Part (a) and (b) results are the same if $\theta = 0$ or $180°$.

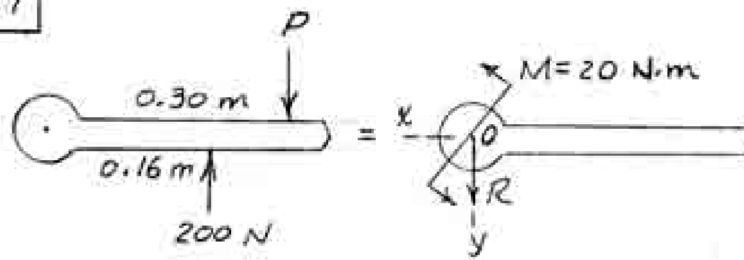
2/66



$M = Fd$: $\quad 60 = 300\,(b \cos 20°)$, $b = 0.213$ m

or $\underline{b = 213 \text{ mm}}$

2/67

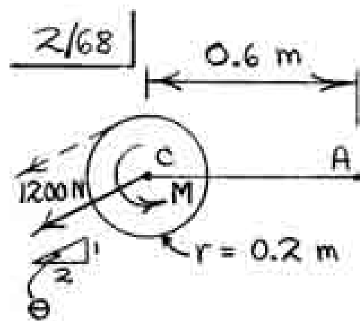


$M = \Sigma Fd;$ $20 = 200(0.16) - 0.30P$, $P = 40$ N

$\underline{P} = 40\underline{j}$ N

$\underline{R} = -200\underline{j} + 40\underline{j} = -160\underline{j}$ N

2/68

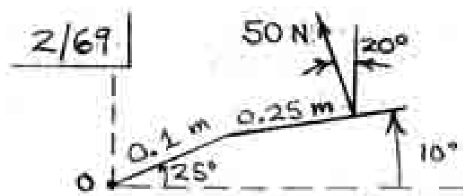


$\circlearrowleft +$ $M = 1200(0.2) = 240\ \text{N}\cdot\text{m}$

$\theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.6°$

$\circlearrowleft +$ $M_A = 1200 \sin 26.6\ (0.6) + 240 = \underline{562\ \text{N}\cdot\text{m}}$

2/69



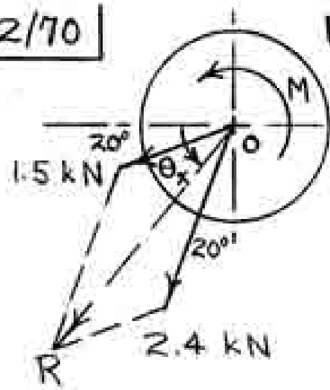
Use principle of moments.

$$\circlearrowleft + \Sigma M_O = 50 \cos 20° [0.1 \cos 25° + 0.25 \cos 10°]$$
$$+ 50 \sin 20° [0.1 \sin 25° + 0.25 \sin 10°]$$
$$= 17.29 \text{ N·m}$$

Force - Couple System at O:

$$\begin{cases} R = 50 \text{ N} \ \nwarrow 110° \\ M_O = 17.29 \text{ N·m} \ \circlearrowleft \end{cases}$$

2/70



$R_x = \Sigma F_x = 1.5 \cos 20° + 2.4 \sin 20°$

$= 2.23 \text{ kN}$

$R_y = \Sigma F_y = 1.5 \sin 20° + 2.4 \cos 20°$

$= 2.77 \text{ kN}$

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{2.23^2 + 2.77^2}$

$= \underline{3.56 \text{ kN}}$

$\theta_x = \tan^{-1}\left(\frac{2.77}{2.23}\right) = \underline{51.1°}$

$M = 1.5\,(200) \cos 20° - 2.4\,(120) \cos 20°$

$= \underline{11.28 \text{ N·m CCW}}$

Initial rotation would be CCW.

2/71

$$\tan 15° = \frac{F}{7000}$$

$$F = 1876 \text{ N}$$

G

A

1 m

0.5 m

7000 N

α

F

R

$(\alpha = 15°)$

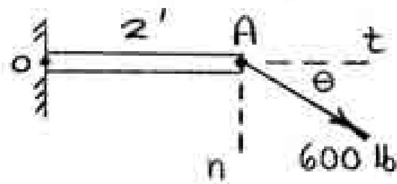
At G:

$$R = \Sigma F = \underline{7250 \text{ N}} \quad 105°$$

$$\curvearrowleft + M_G = 7000(1) + 1876(.5)$$

$$= \underline{7940 \text{ N}\cdot\text{m}}$$

2/72



The equivalent force-couple system at O is

$R_t = 600 \cos\theta$ (lb)

$R_n = 600 \sin\theta$ (lb

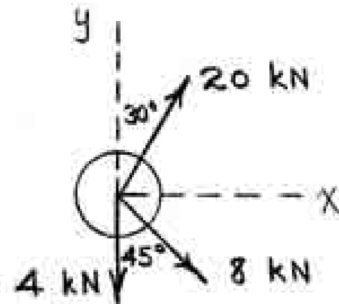
$+\circlearrowleft M_O = 2(600 \sin\theta) = 1200 \sin\theta$ (lb-ft)

Constraints:

$$\begin{cases} 600\cos\theta \le 550, & \theta \ge 23.6° \\ 600 \sin\theta \le 550, & \theta \le 66.4° \\ 1200 \sin\theta < 1000, & \theta \le 56.4° \end{cases}$$

All considered, $\underline{23.6° \le \theta \le 56.4°}$

2/73



$R_x = \Sigma F_x = 20 \sin 30° + 8 \sin 45° = 15.66$ kN

$R_y = \Sigma F_y = 20 \cos 30° - 8 \cos 45° - 4 = 7.66$ kN

$R = \sqrt{R_x^2 + R_y^2} = \underline{17.43 \text{ kN}}$

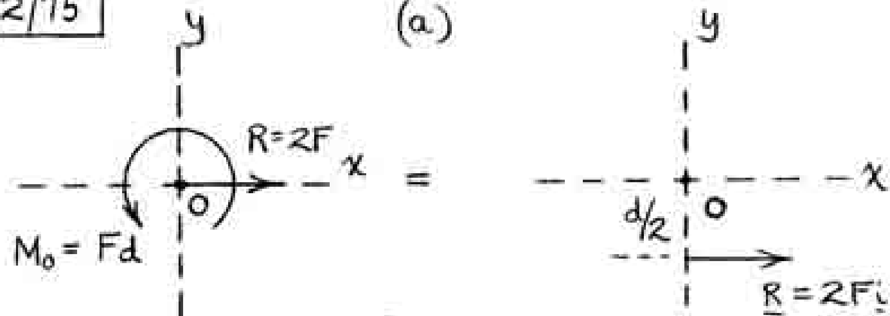
$\theta_x = \tan^{-1}(R_y/R_x) = \underline{26.1°}$

2/74 (a) $\underline{R} = -2F\underline{j}$, $\underline{M}_o = \underline{0}$

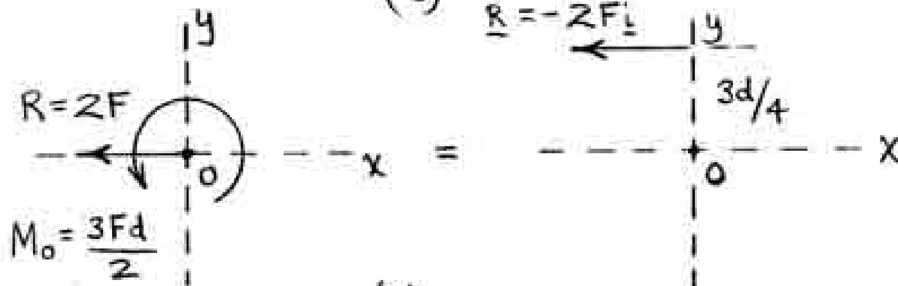
(b) $\underline{R} = \underline{0}$, $\underline{M}_o = Fd\underline{k}$ (+$\underline{k}$ is out)

(c) $\underline{R} = -F\underline{i} + F\underline{j}$, $\underline{M}_o = \underline{0}$

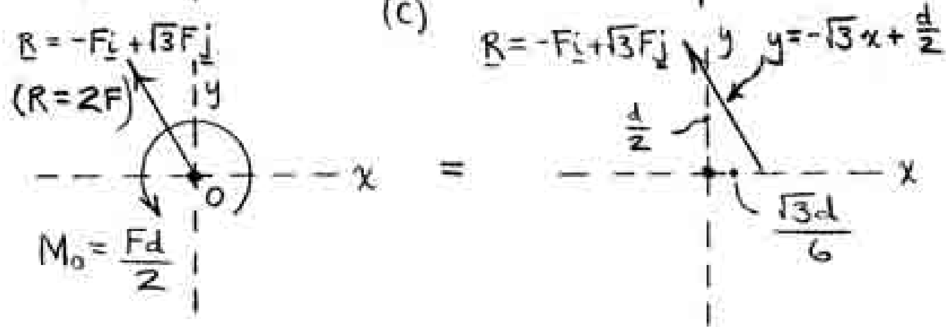
2/75



(a)
y
R=2F
x
=
y
x
O
$M_o = Fd$
d/2
O
$\underline{R} = 2F\underline{i}$

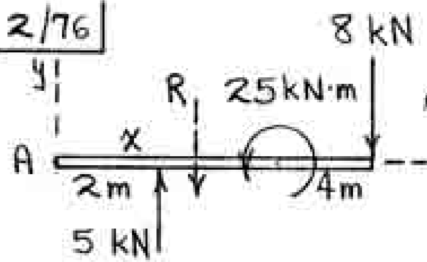


(b)
$\underline{R} = -2F\underline{i}$
y
y
3d/4
R=2F
x
=
x
O
O
$M_o = \frac{3Fd}{2}$



(c)
$\underline{R} = -F\underline{i} + \sqrt{3}F\underline{j}$
$\underline{R} = -F\underline{i} + \sqrt{3}F\underline{j}$
y
$y = -\sqrt{3}x + \frac{d}{2}$
(R=2F)
y
$\frac{d}{2}$
x
=
x
O
$\frac{\sqrt{3}d}{6}$
$M_o = \frac{Fd}{2}$

2/76



$R = \Sigma F_y = 5 - 8 = \underline{-3 \text{ kN}}$

$\circlearrowleft |R| x = M_A : \ 3x = -5(2) - 25 + 8(6)$

$\underline{x = 4.33 \text{ m}}$

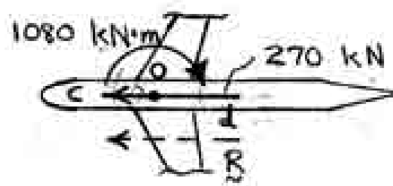
2/77 $M_O = 0$, so

$$\circlearrowleft^{+} M - 400(0.150 \cos 30°) - 320(0.300) = 0$$

$M = 148.0$ N·m

2/78 Force - Couple system at point O:

$$\begin{cases} R = 3(90) = 270 \text{ kN } (\leftarrow) \\ +\circlearrowleft M_O = 12(90) = 1080 \text{ kN·m} \end{cases}$$

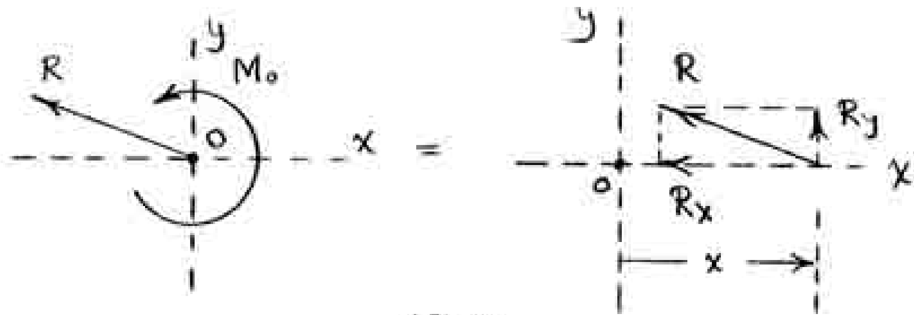


$$d = \frac{M_O}{R} = \frac{1080}{270}$$

$$= \underline{4 \text{ m}}$$

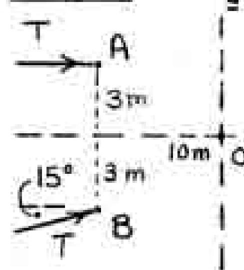
2/79 $\underline{\mathbf{R} = -50\mathbf{i} + 20\mathbf{j} \text{ lb}}$

$\circlearrowleft + \; M_O = -40(10) + 60(20) + 50(10) = 1300 \text{ lb-in.}$



$R_y x = M_O, \quad x = \dfrac{1300}{20} = \underline{65 \text{ in. (off pipe)}}$

2/80



$$\underline{R} = \Sigma \underline{F} = T\underline{i} + T(\cos 15^\circ \underline{i} + \sin 15^\circ \underline{j})$$

$$= 1.966T\underline{i} + 0.259T\underline{j}$$

$$\circlearrowleft M_O = 3T - T\cos 15^\circ (3) + T\sin 15^\circ (10) = 2.69T$$

$$-R_y x = M_O: \quad -0.259T(x) = 2.69T$$

$$x = -10.39 \text{ m}$$

2/81

$R_x = \Sigma F_x = 2\cos 70° + 1.2\left(\frac{4}{5}\right) = 1.644 \text{ kN}$

$R_y = \Sigma F_y = 2\sin 70° - 1.2\left(\frac{3}{5}\right) = 1.159 \text{ kN}$

$\curvearrowleft + \; M_A = -2\cos 70°(0.15) + 2\sin 70°(1.5 + 0.5)$

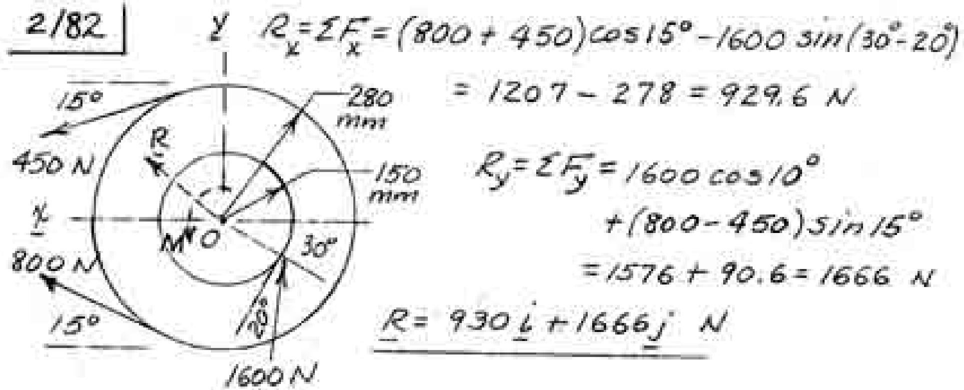
$+1.2\left(\frac{4}{5}\right)(0.15) - 1.2\left(\frac{3}{5}\right)(1.5) - 0.5$

$= 2.22 \text{ kN}\cdot\text{m}$ CCW

So the force-couple system is

$\begin{cases} \underline{R} = 1.644\underline{i} + 1.159\underline{j} \text{ kN} \\ M_A = 2.22 \text{ kN}\cdot\text{m} \text{ CCW} \end{cases}$

2/82



$R_x = \Sigma F_x = (800 + 450)\cos 15° - 1600 \sin(30° - 20°)$

$= 1207 - 278 = 929.6 \text{ N}$

$R_y = \Sigma F_y = 1600 \cos 10° + (800 - 450)\sin 15°$

$= 1576 + 90.6 = 1666 \text{ N}$

$\underline{\mathbf{R} = 930\mathbf{i} + 1666\mathbf{j} \text{ N}}$

$M = \Sigma M_O$ (+ccw); $M = 1600 \cos 20° (0.150) + (450 - 800) 0.280$

$= 225.5 - 98.0 = \underline{127.5 \text{ N·m CCW}}$

so unit is speeding up in CCW dir.

2/83 Equivalent force-couple system at A:

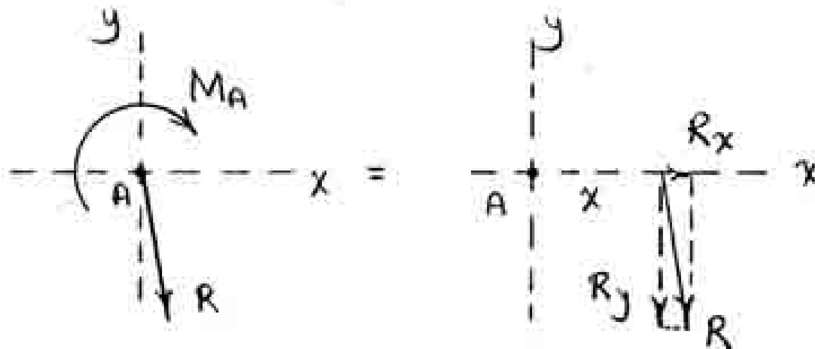
$\underline{R} = -2500\underline{j} - 1200\underline{j} + 800(\sin 30°\underline{i} + \cos 30°\underline{j})$

$= \underline{400\underline{i} - 3010\underline{j}\ \text{lb}}$

$\circlearrowright M_A = 2500(4) + 1200(4 + 4\cos 30° + 3)$

$- 800\sin 30°(2\sin 30°) - 800\cos 30°(4 + 2\cos 30°)$

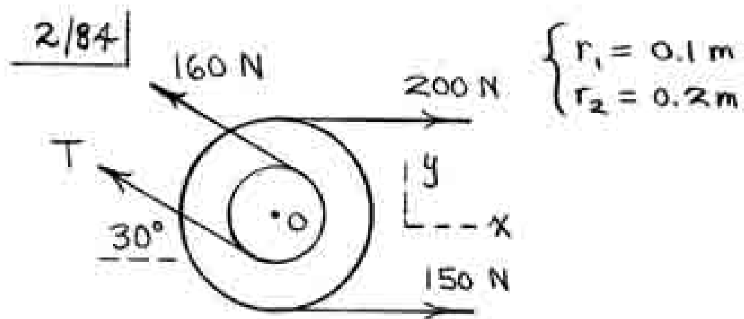
$= \underline{18{,}190\ \text{lb-ft}\quad \text{CW}}$



Condition: $x|R_y| = M_A$

$x = \dfrac{18{,}190}{3010} = \underline{6.05\ \text{ft}}$

2/84



$$+\circlearrowleft \Sigma M_O = 0: \ 200(0.2) - 150(0.2) - 160(0.1) + (0.1)T = 0$$

$$\underline{T = 60 \text{ N}}$$

$$R_x = \Sigma F_x = 200 + 150 - (160 + 60)\cos 30° = 159.5 \text{ N}$$

$$R_y = \Sigma F_y = (160 + 30)\sin 30° = 110 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2} = \underline{193.7 \text{ N}}$$

$$\theta = \tan^{-1}(R_y/R_x) = \underline{34.6°}$$

2/85 $\underline{R} = 45\underline{i} - 15\underline{j}$ lb

$\circlearrowright M_A = 25(30) + 15(60) = 1650$ lb-in.

or $\underline{M}_A = -1650\underline{k}$ lb-in.

For final line of action, $\underline{r} \times \underline{R} = \underline{M}_A$

$(x\underline{i} + y\underline{j}) \times (45\underline{i} - 15\underline{j}) = -1650\underline{k}$

$\Rightarrow -15x - 45y = -1650$ or $y = -\frac{1}{3}x + \frac{110}{3}$

(Axis intercepts: $x = 110''$, $y = {}^{110}/_{3}\,''$)

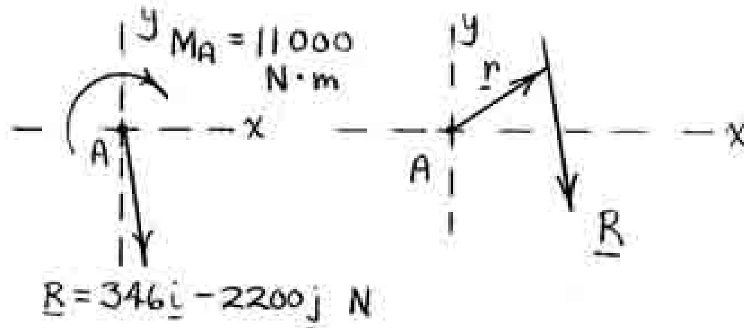
2/86 | Equivalent force-couple system at A:

$\underline{R} = \Sigma \underline{F} = [-2(250) - 3(500)]\underline{j} + 400[\cos 30° \underline{i} - \sin 30° \underline{j}]$

$= \underline{346\underline{i} - 2200\underline{j} \text{ N}}$

$M_A = 500[2.5 + 5 + 7.5] + 250[10] + 400(2.5)$

$= \underline{11\,000 \text{ N}\cdot\text{m} \quad \text{CW}}$



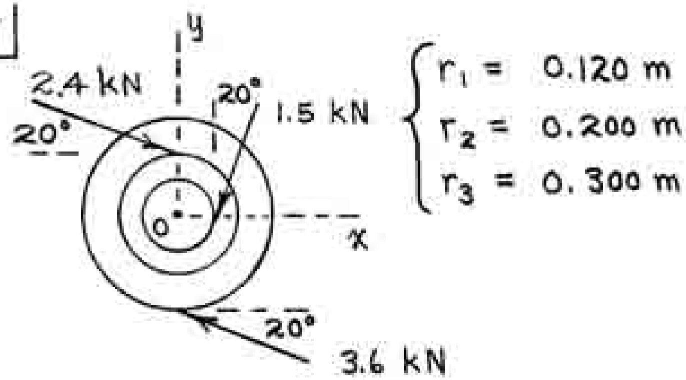
Condition: $\underline{M_A} = \underline{r} \times \underline{R}$

$-11\,000\underline{k} = (x\underline{i} + y\underline{j}) \times (346\underline{i} - 2200\underline{j})$

$= (-2200x - 346y)\underline{k}$

Set $y = 0$ & obtain $\underline{x = 5 \text{ m}}$ (!)

2/87



$\underline{R} = \Sigma \underline{F} = 2.4(c20°\underline{i} - s20°\underline{j}) + 1.5(-s20°\underline{i} - c20°\underline{j})$

$+ 3.6(-c20°\underline{i} + s20°\underline{j}) = -1.641\underline{i} - 0.999\underline{j}$ kN

$\circlearrowright M_O = (2.4(0.2) + 1.5(0.12) + 3.6(0.3))\cos 20° = 1.635$ kN·m

$\underline{r} \times \underline{R} = \underline{M_O}$: $(x\underline{i} + y\underline{j}) \times (-1.641\underline{i} - 0.999\underline{j}) = -1.635$

$\Rightarrow -0.999x + 1.641y = -1.635$

Axis intercepts : $x = 1.637$ m, $y = -0.997$ m

2/88 $\underline{R} = \Sigma \underline{F} = 400(\cos 45^\circ \underline{i} - \sin 45^\circ \underline{j}) + 500(\sin 15^\circ \underline{i} - \cos 15^\circ \underline{j})$

$= 412\underline{i} - 766\underline{j}$ N

$\circlearrowleft + \; M_o = (500-400)(0.060) = 6$ N·m

For the line of action of the standalone force:

$\underline{r} \times \underline{R} = \underline{M}_o$

$(x\underline{i} + y\underline{j}) \times (412\underline{i} - 766\underline{j}) = -6\underline{k}$

$-766x - 412y = -6$

For $x = 0$: $y = 0.01455$ m or $\underline{y = 14.55 \text{ mm}}$

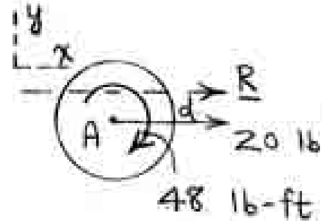
For $y = 0$: $x = 0.00783$ m or $\underline{x = 7.83 \text{ mm}}$

2/89 Force - Couple System at point A:

$\underline{R} = \Sigma \underline{F} = -500\underline{j} + 60\underline{i} - 100\underline{j} - 40\underline{i} + 600\underline{j}$

$= \underline{20\underline{i} \text{ lb}}$

$\curvearrowleft + \; M_A = 2 - 40\left(\frac{15}{12}\right) = -48 \text{ lb-ft}$



$Rd = M_A$

$d = \frac{M_A}{R} = \frac{48}{20}$

$= \underline{2.40 \text{ ft}}$

2/90 Use $\overset{y}{\uparrow}\underset{}{\overset{+}{\curvearrowleft}}\to_x$ system at G:

$$\underline{R} = \Sigma \underline{F} = (80 + 40 + 40 + 50 \sin 30^\circ)\underline{i} + (50 \cos 30^\circ + 70)\underline{j}$$

$$= \underline{185\,\underline{i} + 113.3\,\underline{j}\ \text{lb}}$$

$$M_G = 70(66) + 50 \sin 30^\circ (36) = 5520 \text{ lb-in.}$$

$$= \underline{460 \text{ lb-ft}}\ (\curvearrowleft)$$

For line of action of resultant:

$$\underline{r} \times \underline{R} = \underline{M}_G$$

$$(x\underline{i} + y\underline{j}) \times (185\underline{i} + 113.3\underline{j}) = 460\,\underline{k}$$

$$113.3x - 185y = 460$$

$$\underline{x = 4.06 \text{ ft}} \text{ when } y = 0.$$

2/91 | For a zero force-couple system at point O:

$$\underline{R} = \Sigma \underline{F} = (-F_C \sin 30° + F_D \sin 30°)\,\underline{i} + (50 - 10 - 100 - 50 + F_B + F_C \cos 30° + F_D \cos 30°)\,\underline{j} = \underline{0}$$

$$\Rightarrow F_C = F_D = F$$

$$\circlearrowleft M_O = -10(0.5) + 50(0.7) - 100(1.35) + F_B(2) - 50(2.5) + 2F \cos 30° (2.9) = 0$$

$\underline{F = F_C = F_D = 6.42 \text{ N}}$, $\underline{F_B = 98.9 \text{ N}}$

2/92 | Use the x-y coordinates of the figure:

$\underline{R} = \Sigma \underline{F} = 20(\sin 5° \underline{i} + \cos 5° \underline{j}) + 40(\sin 15° \underline{i} -$

$\cos 15° \underline{j}) = \underline{12.10 \underline{i} - 18.71 \underline{j} \text{ lb}}$

$\underline{M}_O = 6.5(\cos 30° \underline{i} + \sin 30° \underline{j}) \times 40(\sin 15° \underline{i} - \cos 15° \underline{j})$

$+ 6.5(-\cos 30° \underline{i} - \sin 30° \underline{j}) \times 20(\sin 5° \underline{i} + \cos 5° \underline{j})$

$= \underline{-358 \underline{k} \text{ lb-in.}}$

Condition for line of action of resultant:

$\underline{r} \times \underline{R} = \underline{M}_O$

$(x\underline{i} + y\underline{j}) \times (12.10\underline{i} - 18.71\underline{j}) = -358\underline{k}$

$(-18.71x - 12.10y)\underline{k} = -358\underline{k}$

or $-18.71x - 12.10y = -358$

(or $\underline{y = -1.547x + 29.6 \text{ in.}}$)

2/93 $\underline{T} = T\underline{n}_{AB}$

$$\underline{T} = 12\left[\frac{35\underline{i} - 25\underline{j} - 60\underline{k}}{\sqrt{35^2 + 25^2 + 60^2}}\right]$$

$$= \underline{5.69\underline{i} - 4.06\underline{j} - 9.75\underline{k} \ \ \text{kN}}$$

2/94

$$\underline{F} = F\underline{n}_{AB} = 400\left[\frac{-0.2\underline{i} + 0.5\underline{j} - 0.1\underline{k}}{\sqrt{0.2^2 + 0.5^2 + 0.1^2}}\right]$$

$$= \underline{-146.1\underline{i} + 365\underline{j} - 73.0\underline{k}\ \text{N}}$$

Projection onto x-axis $\underline{F_x = -146.1\ \text{N}}$

2/95

$$\underline{T} = T\frac{\underline{CD}}{\overline{CD}} = 1.2\frac{1.5\underline{i} + 3\underline{j} - 4.5\underline{k}}{\sqrt{1.5^2 + 3^2 + 4.5^2}}$$

$$= \underline{0.321\underline{i} + 0.641\underline{j} - 0.962\underline{k} \ \text{kN}}$$

The two indicated coordinate systems are equivalent for the question at hand.

2/96

$$T_{GF} = \underline{T} \cdot \underline{n}_{GF}$$

$$= (0.321\underline{i} + 0.641\underline{j} - 0.962\underline{k}) \cdot \frac{2\underline{i} - 3\underline{k}}{\sqrt{2^2 + 3^2}}$$

$$= \underline{0.978 \text{ kN}}$$

2/97 $F_{hor} = 5 \cos 50° = 3.21$ kN

$$\begin{cases} F_x = 3.21 \cos 65° = 1.358 \text{ kN} \\ F_y = 3.21 \sin 65° = 2.91 \text{ kN} \\ F_z = 5 \sin 50° = 3.83 \text{ kN} \end{cases}$$

So $\underline{F} = 1.358\underline{i} + 2.91\underline{j} + 3.83\underline{k}$ kN

Projection onto y-axis : $F_y = 2.91$ kN

Projection onto OB :

$$F_{OB} = \underline{F} \cdot \underline{n}_{OB}$$

$$= (1.358\underline{i} + 2.91\underline{j} + 3.83\underline{k}) \cdot (\cos 30° \underline{i} + \sin 30° \underline{j})$$

$$= 2.63 \text{ kN}$$

2/98 $\overline{AB} = \sqrt{\overline{1.2}^2 + \overline{0.5}^2 + \overline{0.1}^2} = 1.304$ m

$$\left.\begin{aligned} l &= -1.2/1.304 = -0.920 \\ m &= 0.5/1.304 = 0.383 \\ n &= 0.1/1.304 = 0.0767 \end{aligned}\right\} \underline{T} = 2(-0.920\underline{i} + 0.383\underline{j} + 0.077\underline{k}) \text{ kN}$$

2/99 The coordinates of point B are

$$(x_B, y_B, z_B) = (1.6, -0.8 \sin 30°, 0.8 \cos 30°)$$
$$= (1.6, -0.4, 0.693) \text{ m}$$

The position vector $\underline{BC}$ is

$$\underline{BC} = (0-1.6)\underline{i} + (0.7-(-0.4))\underline{j} + (1.2-0.693)\underline{k}$$
$$= -1.6\underline{i} + 1.1\underline{j} + 0.507\underline{k} \text{ m}$$

The unit vector which characterizes $\underline{BC}$ is

$$\underline{n}_{BC} = \frac{-1.6\underline{i} + 1.1\underline{j} + 0.507\underline{k}}{\sqrt{1.6^2 + 1.1^2 + 0.507^2}}$$
$$= -0.797\underline{i} + 0.548\underline{j} + 0.253\underline{k}$$

Then $\underline{T} = T\underline{n}_{BC}$

$$= 750(-0.797\underline{i} + 0.548\underline{j} + 0.253\underline{k})$$
$$= -598\underline{i} + 411\underline{j} + 189.5\underline{k} \text{ N}$$

2/100

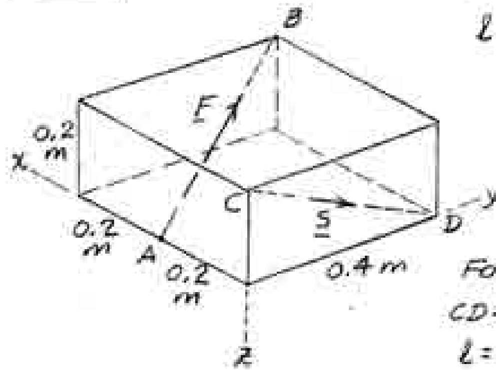
$$\underline{F} = F\,\underline{n}_{AM} = F\,\frac{\overline{AM}}{AM}$$

$$F = 500\left[\frac{16\underline{i} - 10\underline{j} + 8\underline{k}}{\sqrt{16^2 + 10^2 + 8^2}}\right]$$

$$= \underline{500\left[0.781\underline{i} - 0.488\underline{j} + 0.390\underline{k}\right]\ \text{lb}}$$

$\underline{F_x = 390\ \text{lb}}$, $\underline{F_y = -244\ \text{lb}}$, $\underline{F_z = 195.2\ \text{lb}}$

2/101 $F = 2$ kN; For $\underline{F}$, $\overline{AB} = \sqrt{\overline{0.2}^2 + \overline{0.4}^2 + \overline{0.2}^2} = \sqrt{0.24}$ m



$l = \frac{0.2}{\sqrt{0.24}} = \frac{1}{\sqrt{6}}$, $m = \frac{0.4}{\sqrt{0.24}} = \frac{2}{\sqrt{6}}$

$n = -\frac{1}{\sqrt{6}}$

$\underline{F} = \frac{2}{\sqrt{6}}(\underline{i} + 2\underline{j} - \underline{k})$ kN

For unit vector $\underline{s}$

$CD = \sqrt{\overline{0.2}^2 + \overline{0.4}^2} = \sqrt{0.20}$ m

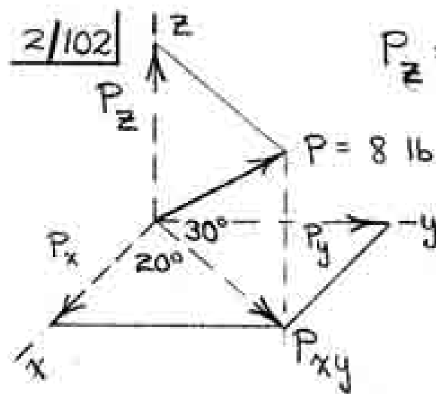
$l = 0$, $m = \frac{0.4}{\sqrt{0.2}} = \frac{2}{\sqrt{5}}$, $n = \frac{1}{\sqrt{5}}$

$\underline{s} = \frac{1}{\sqrt{5}}(2\underline{j} + \underline{k})$

$F_{CD} = \underline{F} \cdot \underline{s} = \frac{2}{\sqrt{6}}(\underline{i} + 2\underline{j} - \underline{k}) \cdot \frac{1}{\sqrt{5}}(2\underline{j} + \underline{k})$

$= \frac{2}{\sqrt{30}}(4 - 1) = \frac{6}{\sqrt{30}} = \underline{\sqrt{6/5}\text{ kN}}$

$\cos\theta = \frac{\underline{F} \cdot \underline{s}}{F} = \frac{\sqrt{6/5}}{2}$, $\underline{\theta = 56.8°}$



$P_z = P \sin 30° = 8 \sin 30°$
$= 4 \text{ lb}$

$P_{xy} = P \cos 30°$
$= 6.93 \text{ lb}$

$P_y = P_{xy} \sin 20°$
$= 2.37 \text{ lb}$

$P_x = P_{xy} \cos 20° = 6.51 \text{ lb}$

$\underline{\mathbf{P} = 6.51\mathbf{i} + 2.37\mathbf{j} + 4.00\mathbf{k} \text{ lb}}$

$\cos\theta_x = P_x/P = 6.51/8$; $\theta_x = 35.5°$

$\cos\theta_y = P_y/P = 2.37/8$; $\theta_y = 72.8°$

$\cos\theta_z = P_z/P = 4/8$; $\underline{\theta_z = 60.0°}$

2/103

$\underline{T} = T\underline{n}_{BC}$

$$\underline{T} = 800\left[\frac{+1\underline{i} - 7\underline{j} + 1.5\underline{k}}{\sqrt{1^2 + 7^2 + 1.5^2}}\right]$$

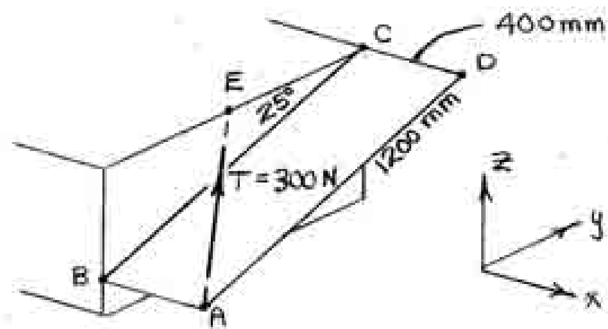
$$= \underline{+110.7\underline{i} - 775\underline{j} + 166.0\underline{k} \text{ lb}}$$

$\cos\theta_x = \frac{+1}{7.23}$, $\theta_x = 82.0°$

$\cos\theta_y = \frac{-7}{7.23}$, $\theta_y = 165.6°$

$\cos\theta_z = \frac{1.5}{7.23}$, $\underline{\theta_z = 78.0°}$

2/104



$$\underline{T} = T\underline{n}_{AE} = 300\left[\frac{-400\underline{i} + 544\underline{j} + 507\underline{k}}{\sqrt{400^2 + 544^2 + 507^2}}\right]$$

$$= 300\left[-0.474\underline{i} + 0.644\underline{j} + 0.601\underline{k}\right] \text{ N}$$

$\underline{n}_{BC} = \cos 25° \underline{j} + \sin 25° \underline{k}$

Carry out $T_{BC} = \underline{T} \cdot \underline{n}_{BC}$ to obtain

$$T_{BC} = 251 \text{ N}$$

2/105

$x_B - x_A = 120 - 90\sin 40° = 62.1$ mm

$y_B - y_A = 0 - (-90\cos 40°) = 68.9$ mm

$z_B - z_A = 0 - 140 = -140$ mm

$\overline{AB} = \sqrt{62.1^2 + 68.9^2 + 140^2}$

$= 168.0$ mm

$l = \frac{62.1}{168.0} = 0.370$

$m = \frac{68.9}{168.0} = 0.410$

$n = \frac{-140}{168.0} = -0.833$

$\therefore \quad \underline{\mathbf{T} = 500\,(0.370\mathbf{i} + 0.410\mathbf{j} - 0.833\mathbf{k})\text{ N}}$

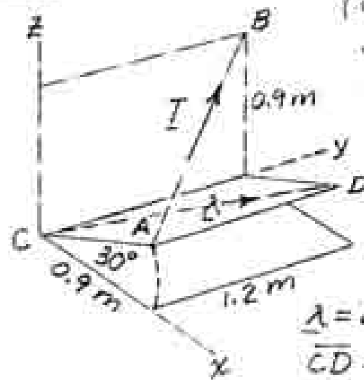
$\overline{DC} = \sqrt{140^2 + 100^2} = 172.0$ mm

$\mathbf{n}_{DC} = 0\mathbf{i} + \frac{100}{172.0}\mathbf{j} - \frac{140}{172.0}\mathbf{k} = 0.581\mathbf{j} - 0.814\mathbf{k}$

$T_{DC} = \mathbf{T}\cdot\mathbf{n}_{DC} = 500\,(0.371\mathbf{i} + 0.410\mathbf{j} - 0.833\mathbf{k})\cdot(0.581\mathbf{j} - 0.814\mathbf{k})$

$= \underline{458 \text{ N}}$

2/106 $T = 150\ N$



Coordinates of A are

$(0.9\cos 30°, 0, 0.9\sin 30°)$

or $(0.779, 0, 0.45)$ m

$\overline{AB} = \sqrt{0.779^2 + 1.2^2 + (0.9 - 0.45)^2}$

$= 1.50\ m$

$l = -0.779/1.5,\ m = 1.2/1.5,\ n = \frac{0.45}{1.5}$

$\underline{T} = \frac{100}{1.5}(-0.779\underline{i} + 1.2\underline{j} + 0.45\underline{k})\ N$

$\underline{\lambda}$ = unit vector along CD

$\overline{CD} = \sqrt{0.9^2 + 1.2^2} = 1.5\ m$

$l = 0.779/1.5,\ m = 1.2/1.5,\ n = 0.45/1.5$

$\lambda = \frac{1}{1.5}(0.779\underline{i} + 1.2\underline{j} + 0.45\underline{k})$

$T_{CD} = \underline{T}\cdot\underline{\lambda} = \frac{100}{1.5}(-0.779\underline{i} + 1.2\underline{j} + 0.45\underline{k})\cdot\frac{1}{1.5}(0.779\underline{i} + 1.2\underline{j} + 0.45\underline{k})$

$= \frac{100}{2.25}(-0.779^2 + 1.2^2 + 0.45^2) = \frac{100}{2.25}(-0.6075 + 1.44 + 0.2025)$

$= \frac{400}{9}(1.035) = \underline{46.0\ N}$

▸2/107

$\theta = \tan^{-1} \frac{4.5}{30} = 8.53°$

$T_{xy} = 200 \cos 15° = 193.2$ lb

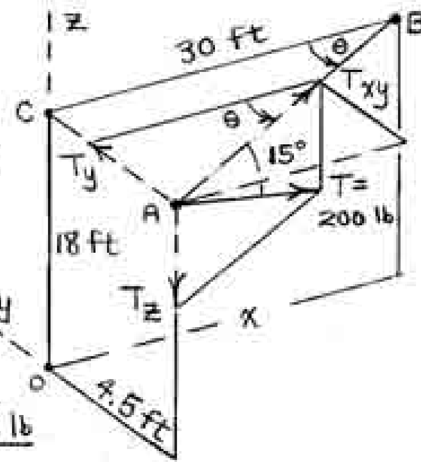
$T_x = T_{xy} \cos\theta = 191.0$ lb

$T_y = T_{xy} \sin\theta = 28.7$ lb

$T_z = -T \sin 15° = -51.8$ lb

$T_{xz} = \sqrt{T_x^2 + T_z^2} = \underline{197.9 \text{ lb}}$

$\underline{\mathbf{T} = 191.0\mathbf{i} + 28.7\mathbf{j} - 51.8\mathbf{k} \text{ lb}}$



▸2/108 The position of point A is

$$\underline{r}_A = 10\cos 15°\underline{i} + L\underline{j} + 10\sin 15°\underline{k}$$

$$= 9.66\underline{i} + L\underline{j} + 2.59\underline{k} \text{ in.}$$ [L = distance from O to disk center]

$$\underline{r}_B = 8\cos 30°\underline{i} + (L+36)\underline{j} - 8\sin 30°\underline{k}$$

$$= 6.93\underline{i} + (L+36)\underline{j} - 4\underline{k} \text{ in.}$$

$$\underline{r}_{AB} = \underline{r}_B - \underline{r}_A = -2.73\underline{i} + 36\underline{j} - 6.59\underline{k} \text{ in.}$$

$$r_{AB} = \sqrt{2.73^2 + 36^2 + 6.59^2} = 36.7 \text{ in.}$$ | unstretched length is $\sqrt{(8-10)^2 + 36^2} = 36.1$ in.

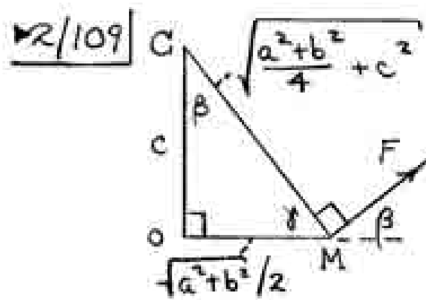
The spring force is $F = k\delta = 15(36.7 - 36.1) = 9.66$ lb

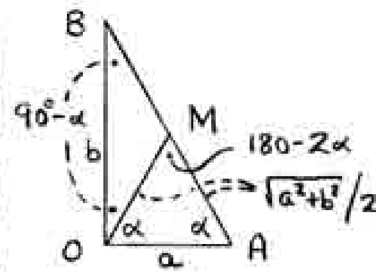
As a vector: $\underline{F} = F\underline{n}_{AB} = F\dfrac{\underline{r}_{AB}}{r_{AB}}$

$$\underline{F} = 9.66\left[\frac{-2.73\underline{i} + 36\underline{j} - 6.59\underline{k}}{36.7}\right]$$

$$= \underline{-0.719\underline{i} + 9.48\underline{j} - 1.734\underline{k} \text{ lb}}$$

2/109





$\tan\gamma = \frac{c}{\sqrt{a^2+b^2}/2} = \frac{2c}{\sqrt{a^2+b^2}}$

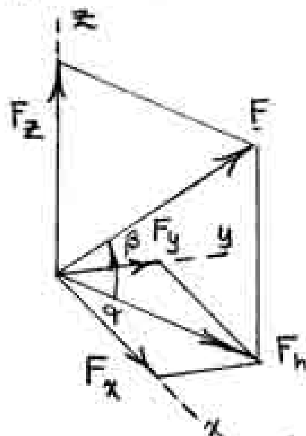
$\gamma + 90° + \beta = 180°$

$\beta = 90° - \gamma = 90° - \tan^{-1}\frac{2c}{\sqrt{a^2+b^2}}$

$\tan\alpha = \frac{b}{a}$

$\cos\alpha = \frac{a}{\sqrt{a^2+b^2}}$

$\sin\alpha = \frac{b}{\sqrt{a^2+b^2}}$



$F_z = F\sin\beta$

$F_h = F\cos\beta$

$F_x = F_h\cos\alpha = F\cos\beta\cos\alpha$

$F_y = F_h\sin\alpha = F\cos\beta\sin\alpha$

Now simplify $\sin\beta$ & $\cos\beta$ expressions:

$\sin\beta = \sin\left[90° - \tan^{-1}\frac{2c}{\sqrt{a^2+b^2}}\right]$

$= \sin 90° \cos\left[\tan^{-1}\frac{2c}{\sqrt{a^2+b^2}}\right] - \cancelto{0}{\cos 90°} \sin\left[\tan^{-1}\frac{2c}{\sqrt{a^2+b^2}}\right]$

$= \frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2+4c^2}}$ (Also see that $\sin\beta = \cos\gamma$!)

$\cos\beta = \sin\gamma = \frac{c}{\sqrt{\frac{a^2+b^2}{4}+c^2}} = \frac{2c}{\sqrt{a^2+b^2+4c^2}}$

Finally,

$F_x = F\frac{2c}{\sqrt{a^2+b^2+4c^2}}\frac{a}{\sqrt{a^2+b^2}} = \frac{2acF}{\sqrt{a^2+b^2}\sqrt{a^2+b^2+4c^2}}$

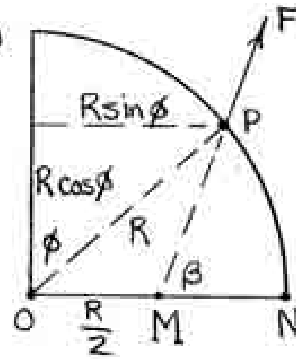
$F_y = F\frac{2c}{\sqrt{a^2+b^2+4c^2}}\frac{b}{\sqrt{a^2+b^2}} = \frac{2bcF}{\sqrt{a^2+b^2}\sqrt{a^2+b^2+4c^2}}$

$F_z = F\sqrt{\frac{a^2+b^2}{a^2+b^2+4c^2}}$

▸2/110

$F_x = F_{xy}\cos\theta$, $F_y = F_{xy}\sin\theta$

$F_z = F\sin\beta$, $F_{xy} = F\cos\beta$



$$\tan\beta = \frac{R\cos\phi}{R\sin\phi - \frac{R}{2}} = \frac{2\cos\phi}{2\sin\phi - 1}$$

So $$\sin\beta = \frac{2\cos\phi}{\sqrt{(2\sin\phi - 1)^2 + (2\cos\phi)^2}}$$

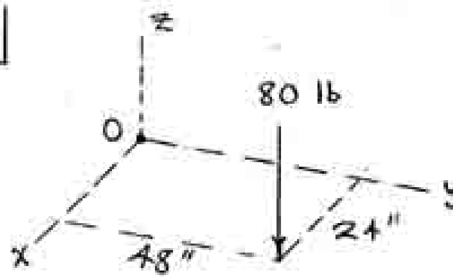
$$\cos\beta = \frac{2\sin\phi - 1}{\sqrt{(2\sin\phi - 1)^2 + (2\cos\phi)^2}}$$

Note that $\sqrt{(2\sin\phi - 1)^2 + (2\cos\phi)^2} = \sqrt{5 - 4\sin\phi}$

So $$\underline{F} = F\left[\cos\theta\cos\beta\,\underline{i} + \sin\theta\cos\beta\,\underline{j} + \sin\beta\,\underline{k}\right]$$

$$= \frac{F}{\sqrt{5 - 4\sin\phi}}\left[(2\sin\phi - 1)(\cos\theta\,\underline{i} + \sin\theta\,\underline{j}) + 2\cos\phi\,\underline{k}\right]$$

2/111



$$\begin{cases} M_{O_x} = -80\left(\frac{48}{12}\right) = -320 \text{ lb-ft} \\ M_{O_y} = 80\left(\frac{24}{12}\right) = 160 \text{ lb-ft} \\ M_{O_z} = 0 \end{cases}$$

$$\therefore \underline{\underline{M}_O = -320\underline{i} + 160\underline{j} \text{ lb-ft}}$$

$$M_O = \sqrt{320^2 + 160^2} = \underline{358 \text{ lb-ft}}$$

2/112

$$\underline{M}_O = F(-c\underline{i} + a\underline{k})$$

$$\underline{M}_A = Fa\underline{k}$$

2/113 $\underline{M} = -150(0.250 + 0.250)\underline{i} + 150(0.150)\underline{j}$

$= -75\underline{i} + 22.5\underline{j}$ N·m

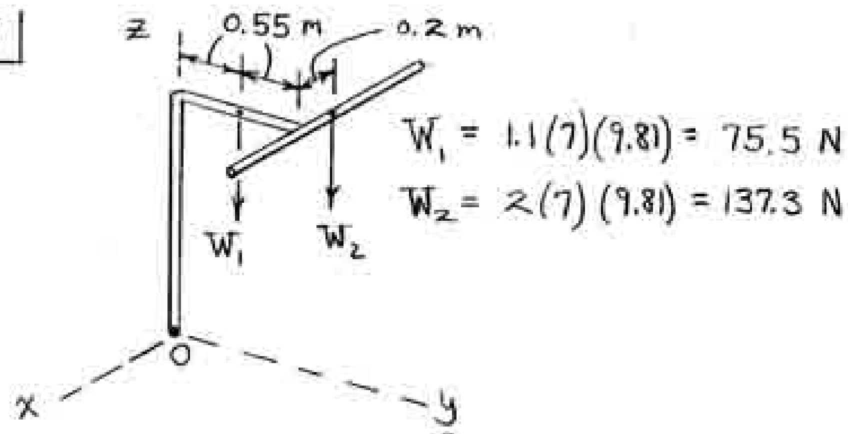
2/114

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

$$= (-6\mathbf{i} + 0.8\mathbf{j} + 1.2\mathbf{k}) \times (-400\mathbf{j})$$

$$= \underline{480\mathbf{i} + 2400\mathbf{k} \text{ N}\cdot\text{m}}$$

2/115



$W_1 = 1.1(7)(9.81) = 75.5$ N

$W_2 = 2(7)(9.81) = 137.3$ N

$$\begin{cases} M_{O_x} = -75.5(0.55) - 137.3(1.1) = -192.6 \text{ N}\cdot\text{m} \\ M_{O_y} = -137.3(0.2) = -27.5 \text{ N}\cdot\text{m} \\ M_{O_z} = 0 \end{cases}$$

$$\therefore \quad \underline{M}_O = -192.6\underline{i} - 27.5\underline{j} \text{ N}\cdot\text{m}$$

$$M_O = 194.6 \text{ N}\cdot\text{m}$$

2/116 $\overline{AB} = \sqrt{0.8^2 + 1.5^2 + 2^2} = 2.62$ m

$\underline{T} = \frac{1.2}{2.62}(0.8\underline{i} + 1.5\underline{j} - 2\underline{k})$ kN

Take $\underline{r} = \overrightarrow{OA} = 1.6\underline{i} + 2\underline{k}$ m

$$\underline{M}_O = \underline{r} \times \underline{T} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1.6 & 0 & 2 \\ 0.8 & 1.5 & -2 \end{vmatrix} \frac{1.2}{2.62}$$

$$\underline{M}_O = 0.457(-3\underline{i} + 4.8\underline{j} + 2.40\underline{k}) \text{ kN·m}$$

$$M_O = |\underline{M}_O| = 0.457\sqrt{3^2 + 4.8^2 + 2.40^2} = \underline{2.81 \text{ kN·m}}$$

2/117 From the solution to Prob. 2/99, the force is $\underline{R} = \underline{T} = -598\underline{i} + 411\underline{j} + 189.5\underline{k}$ N

The moment associated with the couple is $\underline{M}_O = \underline{r}_{OC} \times \underline{T}$, where $\underline{r}_{OC} = 0.7\underline{j} + 1.2\underline{k}$ m

Carry out the cross product to obtain

$$\underline{M}_O = -361\underline{i} - 718\underline{j} + 419\underline{k} \text{ N}\cdot\text{m}$$

2/118 $\underline{T} = 400\left[\frac{6\underline{i} + 2\underline{j} - 9\underline{k}}{\sqrt{6^2 + 2^2 + 9^2}}\right]$

$= 218\underline{i} + 72.7\underline{j} - 327\underline{k}$ N

$\underline{r}_{CA} = 6\underline{j} + 3\underline{k}$ m

Carry out $\underline{M}_C = \underline{r}_{CA} \times \underline{T}$ to obtain

$\underline{M}_C = -2180\underline{i} + 655\underline{j} - 1309\underline{k}$ N·m

$M_C = \sqrt{M_x^2 + M_y^2 + M_z^2} = \underline{2630 \text{ N·m}}$

2/119

From Prob. 2/101 unit vector $\underline{s}$ is $\underline{s} = \frac{1}{\sqrt{5}}(2\underline{j} + \underline{k})$

& $\underline{F} = \frac{F}{\sqrt{6}}(\underline{i} + 2\underline{j} - \underline{k})$

Take $\underline{r} = \overrightarrow{DA}$

$= 0.2\underline{i} - 0.4\underline{j}$ m

$M_{CD} = \underline{r} \times \underline{F} \cdot \underline{s}$

$= \left[(0.2\underline{i} - 0.4\underline{j}) \times \frac{F}{\sqrt{6}}(\underline{i} + 2\underline{j} - \underline{k})\right] \cdot \frac{1}{\sqrt{5}}(2\underline{j} + \underline{k})$

So

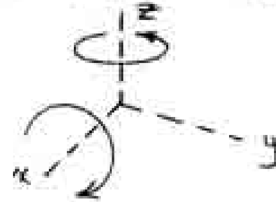
$$50 = \frac{F}{\sqrt{30}} \begin{vmatrix} 0.2 & -0.4 & 0 \\ 1 & 2 & -1 \\ 0 & 2 & 1 \end{vmatrix}$$

$$= \frac{F}{\sqrt{30}}(0.8 + 0.4) = \frac{1.2F}{\sqrt{30}}, \quad F = \frac{50\sqrt{30}}{1.2} = \underline{228 \text{ N}}$$

2/120 $\underline{M} = 1.2(40)\,\underline{k} - 1.2(50)\,\underline{i}$

$= \underline{-60\,\underline{i} + 48\,\underline{k} \text{ lb-in.}}$

The spacecraft will begin to rotate about its x- and z axes.



2/121 $\underline{F} = 50(\cos 15°\underline{i} - \sin 15°\underline{k})$ lb

$\underline{r}_{OC} = -8\sin 15°\underline{i} + 7\underline{j} - (6 + 8\cos 15°)\underline{k}$

Carry out $\underline{M}_O = \underline{r}_{OC} \times \underline{F}$ to obtain

$\underline{M}_O = -90.6\underline{i} - 690\underline{j} - 338\underline{k}$ lb-in.

$M_{OA} = M_{O_y} = -690$ lb-in.

2/122

$$\begin{cases} x_B - x_A = 0 - (-14\cos 60°) = 7 \text{ in.} \\ y_B - y_A = 14 - 14\sin 60° = 1.876 \text{ in.} \\ z_B - z_A = 0 - 16 = -16 \text{ in.} \end{cases}$$

$$\overline{AB} = \sqrt{7^2 + 1.876^2 + 16^2} = 17.56 \text{ in.}$$

$$\underline{T} = 120\left(\frac{7\underline{i} + 1.876\underline{j} - 16\underline{k}}{17.56}\right) = 47.8\underline{i} + 12.81\underline{j} - 109.3\underline{k} \text{ lb}$$

$$\underline{r}_{OB} = 14\underline{j} \text{ in.}$$

Carry out $\underline{M}_O = \underline{r}_{OB} \times \underline{T}$ to obtain

$$\underline{M}_O = -1530\underline{i} - 670\underline{k} \text{ lb-in.}$$

2/123 $\underline{F} = 300(\sin 60° \underline{j} - \cos 60° \underline{k})$ N

$\underline{r}_{OA} = 0.115\underline{i} + 0.350\cos 40° \underline{j} + 0.350 \sin 40° \underline{k}$ m

Carry out $\underline{M}_O = \underline{r}_{OA} \times \underline{F}$ to obtain

$\underline{M}_O = -98.7\underline{i} + 17.25\underline{j} + 29.9\underline{k}$ N·m

$M_x = -98.7$ N·m

2/124

$$M_O = (250 \sin 60°)12 + (250 \cos 60°) \sin 40° (8 - 4.2)$$

$$= \underline{2900 \text{ lb-in.}}$$ (CW)

2/125 $\underline{R} = 50(-\cos 40° \underline{i} - \sin 40° \underline{j})$

$= -38.3\underline{i} - 32.1\underline{j}$ N

$\underline{M}_O = (50 \sin 40°)(20)\underline{i} - (50 \cos 40°)(20)\underline{j}$

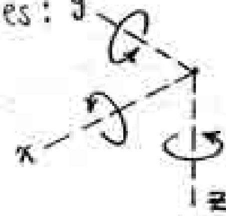
$+ 50(125)\underline{k}$

$= 643\underline{i} - 766\underline{j} + 6250\underline{k}$ N·mm

2/126 $\underline{M} = (1700)(2)\underline{i} - (1700)(30)\underline{j} - (1700)(30)\underline{k}$

$$= \underline{3400\,\underline{i} - 51\,000\,\underline{j} - 51{,}000\,\underline{k}\ \text{N·m}}$$

The orbiter would acquire rotational motion about all three axes:



2/127

$$\mathbf{M}_O = 0\mathbf{i} - (200)(0.2 + 0.125 \sin 20°)\,\mathbf{j}$$

$$-200\,(0.125 \cos 20° - 0.070)\,\mathbf{k}$$

$$= -48.6\mathbf{j} - 9.49\mathbf{k} \quad \text{N·m}$$

There would be no z-component of $\mathbf{M}_O$ if

$d \cos 20° - 70 = 0$, $d = 74.5$ mm

2/128 $\underline{M}_O = \underline{r}_{OB} \times \underline{T}$, $\underline{r}_{OB} = 6\underline{i} + 13\underline{j}$ m

$$\underline{T} = T\underline{n}_{AB} = 24\left[\frac{6\underline{i} - 5\underline{j} - 30\underline{k}}{\sqrt{6^2 + 5^2 + 30^2}}\right]$$

$$= 4.65\underline{i} - 3.87\underline{j} - 23.2\,\underline{k} \text{ kN}$$

Carry out $\underline{r}_{OB} \times \underline{T}$ to obtain

$$\underline{M}_O = -302\underline{i} + 139.4\underline{j} - 83.6\underline{k} \text{ N·m}$$

2/129

$$\underline{T} = T\left[\frac{-0.35\underline{i} - 0.45\cos 20°\,\underline{j} + (0.4 + 0.45\sin 20°)\,\underline{k}}{\sqrt{(0.35)^2 + (0.45\cos 20°)^2 + (0.4 + 0.45\sin 20°)^2}}\right]$$

$$= 143.4\left[-0.449\underline{i} - 0.542\underline{j} + 0.710\underline{k}\right] \text{ N}$$

Moment of this force about the x-axis is

$$M_{O_x} = (0.710)(143.4)(0.45\cos 20°) - 0.542(143.4)(0.45\sin 20°) = \underline{31.1 \text{ N}\cdot\text{m}}$$

The moment of the weight W of the 15-kg plate about the x-axis is

$$(M_{O_x})_W = -15(9.81)\,\frac{0.45\cos 20°}{2} = \underline{-31.1 \text{ N}\cdot\text{m}}$$

The moment of $\underline{T}$ about the line OB is zero, because $\underline{T}$ intersects OB.

2/130 Moment of couple is $240(\underline{j}\cos 30° - \underline{k}\sin 30°)$

$= 207.8\underline{j} - 120\underline{k}$ N·m

Moment of force is

$1200\cos 30°(-0.250\underline{i} + 0.200\underline{k}) + 1200\sin 30°(0.200\underline{j})$

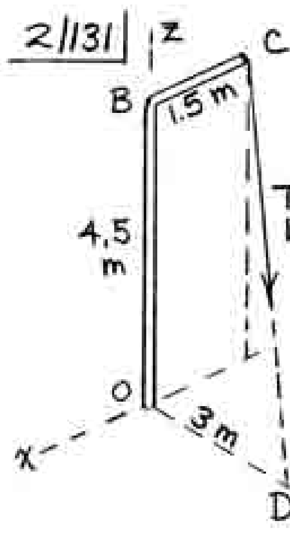
$= -259.8\underline{i} + 120\underline{j} + 207.8\underline{k}$ N·m

Thus total moment is

$\underline{M}_O = -259.8\underline{i} + 327.8\underline{j} + 87.8\underline{k}$ N·m

or $\underline{M}_O = -260\underline{i} + 328\underline{j} + 88\underline{k}$ N·m

2/131



From the solution to Prob. 2/95,

$\underline{T} = 0.321\underline{i} + 0.641\underline{j} - 0.962\underline{k}$ kN

(a) $\underline{M}_O = \underline{r}_{OD} \times \underline{T}$, $\underline{r}_{OD} = 3\underline{j}$ m

$\underline{M}_O = -2.89\underline{i} - 0.962\underline{k}$ kN·m

Could also use

$\underline{M}_O = \underline{r}_{OC} \times \underline{T}$, where

$\underline{r}_{OC} = -1.5\underline{i} + 4.5\underline{k}$ N

Result for $\underline{M}_O$ is the same.

(b) About z-axis:

$\underline{M}_z = -1.5\,T_y\,\underline{k} = -1.5\,(0.641)\underline{k} = -0.962\underline{k}$ kN·m

Also, $\underline{M}_z = \underline{M}_O \cdot \underline{k}\;\underline{k} = (-2.89\underline{i} - 0.962\underline{k}) \cdot \underline{k}\;\underline{k}$

$= -0.962\underline{k}$ kN·m

2/132 $\underline{F} = 180\left(-\cos 60^\circ \underline{i} - \sin 60^\circ \underline{j}\right)$ lb

$\underline{r}_{BA} = +15\sin 45^\circ \underline{i} - (15\cos 45^\circ + 13)\underline{j} - 12\underline{k}$ in.

$\underline{r}_{CA} = (+15\sin 45^\circ - 6)\underline{i} - (15\cos 45^\circ + 13)\underline{j}$ in.

$\underline{M}_B = \underline{r}_{BA} \times \underline{F}$, $\underline{M}_C = \underline{r}_{CA} \times \underline{F}$

Carry out to obtain

$\underline{M}_B = -1871\underline{i} + 1080\underline{j} - 3780\underline{k}$ lb-in.

$\underline{M}_C = -2840\underline{k}$ lb-in.

The unit vector which characterizes line BC is

$$\underline{n}_{BC} = \frac{6\underline{i} - 12\underline{k}}{\sqrt{6^2 + 12^2}} = 0.447\underline{i} - 0.894\underline{k}$$

$M_{BC} = \underline{M}_B \cdot \underline{n}_{BC} = 2540$ lb-in.

($\underline{M}_C \cdot \underline{n}_{BC} = 2540$ lb-in., also)

2/133 Using the coordinates of the figure:

$$\underline{M}_A = \underline{r} \times \underline{F} \quad , \quad \underline{F} = -1.8\,\underline{k} \text{ lb}$$

$$\underline{r} = [(2+1)\cos 30°]\,\underline{i} + 3\,\underline{j} + [(2+1)\sin 30°]\,\underline{k} \text{ in.}$$

$$\therefore \quad \underline{M}_A = -5.40\,\underline{i} + 4.68\,\underline{j} \text{ lb-in.}$$

$$\underline{M}_{AB} = (\underline{M}_A \cdot \underline{n}_{AB})\,\underline{n}_{AB} \, , \quad \underline{n}_{AB} = \cos 30°\,\underline{i} + \sin 30°\,\underline{k}$$

$$\therefore \quad \underline{M}_{AB} = -4.05\,\underline{i} - 2.34\,\underline{k} \text{ lb-in.}$$

2/134 $\underline{M}_o = \underline{r}_{OA} \times \underline{F}$

$\underline{r}_{OA} = (0.050 + 0.130 \sin 60°)\underline{i}$

$+ (-0.140 - 0.130 \cos 60°)\underline{j} + 0.150\underline{k}$

$= 0.1626\underline{i} - 0.205\underline{j} + 0.150\underline{k}$ m

$\underline{F} = 600(\cos 45° \sin 60° \underline{i} - \cos 45° \cos 60° \underline{j} + \sin 45° \underline{k})$

$= 600(0.612\underline{i} - 0.354\underline{j} + 0.707\underline{k})$

$= 367\underline{i} - 212\underline{j} + 424\underline{k}$ N

Carry out $\underline{M}_o = \underline{r}_{OA} \times \underline{F}$ to obtain

$\underline{M}_o = -55.2\underline{i} - 13.86\underline{j} + 40.8\underline{k}$ N·m

2/135

$$\underline{R} = \underline{W} + \underline{L} + \underline{D}$$

$$= -5\underline{j} + L\left(-\sin\theta\,\underline{j} - \cos\theta\,\underline{k}\right) - 1.7\,\underline{i}$$

$$= -1.7\underline{i} - (5 + L\sin\theta)\underline{j} - L\cos\theta\,\underline{k}$$

$$\begin{cases} -L\cos\theta = -0.866 \\ -(5 + L\sin\theta) = -5.500 \end{cases} \Rightarrow \quad \begin{matrix} \theta = 30° \\ \underline{L = 1 \text{ oz}} \end{matrix}$$

$$R = \left[(1.7)^2 + (5 + 1\sin 30°)^2 + (1\cos 30°)^2\right]^{1/2}$$

$$= \underline{5.82 \text{ oz}}$$

2/136

$$\begin{cases} \underline{R} = -3F\underline{k} \\ \underline{M}_O = -\frac{\sqrt{3}}{2} bF\underline{i} \end{cases}$$

$$\underline{R} \cdot \underline{M}_O = 0 \quad \text{so} \quad \underline{R} \perp \underline{M}_O$$

2/137

$$\mathbf{R} = -2F\mathbf{k} + F\mathbf{k} + F(\cos 30^\circ \mathbf{k} + \sin 30^\circ \mathbf{j})$$

$$= \frac{F}{2}\mathbf{j} + F\left(\frac{\sqrt{3}}{2} - 1\right)\mathbf{k} = \underline{F\left(\frac{1}{2}\mathbf{j} + \left(\frac{\sqrt{3}}{2} - 1\right)\mathbf{k}\right)}$$

$$\mathbf{M}_O = 2Fb\mathbf{j} + Fb\mathbf{i} + \frac{F}{2}(2b)\mathbf{k} + \frac{\sqrt{3}}{2}Fb\mathbf{i} - \frac{\sqrt{3}}{2}F(2b)\mathbf{j}$$

$$= \underline{Fb\left[\left(1 + \frac{\sqrt{3}}{2}\right)\mathbf{i} + \left(2 - \sqrt{3}\right)\mathbf{j} + \mathbf{k}\right]}$$

$$\mathbf{R} \cdot \mathbf{M}_O = \left[\frac{1}{2}(2 - \sqrt{3}) + \left(\frac{\sqrt{3}}{2} - 1\right)(1)\right]F^2 b = 0, \text{ so}$$

$\underline{\mathbf{R} \perp \mathbf{M}_O.}$

2/138 $\underline{R} = (1.2 - 1.2 - 1.2)\underline{j} = -1.2\underline{j}$ lb

$\underline{M}_G = 1.2(3)(20)\underline{k} + (1.2 - 1.2 - 1.2)(25)\underline{i}$

$\underline{M}_G = -30\underline{i} + 72\underline{k}$ lb-in.

2/139

$$\underline{R} = (200 + 800)\underline{i} + 1200(\cos 10° \underline{j} - \sin 10° \underline{i})$$

$$= \underline{792\underline{i} + 1182\underline{j} \text{ N}}$$

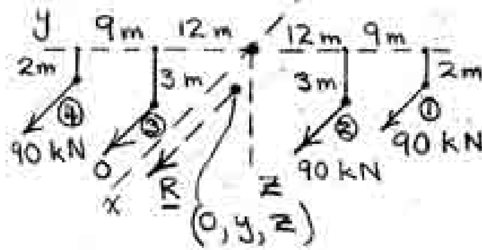
$$\underline{M}_O = [(200-800)(0.1) + (1200 \cos 10°)(0.075)]\underline{k}$$

$$+ [-(200+800)(0.220 + 0.330) + 1200 \sin 10° (0.220)]\underline{j}$$

$$+ [1200 \cos 10° (0.220)]\underline{i}$$

$$= \underline{260\underline{i} - 504\underline{j} + 28.6\underline{k} \text{ N·m}}$$

2/140



$R = \Sigma F = 3(90) = 270 \text{ kN}$

$\Sigma M_z = -Ry = 90(21) + 90(12) - 90(21)$, $\underline{y = -4 \text{ m}}$

$\Sigma M_y = Rz = 2(90)(2) + 1(90)(3)$, $\underline{z = 2.33 \text{ m}}$

2/141 At O: $\underline{R} = \Sigma \underline{F} = (200+400)\underline{j} = \underline{600\underline{j}\ \text{lb}}$

$\underline{M}_O = 600(8)\underline{k} + 400(3)\underline{i} = \underline{1200\underline{i} + 4800\underline{k}\ \text{lb-ft}}$

$\underline{R} \cdot \underline{M}_O = 0 \Rightarrow \underline{R} \perp \underline{M}_O$ (Loading system can be represented by single force)

Let P have coordinates $(x, 0, z)$ and let $\underline{R}$ act at P.

$\underline{r}_{OP} \times \underline{R} = \underline{M}_O$: $(x\underline{i} + z\underline{k}) \times 600\underline{j} = 1200\underline{i} + 4800\underline{k}$

$600x\,\underline{k} - 600z\,\underline{i} = 1200\underline{i} + 4800\underline{k}$

$\Rightarrow \underline{x = 8 \text{ ft}, \quad z = -2 \text{ ft}}$

2/142 The two 160-N forces constitute a couple $160(0.250)\underline{j} = 40\underline{j}$ N·m

$\underline{R} = \Sigma\underline{F} = 120\underline{i} - 180\underline{j} - 100\underline{k}$ N

$\underline{M} = \Sigma\underline{M}_A = [120(0.25) + 100(0.3) + 40]\underline{j} + 50\underline{k}$ N·m

$= 100\underline{j} + 50\underline{k}$ N·m

2/143

$R_x = \Sigma F_x = -20$ kN

$R_y = \Sigma F_y = -40 \cos\left(\tan^{-1}\frac{1}{3}\right) = -37.9$ kN

$R_z = \Sigma F_z = 40 \sin\left(\tan^{-1}\frac{1}{3}\right) = 12.65$ kN

$\Rightarrow \underline{R} = -20\underline{i} - 37.9\underline{j} + 12.65\underline{k}$ kN

$M_{A_x} = 0$

$M_{A_y} = 20(1) + 40\frac{1}{\sqrt{3^2+1^2}}(2) = 45.3$ kN·m

$M_{A_z} = 40\frac{3}{\sqrt{3^2+1^2}}(2) - 35 = 40.9$ kN·m

$\Rightarrow \underline{M} = 45.3\underline{j} + 40.9\underline{k}$ kN·m

2/144 $R = \Sigma F_z = 70 + 30 - 80 - 60 - 50 = -90 \text{ lb}$

$-|R|y = \Sigma M_x: \ -90y = 30(12) + 70(12) - 60(6) - 50(12)$

$\underline{y = -2.67 \text{ in.}}$

$|R|x = \Sigma M_y: \ 90x = 80(10) - 30(10) - 50(8)$

$\underline{x = 1.111 \text{ in.}}$

2/145 $R_x = -120 \text{ N}$, $R_y = 0$, $R_z = -160 \text{ N}$

$R = \sqrt{\overline{120}^2 + \overline{160}^2} = 200 \text{ N}$, $\underline{\mathbf{R} = -120\mathbf{i} - 160\mathbf{k} \text{ N}}$

$M_x = 25 - 160(0.2) = -7 \text{ N·m}$

$M_y = 160(0.075) - 120(0.100 - 0.075) = 9 \text{ N·m}$

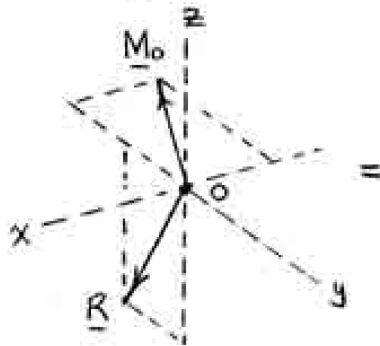
$M_z = 120(0.2) = 24 \text{ N·m}$

$M = \sqrt{7^2 + 9^2 + \overline{24}^2} = 25.5 \text{ N·m}$

$\mathbf{M} = -7\mathbf{i} + 9\mathbf{j} + 24\mathbf{k} \text{ N·m}$

2/146

$$\begin{cases} \underline{R} = -20\underline{j} - 40\underline{k} \text{ lb} \quad (= 44.7(-0.447\underline{j} - 0.894\underline{k})) \\ \underline{M}_o = -40(1.4)\underline{i} - 40(8)\underline{j} \\ \quad = -56\underline{i} - 320\underline{j} \end{cases}$$



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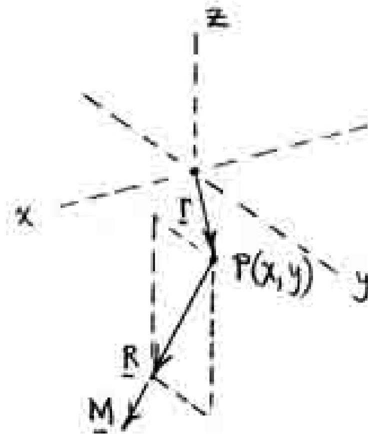


$\underline{M}_o$: $-56\underline{i} - 320\underline{j} = \underline{r} \times \underline{R} + \underline{M} = (x\underline{i} + y\underline{j}) \times (-20\underline{j} - 40\underline{k})$
$+ M(-0.447\underline{j} - 0.894\underline{k})$

Equate coefficients:

$$\begin{cases} \underline{i}: -56 = -40y \\ \underline{j}: -320 = 40x - 0.447M \\ \underline{k}: 0 = -20x - 0.894M \end{cases}$$

Solution:

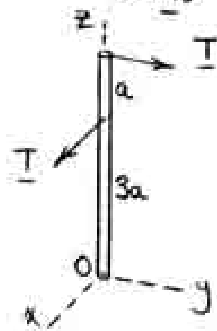
$x = -6.4$ in.

$y = 1.4$ in.

2/147

$\underline{R} = \Sigma \underline{F} = T\underline{i} + T\underline{j} = \sqrt{2}T\left[\frac{1}{\sqrt{2}}\underline{i} + \frac{1}{\sqrt{2}}\underline{j}\right]$

$\Sigma \underline{M}_O = 3aT\underline{j} - 4aT\underline{i}$



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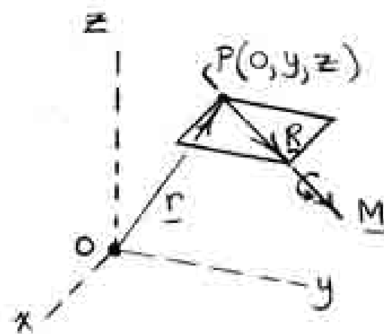


$\Sigma \underline{M}_O = \underline{r} \times \underline{R} + \underline{M}$

$3aT\underline{j} - 4aT\underline{i} = (y\underline{j} + z\underline{k}) \times (T\underline{i} + T\underline{j}) + M\left(\frac{1}{\sqrt{2}}\underline{i} + \frac{1}{\sqrt{2}}\underline{j}\right)$

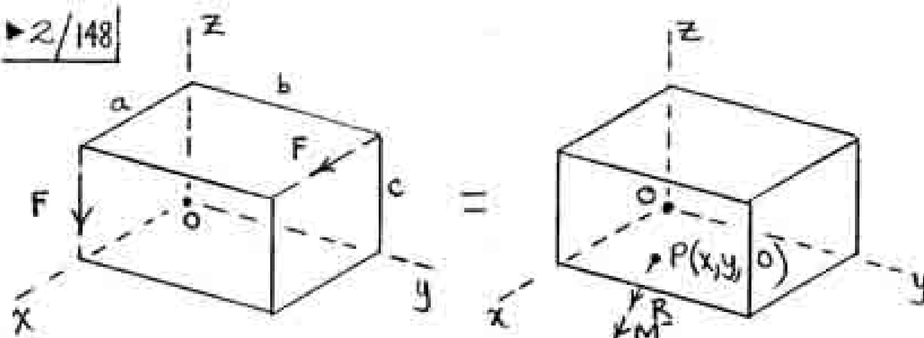
$$\Rightarrow \begin{cases} -4aT = -zT + \frac{M}{\sqrt{2}} \\ 3aT = zT + \frac{M}{\sqrt{2}} \\ 0 = -yT \end{cases}$$

So $\begin{cases} y = 0 \\ z = \frac{7}{2}a \\ M = -\frac{\sqrt{2}}{2}aT \end{cases}$

So $\underline{M} = -\frac{\sqrt{2}}{2}aT\left(\frac{1}{\sqrt{2}}\underline{i} + \frac{1}{\sqrt{2}}\underline{j}\right)$

$= \underline{\frac{-aT}{2}(\underline{i} + \underline{j})}$ (a negative wrench)

▸2/148



$$\mathbf{R} = \Sigma\mathbf{F} = F\mathbf{i} - F\mathbf{k} = \sqrt{2}\,F\left(\frac{1}{\sqrt{2}}\mathbf{i} - \frac{1}{\sqrt{2}}\mathbf{k}\right)$$

$$\Sigma\mathbf{M}_O = aF\mathbf{j} + cF\mathbf{j} - bF\mathbf{k} = F\left((a+c)\mathbf{j} - b\mathbf{k}\right)$$

$$= (x\mathbf{i} + y\mathbf{j}) \times \mathbf{R} + \mathbf{M}$$

$$= (x\mathbf{i} + y\mathbf{j}) \times (F\mathbf{i} - F\mathbf{k}) + M\left(\frac{1}{\sqrt{2}}\mathbf{i} - \frac{1}{\sqrt{2}}\mathbf{k}\right)$$

$$= \left(-Fy + \frac{M}{\sqrt{2}}\right)\mathbf{i} + (Fx)\mathbf{j} + \left(-Fy - \frac{M}{\sqrt{2}}\right)\mathbf{k}$$

Equate coefficients:

$$0 = -Fy + \frac{M}{\sqrt{2}}$$

$$F(a+c) = Fx$$

$$-Fb = -Fy - \frac{M}{\sqrt{2}}$$

Solution:

$$x = a + c$$

$$y = \frac{b}{2}$$

$$M = \frac{Fb\sqrt{2}}{2}$$

$$\mathbf{M} = \frac{Fb\sqrt{2}}{2}\left(\frac{1}{\sqrt{2}}\mathbf{i} - \frac{1}{\sqrt{2}}\mathbf{k}\right) = \frac{Fb}{2}(\mathbf{i} - \mathbf{k})$$

▸2/149 $\underline{R} = \Sigma \underline{F} = 100\underline{i} + 100\underline{j}$ N

Direction cosines $l = 1/\sqrt{2}$, $m = 1/\sqrt{2}$, $n = 0$

Let $P = P(x, 0, z)$

$$\underline{M}_P = 100z\underline{i} + 100(0.4 - x)\underline{k} + 100(0.4 - z)\underline{j}$$
$$- 100(0.3)\underline{k} - 20\underline{j}$$
$$= 100z\underline{i} + 100(0.2 - z)\underline{j} + 100(0.1 - x)\underline{k} \quad \text{N}\cdot\text{m}$$

Let $M = |\underline{M}_P|$. Equate direction cosines of $\underline{R}$ & $\underline{M}_P$ to obtain

$$\frac{100z}{M} = \frac{1}{\sqrt{2}}, \quad \frac{100(0.2 - z)}{M} = \frac{1}{\sqrt{2}} \quad \frac{100(0.1 - x)}{M} = 0$$

Solution : $\begin{cases} x = 0.1 \text{ m} \\ z = 0.1 \text{ m} \\ \underline{M = 10\sqrt{2} \text{ N}\cdot\text{m}} \end{cases}$

$$\underline{M} = 10\sqrt{2}\left(\frac{1}{\sqrt{2}}\underline{i} + \frac{1}{\sqrt{2}}\underline{j}\right) = \underline{10\underline{i} + 10\underline{j} \text{ N}\cdot\text{m}}$$

▸2/150 $\underline{R} = \Sigma \underline{F} = -20\underline{i} - 40\frac{3}{\sqrt{3^2+1^2}}\underline{j} + 40\frac{1}{\sqrt{3^2+1^2}}\underline{k}$

$= -20\underline{i} - 37.9\underline{j} + 12.65\,\underline{k}$ kN

$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \underline{44.7 \text{ kN}}$

Direction cosines of $\underline{R}$: $\begin{cases} l = -\frac{20}{44.7} = -0.447 \\ m = \frac{-37.9}{44.7} = -0.849 \\ n = \frac{12.65}{44.7} = 0.283 \end{cases}$

Let $P = P(x, y, 0)$:

$\underline{M} = \Sigma \underline{M}_P = 20(-y\underline{k} + \underline{j}) + 37.9(x\underline{k} - \underline{i}) + 12.65[(3-y)\underline{i} + x\underline{j}]$

$- 35\underline{k} = -12.65y\,\underline{i} + (20 + 12.65x)\underline{j} + (37.9x - 20y - 35)\underline{k}$ kN·m

Equate direction cosines of $\underline{R}$ and $\underline{M}$:

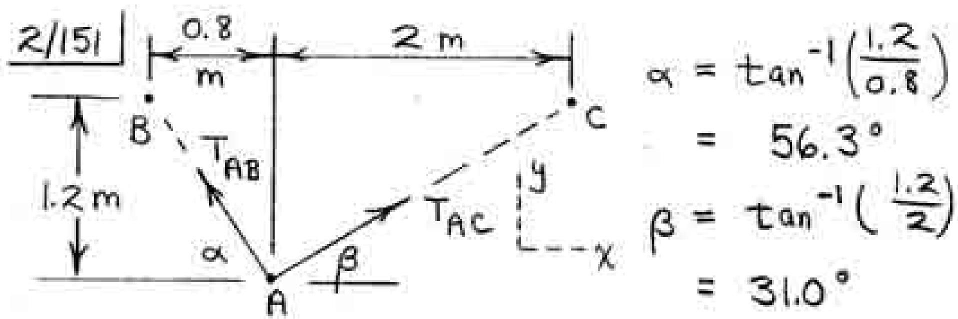
$\left.\begin{array}{l} -0.447 = -12.65y\,/M \\ -0.849 = (20 + 12.65x)/M \\ 0.283 = (37.9x - 20y - 35/M \end{array}\right\}$ Solution:

$\underline{x = 0.221 \text{ m}}$

$\underline{y = -0.950 \text{ m}}$

$\underline{M = -26.9 \text{ kN·m}}$

$\underline{M}$ and $\underline{R}$ have opposite directions.



$\alpha = \tan^{-1}\left(\frac{1.2}{0.8}\right) = 56.3°$

$\beta = \tan^{-1}\left(\frac{1.2}{2}\right) = 31.0°$

$\underline{T}_{AB} = T_{AB}\,\underline{n}_{AB} = 0.858(60)(9.81)\left[-\cos 56.3°\underline{i} + \sin 56.3°\underline{j}\right]$

$= \underline{-280\underline{i} + 420\underline{j} \text{ N}}$

$\underline{T}_{AC} = T_{AC}\,\underline{n}_{AC} = 0.555(60)(9.81)\left[\cos 31.0°\underline{i} + \sin 31.0°\underline{j}\right]$

$= \underline{280\underline{i} + 168.1\underline{j} \quad \text{N}}$

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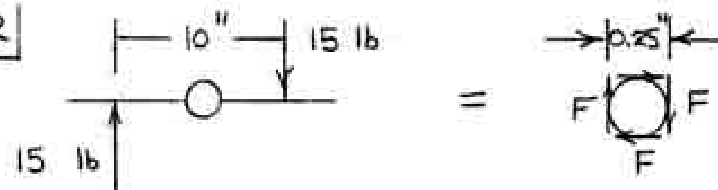

$M = Fd = 15(10) = 2F(0.25)$, $\underline{F = 300 \text{ lb}}$

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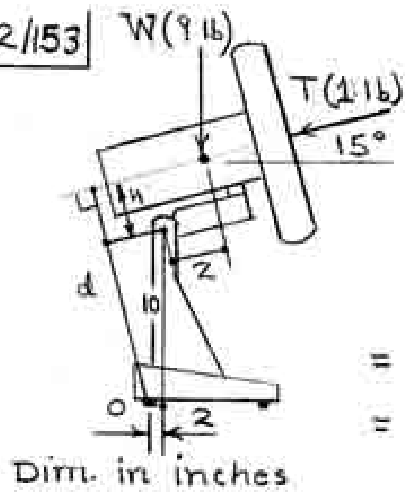

$M_O = Fd$

$= 1(4 + 10\cos 15° - 2\sin 15°)$

$= 13.14$ lb-in. (↶)

$M_{O_W} = W d_W$

$= 9(2 + 2\cos 15° - 4\sin 15°)$

$= 26.1$ lb-in. (↷)

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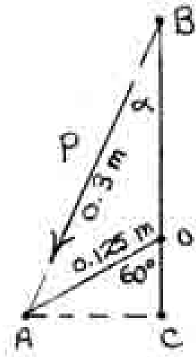

$AC = 0.125 \sin 60° = 0.1083 \text{ m}$

$\alpha = \sin^{-1} \frac{0.1083}{0.300} = 21.2°$

$BC = 0.300 \cos \alpha = 0.280 \text{ m}$

$BO = 0.280 - 0.125 \cos 60° = 0.217 \text{ m}$

$\circlearrowleft + \; M_O = 720 = P \sin \alpha \, (BO) = P \sin 21.2° \, (0.217)$

$\underline{P = 9.18 \text{ kN}}$

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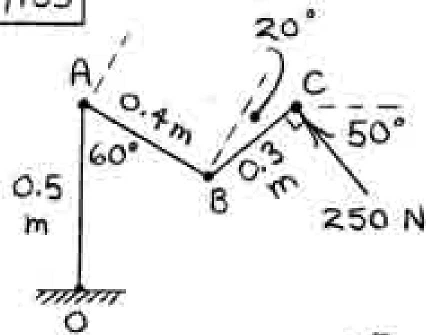

$$\curvearrowleft + \; M_O = 250\cos 50^\circ\left[0.5 - 0.4\cos 60^\circ + 0.3\sin 40^\circ\right]$$

$$+ 250\sin 50^\circ\left[0.4\sin 60^\circ + 0.3\cos 40^\circ\right]$$

$$= \underline{189.6\ \text{N}\cdot\text{m}\ \ \text{CCW}}$$

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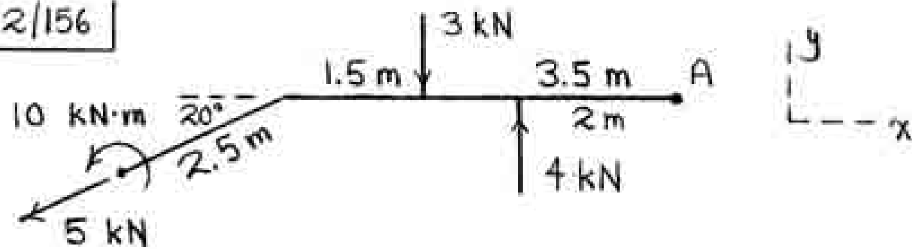


$R_x = \Sigma F_x = -5\cos 20° = -4.70\ \text{kN}$

$R_y = \Sigma F_y = -5\sin 20° + 4 - 3 = -0.710\ \text{kN}$

$R = \sqrt{R_x^2 + R_y^2} = 4.75\ \text{kN}$

$\circlearrowleft + \Sigma M_A = 10 + 5\sin 20°(5) + 3(3.5) - 4(2)$

$= \underline{21.1\ \text{kN}\cdot\text{m}}$

2/157 At A: $R = \Sigma F = 200 + 180 - 300 = \underline{80 \text{ lb } (\downarrow)}$

$M_A = 200(8) + 180(28) - 300(18) = \underline{1240 \text{ lb-in.}}$

80 lb, A, x, 1240 lb-in. = A, x, 80 lb

$80x = 1240$, $\underline{x = 15.5 \text{ in.}}$

2/158 Coordinates of A: $(x_A, y_A, z_A) = (0, r, 0)$

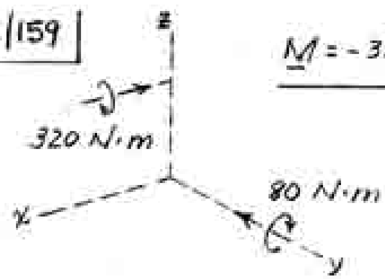
Coordinates of B: $(x_B, y_B, z_B) = (h, r\cos\theta, r\sin\theta)$

So $\underline{r}_{AB} = h\underline{i} + (r\cos\theta - r)\underline{j} + r\sin\theta\,\underline{k}$

and $\underline{F} = F\underline{n}_{AB} = F\left[\dfrac{h\underline{i} + r(\cos\theta - 1)\underline{j} + r\sin\theta\,\underline{k}}{\sqrt{h^2 + [r(\cos\theta-1)]^2 + [r\sin\theta]^2}}\right]$

$$= F\left[\frac{h\underline{i} + r(\cos\theta - 1)\underline{j} + r\sin\theta\,\underline{k}}{\sqrt{h^2 + 2r^2(1-\cos\theta)}}\right]$$

2/159



$\underline{M} = -320\underline{i} - 80\underline{j}$ N·m

$$\cos\theta_x = \frac{M_x}{|\underline{M}|} = \frac{-320}{\sqrt{320^2 + 80^2}} = \underline{-0.970}$$

2/160 $\underline{R} = P(0.6\underline{j} + 0.8\underline{k})$

$$\begin{cases} M_{Bx} = Pb(-0.6 + 0.8) = 0.2Pb \\ M_{By} = 0.8Pb \\ M_{Bz} = -0.6Pb \end{cases}$$

So $\underline{M}_B = Pb(0.2\underline{i} + 0.8\underline{j} - 0.6\underline{k})$

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$\underline{M}_{100} = -100\,(0.200)\,\underline{i} = -20\underline{i}$ N·m

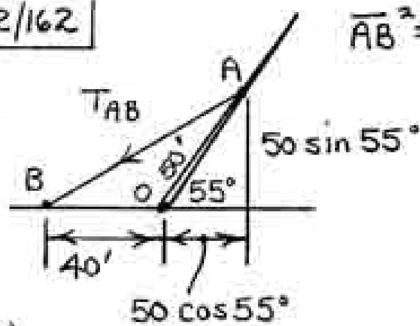
$\underline{M}_{80} = 80\,(0.180 \cos 20°)(-\underline{j} \sin 30° - \underline{k} \cos 30°)$

$= -6.77\underline{j} - 11.72\underline{k}$ N·m

$\underline{M}_{120} = -120\,(0.300 \cos 45°)\underline{k} = -25.5\,\underline{k}$ N·m

So $\underline{M} = -20\underline{i} - 6.77\underline{j} - 37.2\underline{k}$ N·m

2/162



$\overline{AB}^2 = 40^2 + 50^2 - 2(40)(50)\cos 125°$

$\overline{AB} = 80.0'$

(a)

$$\underline{T}_{AB} = 600\left[-\frac{40 + 50\cos 55°}{80.0}\cos 40°\,\underline{i} - \frac{40+50\cos 55°}{80.0}\sin 40°\,\underline{j} - \frac{50\sin 55°}{80.0}\,\underline{k}\right]$$

$$= \underline{-395\,\underline{i} - 331\,\underline{j} - 307\,\underline{k} \text{ lb}}$$

(b) Carry out $\underline{M}_O = \underline{r}_{OB} \times \underline{T}_{AB}$, where $\underline{r}_{OB} = 40'(-\cos 40°\,\underline{i} - \sin 40°\,\underline{j})$: $\underline{M}_O = \underline{7900\,\underline{i} - 9420\,\underline{j} \text{ lb-ft}}$

$\underline{M_{O_x} = 7900 \text{ lb-ft}}$, $\underline{M_{O_y} = -9420 \text{ lb-ft}}$, $\underline{M_{O_z} = 0}$

(c) $\underline{T}_{AO} = \underline{T}_{AB} \cdot \underline{n}_{AO}$, where $\underline{n}_{AO} = -\cos 55°\cos 40°\,\underline{i} - \cos 55°\sin 40°\,\underline{j} - \sin 55°\,\underline{k}$. Carry out to obtain $\underline{T_{AO} = 547 \text{ lb}}$

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$$\underline{R} = \Sigma \underline{F} = 500\cos 45^\circ(\underline{i}) + 400\underline{j} - (600 + 500\sin 45^\circ)\underline{k}$$

$$= 354\underline{i} + 400\underline{j} - 954\underline{k} \quad \text{lb}$$

$$R = \sqrt{\overline{354}^2 + \overline{400}^2 + \overline{954}^2} = \underline{1093 \text{ lb}}$$

$$\underline{M} = [500\cos 45^\circ(3) - 600(3) - 400(10)]\underline{i}$$

$$+ [500\cos 45^\circ(6) + 500\sin 45^\circ(7) + 600(6)]\underline{j}$$

$$+ [500\sin 45^\circ(3) + 400(3)]\underline{k}$$

$$= -4739\underline{i} + 8196\underline{j} + 2261\underline{k} \quad \text{lb-ft}$$

$$M = \sqrt{\overline{4739}^2 + \overline{8196}^2 + \overline{2261}^2} = \underline{9730 \text{ lb-ft}}$$

2/164 $\underline{R} = 800\left[-\sin 30^\circ \cos 20^\circ \underline{i} + \sin 30^\circ \sin 20^\circ \underline{j} + \cos 30^\circ \underline{k}\right]$

$$= \underline{-376\underline{i} + 137\underline{j} + 693\underline{k} \quad \text{N}}$$

$\underline{M}_o = \underline{r}_{OB} \times \underline{F}$

$\underline{r}_{OB} = \left[300 \sin 20^\circ \underline{i} + 300 \cos 20^\circ \underline{j} + 250\underline{k}\right] \text{mm}$

$\underline{\underline{M}_o = 161\underline{i} - 165\underline{j} + 120\underline{k} \quad \text{N}\cdot\text{m}}$

*2/165

$\Sigma F_x = 0: -360 - 240 \sin\theta + T \sin 30° + 400 \cos 30° = 0$ (1)

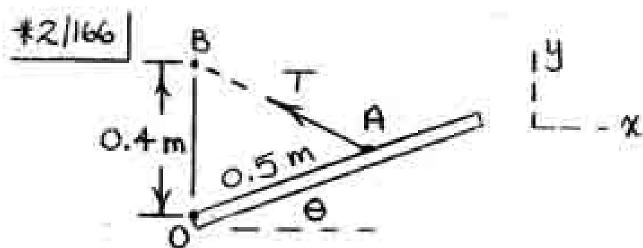
$\Sigma F_y = 600: 240 \cos\theta + T \cos 30° + 400 \sin 30° = 600$ (2)

Numerical solution of Eqs. (1) & (2):

$\underline{\theta = 21.7°, \quad T = 204 \text{ lb}}$

(We could eliminate T between Eqs. (1) & (2), but the resulting equation is still transcendental.)

*2/166



$$\underline{n}_{AB} = \frac{\underline{r}_{AB}}{r_{AB}} = \frac{\underline{r}_B - \underline{r}_A}{r_{AB}}$$

$$\underline{r}_B - \underline{r}_A = 0.4\underline{j} - 0.5(\cos\theta\,\underline{i} + \sin\theta\,\underline{j})$$

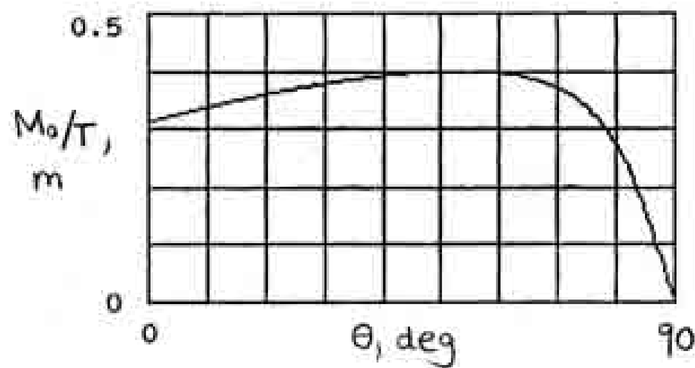
$$= -0.5\cos\theta\,\underline{i} + (0.4 - 0.5\sin\theta)\underline{j}$$

$$\therefore \underline{n}_{AB} = \frac{-0.5\cos\theta\,\underline{i} + (0.4-0.5\sin\theta)\underline{j}}{\sqrt{(0.5\cos\theta)^2 + (0.4-0.5\sin\theta)^2}}$$

Then $\underline{T} = T\underline{n}_{AB}$ and $\underline{M}_o = \underline{r}_{OB} \times \underline{T} = 0.4\underline{j} \times \underline{T}$

Carry out & obtain

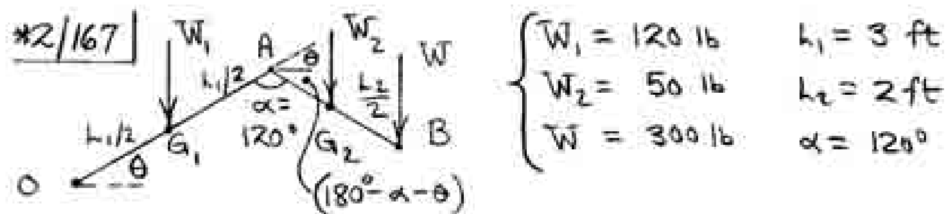
$$\frac{M_o}{T} = \frac{0.2\cos\theta}{\sqrt{0.41 - 0.4\sin\theta}} \quad \text{(in m)}$$



$$\left(\frac{M_o}{T}\right)_{min} = 0 \ @ \ \theta = 90°$$

$$\left(\frac{M_o}{T}\right)_{max} = 0.4 \text{ m} \ @ \ \theta = 53.1°$$

*2/167



$$\begin{cases} W_1 = 120 \text{ lb} & L_1 = 3 \text{ ft} \\ W_2 = 50 \text{ lb} & L_2 = 2 \text{ ft} \\ W = 300 \text{ lb} & \alpha = 120° \end{cases}$$

$$+\circlearrowleft M_O = W_1 \frac{L_1}{2} \cos\theta + W_2\left(L_1 \cos\theta + \frac{L_2}{2} \cos\left(180° - \alpha - \theta\right)\right) + W\left(L_1 \cos\theta + L_2 \cos\left(180° - \alpha - \theta\right)\right)$$

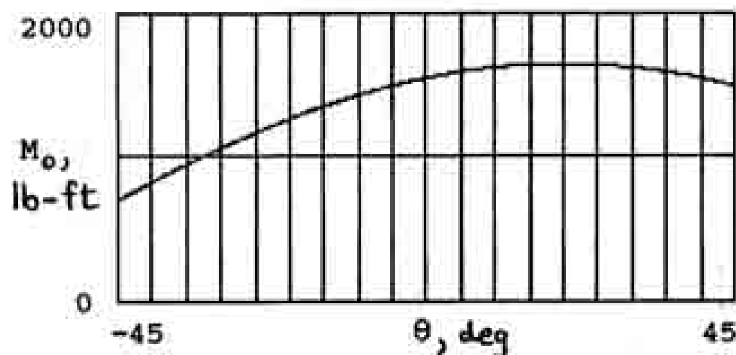
With the above numbers:

$$\underline{M_O = 1230 \cos\theta + 650 \cos\left(60° - \theta\right)} \quad (\text{in lb-ft})$$

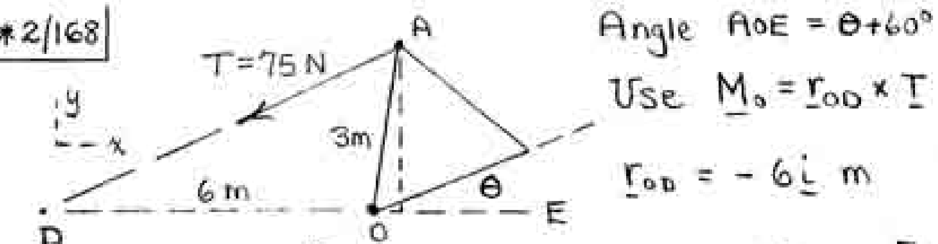
(see plot below)

For $(M_O)_{max}$: $\frac{dM_O}{d\theta} = -1230 \sin\theta + 650 \sin\left(60° - \theta\right) = 0$

Numerical solution: $\underline{\theta = 19.90°}$, $\underline{(M_O)_{max} = 1654 \text{ lb-ft}}$



*2/168



Angle AOE = θ + 60°

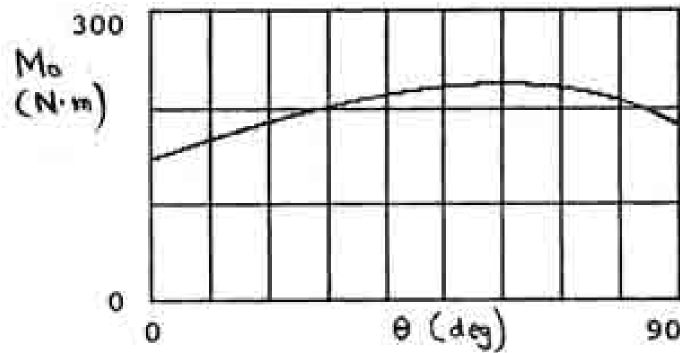
Use $\underline{M}_O = \underline{r}_{OD} \times \underline{T}$

$\underline{r}_{OD} = -6\underline{i}$ m

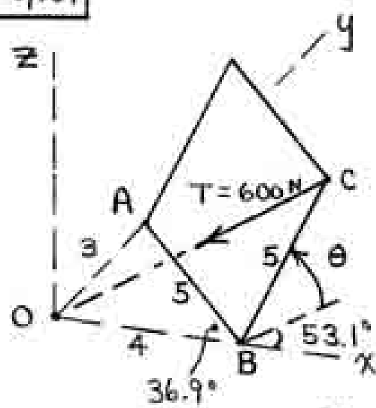
$$\underline{T} = T\underline{n}_{AD} = 75\left[\frac{-(6+3\cos(\theta+60°))\underline{i} - 3\sin(\theta+60°)\underline{j}}{\sqrt{[6+3\cos(\theta+60°)]^2+[3\sin(\theta+60°)]^2}}\right] \text{N}$$

$$\underline{M}_O = \underline{r}_{OD} \times \underline{T} = \frac{1350\sin(\theta+60°)\ \underline{k}}{\sqrt{45+36\cos(\theta+60°)}} \text{ N·m}$$

M_O is a max @ $\underline{\theta = 60°}$: $\underline{M_O = 225 \text{ N·m}}$



*** 2/169**



$C = C(4 + 5\cos 53.1° \cos\theta,\ 5\sin 53.1° \cos\theta,\ 5\sin\theta)$ m

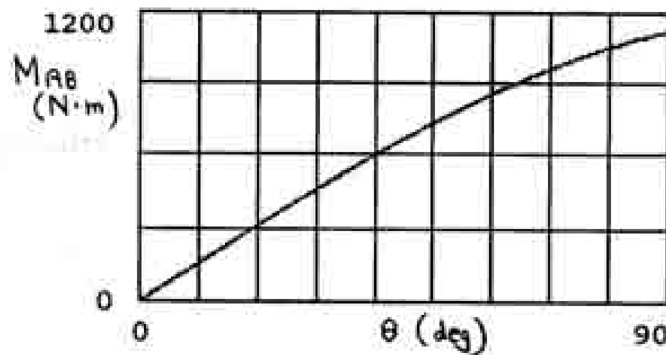
$\underline{n}_{AB} = 0.8\underline{i} - 0.6\underline{j}$

$$\underline{T} = T\underline{n}_{CO} = 600\left[\frac{-(4+3\cos\theta)\underline{i} - (4\cos\theta)\underline{j} - (5\sin\theta)\underline{k}}{\sqrt{(4+3\cos\theta)^2 + (4\cos\theta)^2 + (5\sin\theta)^2}}\right]$$

$$= \frac{600\left[-(4+3\cos\theta)\underline{i} - (4\cos\theta)\underline{j} - (5\sin\theta)\underline{k}\right]}{\sqrt{41 + 24\cos\theta}} \text{ N}$$

$$\underline{M}_B = \underline{r}_{OB} \times \underline{T} = \frac{600}{\sqrt{41+24\cos\theta}}\left(-20\sin\theta\,\underline{j} + 16\cos\theta\,\underline{k}\right) \text{ N·m}$$

Now, $$M_{AB} = \underline{M}_B \cdot \underline{n}_{AB} = \underline{\frac{7200\sin\theta}{\sqrt{41 + 24\cos\theta}} \text{ N·m}}$$



*2/170 Length of spring $L = \sqrt{x^2 + 0.1^2 + 0.15^2}$

$$= \sqrt{x^2 + 0.0325} \text{ m}$$

Deflection $\delta = L - 0.15 = \sqrt{x^2 + 0.0325} - 0.15$ m

Spring force $F = k\delta = 200\left[\sqrt{x^2+0.0325} - 0.15\right]$

As a vector, $\underline{F} = F\underline{n}_{AO}$, where

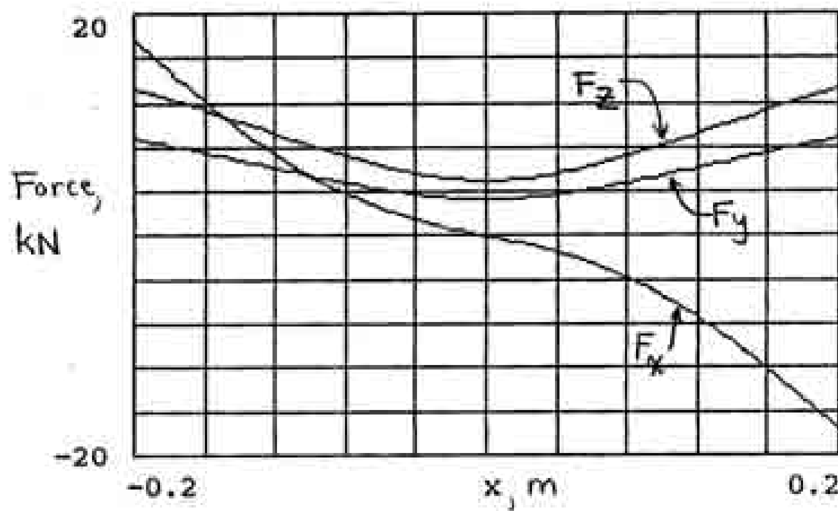
$$\underline{n}_{AO} = \left[\frac{-x_A\underline{i} + 0.1\underline{j} + 0.15\underline{k}}{\sqrt{x^2 + 0.0325}}\right]$$

So the required force components are

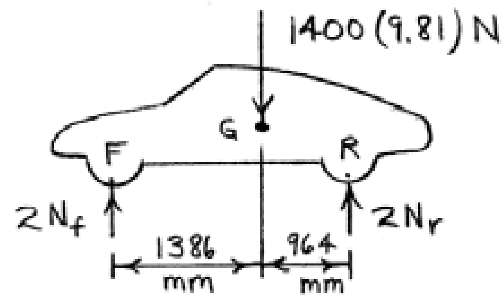
$$F_x = 200\left[\sqrt{x^2+0.0325} - 0.15\right]\left[\frac{-x_A}{\sqrt{x^2+0.0325}}\right]$$

$$F_y = 200\left[\sqrt{x^2+0.0325} - 0.15\right]\left[\frac{0.1}{\sqrt{x^2+0.0325}}\right]$$

$$F_z = 200\left[\sqrt{x^2+0.0325} - 0.15\right]\left[\frac{0.15}{\sqrt{x^2+0.0325}}\right]$$



3/1



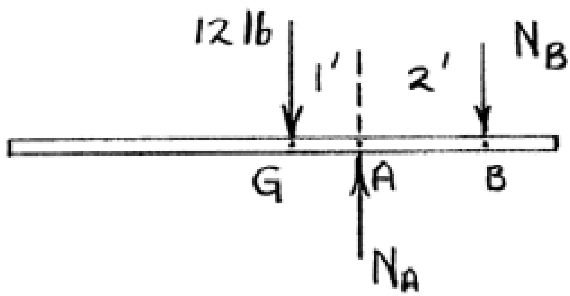
$+\uparrow \Sigma F = 0: \quad 2N_f + 2N_r - 1400(9.81) = 0$

$\circlearrowleft + \Sigma M_F = 0: \quad -1400(9.81)(1386) + 2N_r(1386 + 964) = 0$

Solution : $\begin{cases} N_f = 2820 \text{ N} \\ N_r = 4050 \text{ N} \end{cases}$

Assumes G midway between left and right wheels.

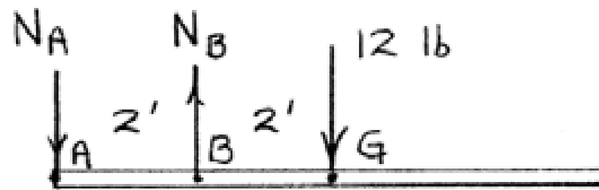
3/2



$\curvearrowleft + \Sigma M_B = 0: \quad 12(3) - N_A(2) = 0$

$N_A = 18 \text{ lb}$

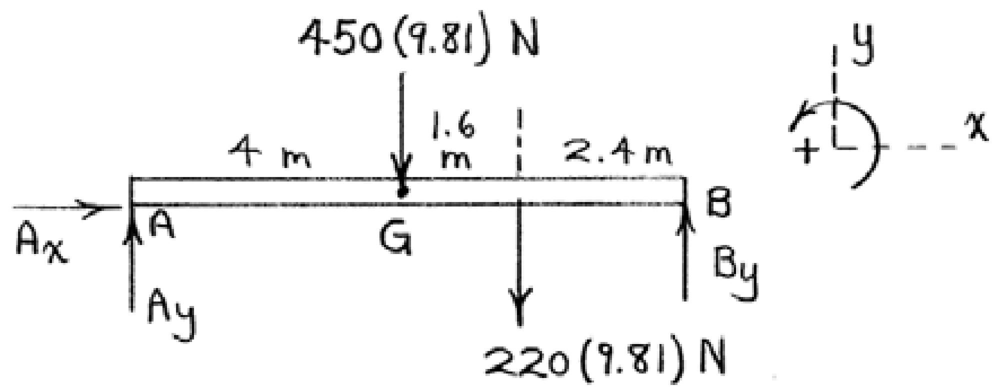
3/3



$\curvearrowleft + \; \Sigma M_A = 0: \quad N_B(2) - 12(4) = 0, \quad \underline{N_B = 24 \text{ lb}}$

$+\uparrow \; \Sigma F = 0: \quad -N_A + 24 - 12 = 0, \quad \underline{N_A = 12 \text{ lb}}$

3/4



From $\Sigma F_x = 0$, $A_x = 0$

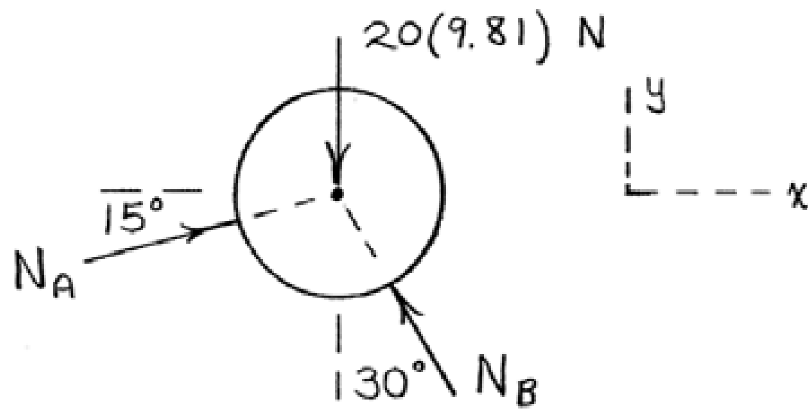
$\Sigma M_A = 0$: $-450(9.81)4 - 220(9.81)(5.6)$

$+ B_y(8) = 0$, $\underline{B_y = 3720\ \text{N}}$

$\Sigma F_y = 0$: $A_y - 450(9.81) - 220(9.81) + 3720 = 0$

$\underline{A_y = 2850\ \text{N}}$

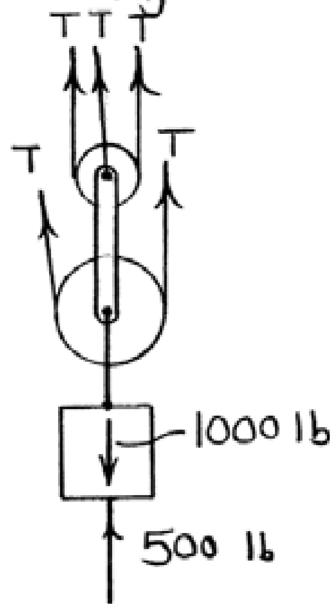
3/5



$$\begin{cases} \Sigma F_x = 0: \quad N_A \cos 15° - N_B \sin 30° = 0 & (1) \\ \Sigma F_y = 0 \quad N_A \sin 15° + N_B \cos 30° - 20(9.81) = 0 & (2) \end{cases}$$

Solution : $\begin{cases} N_A = 101.6 \text{ N} \\ N_B = 196.2 \text{ N} \end{cases}$

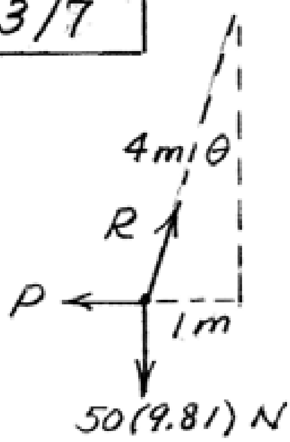
3/6 | FBD of 1000-lb weight and lower pair of pulleys:

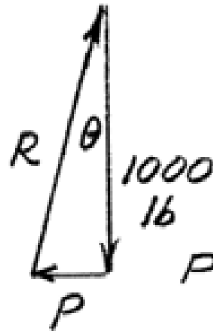


$\uparrow \Sigma F = 0$: $5T + 500 - 1000 = 0$, $\underline{T = 100 \text{ lb}}$

(We assume that the nonverticality of some of the cables is negligible.)

3/7





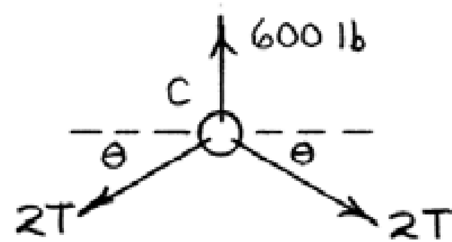
$P = 50(9.81) \tan\theta$

$\sin\theta = 1/4$

$\tan\theta = 1/\sqrt{4^2 - 1^2} = 0.258$

$P = 50(9.81)(0.258) = \underline{126.6 \text{ N}}$

3/8 FBD of junction ring C:

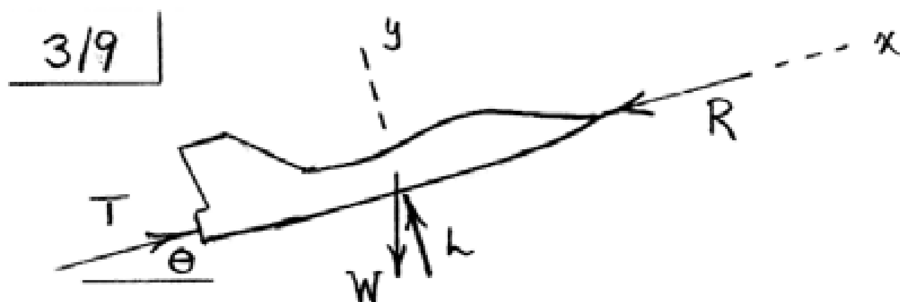


$$\theta = \tan^{-1}\frac{10}{36/2}$$

$$= 29.1°$$

$\uparrow + \Sigma F = 0$: $600 - 4T\sin 29.1° = 0$

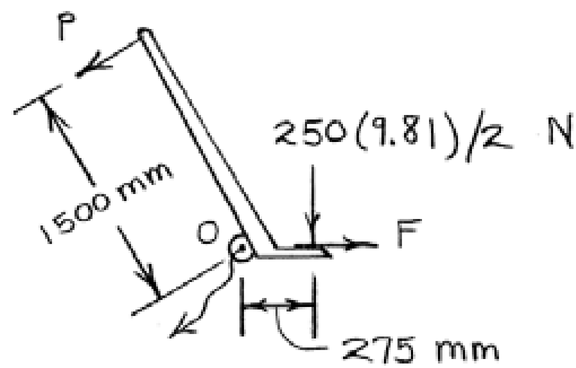
$\underline{T = 309 \text{ lb}}$



$$\Sigma F_x = 0: \quad T - R - W\sin\theta = 0$$

$$n = \frac{T-R}{W} = \underline{\sin\theta}$$

3/10

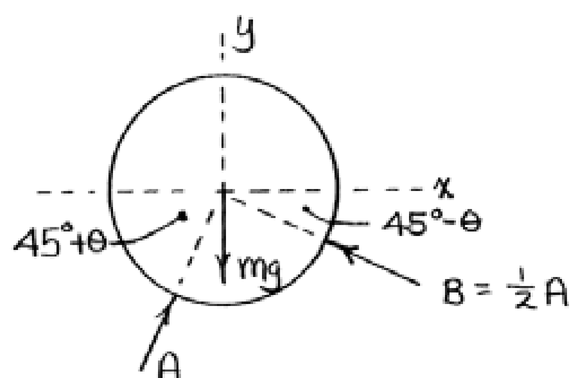


$\circlearrowleft + \Sigma M_O = 0: \quad P(1500) - \frac{1}{2}(250)(9.81)(275) = 0$

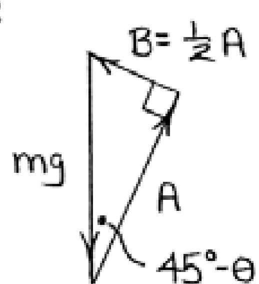
$\underline{P = 225 \text{ N}}$

(assumes that the moment of the friction force F is small compared to the other moments)

3/11



$\Sigma \underline{F} = \underline{0}$:

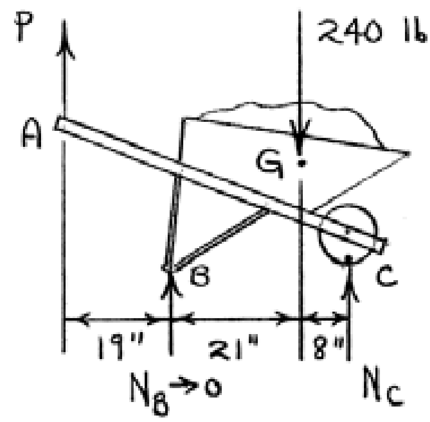


$$\tan(45° - \theta) = \frac{A/2}{A} = \frac{1}{2}$$

$$45° - \theta = 26.6°$$

$$\underline{\theta = 18.43°}$$

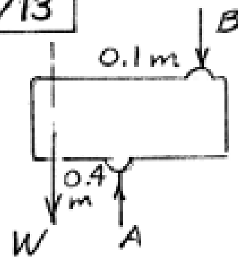
3/12



$$\curvearrowleft + \Sigma M_C = 0: \quad P(48) - 240(8) = 0$$

$$\underline{P = 40 \text{ lb}}$$

3/13



$W = 300(9.81) = 2943 \text{ N}$

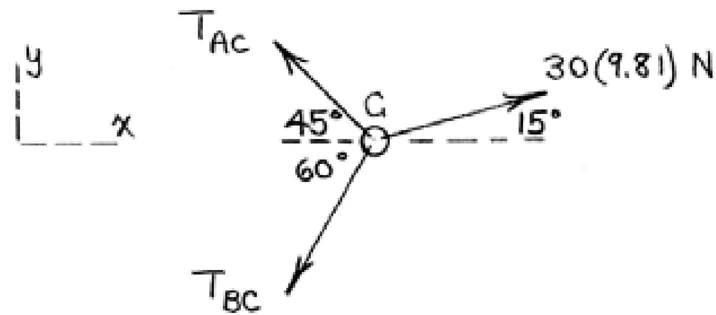
$\Sigma M_A = 0; \quad 2943(0.4) - B(0.6) = 0$

$\underline{B = 1962 \text{ N}}$ or $\underline{1.962 \text{ kN}}$

$\Sigma F = 0; \quad A = 2943 + 1962$

$\underline{= 4910 \text{ N}}$ or $\underline{4.91 \text{ kN}}$

3/14 FBD of junction ring C:

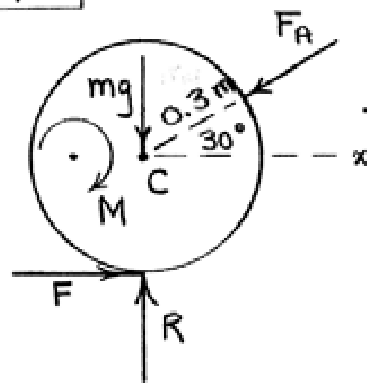


$$\begin{cases} \Sigma F_x = 0: -T_{AC}\cos 45° - T_{BC}\cos 60° + 30(9.81)\cos 15° = 0 \\ \Sigma F_y = 0: T_{AC}\sin 45° - T_{BC}\sin 60° + 30(9.81)\sin 15° = 0 \end{cases}$$

Solve simultaneously to obtain

$$\begin{cases} T_{AC} = 215 \text{ N} \\ T_{BC} = 264 \text{ N} \end{cases}$$

3/15



$mg = 100(9.81) = 981$ N

$M = 60$ N·m

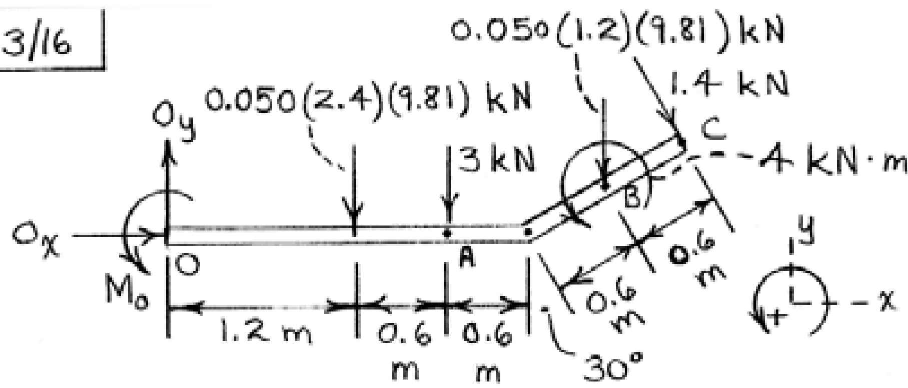
$\circlearrowleft + \; \Sigma M_C = 0: \; 60 - 0.3F = 0$

$F = 200$ N

$\Sigma F_x = 0: \; -F_A \cos 30° + 200 = 0$

$F_A = 231$ N

3/16



$\Sigma F_x = 0: \quad O_x + 1.4 \sin 30° = 0$

$\underline{O_x = -0.7 \text{ kN}}$

$\Sigma F_y = 0: \quad O_y - 0.050(2.4)(9.81) - 3 - 1.4 \cos 30°$

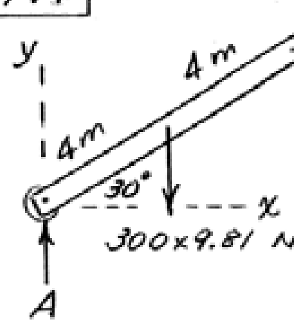
$-0.050(1.2)(9.81) = 0, \quad \underline{O_y = 5.98 \text{ kN}}$

$\Sigma M_o = 0: \quad M_o - 0.050(2.4)(9.81)(1.2) - 3(1.8)$

$-0.050(1.2)(9.81)(2.4 + 0.6 \cos 30°) + 4$

$-1.4(2.4 \cos 30° + 1.2) = 0, \quad \underline{M_o = 9.12 \text{ kN·m}}$

3/17



$\Sigma M_B = 0;$

$300(9.81)\,4\cos 30° - 8A\cos 30° = 0,$

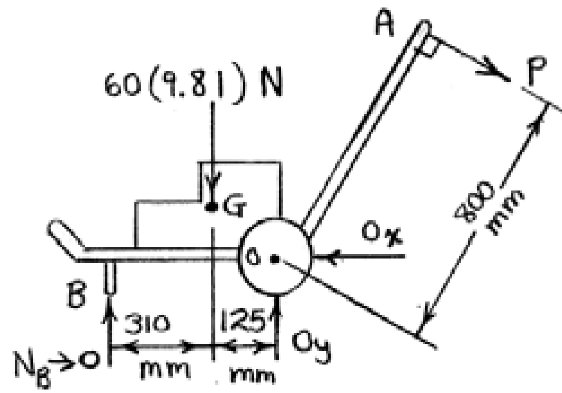
$\underline{A = 1472\ \text{N}}$

$\Sigma F_y = 0;\ B\cos 30° + 1472 - 300(9.81) = 0$

$B = 1699\ \text{N}$

$\Sigma F_x = 0;\ T - 1699\sin 30° = 0,\quad \underline{T = 850\ \text{N}}$

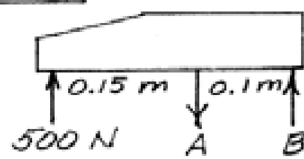
3/18



$$\circlearrowleft + \Sigma M_O = 0: \quad 60(9.81)(125) - P(800) = 0$$

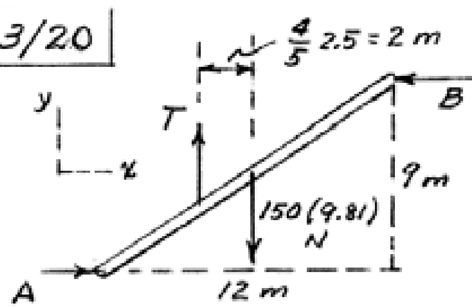
$$\underline{P = 92.0 \text{ N}}$$

3/19



$\Sigma M_B = 0;\quad 500(0.25) - 0.1A = 0$

$\underline{A = 1250\ N}$



$\Sigma F_y = 0$; $T - 150(9.81) = 0$

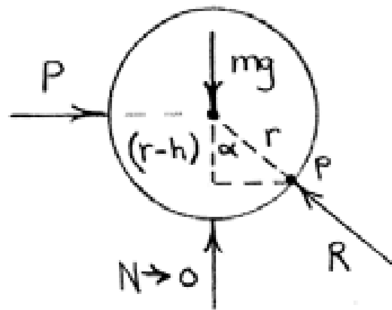
$T = 1472$ N

$\Sigma F_x = 0$; $A = B$

$\Sigma M = 0$; $1472(2) - A(9) = 0$

$\underline{A = B = 327 \text{ N}}$

3/21



$$\cos\alpha = \frac{r-h}{r}$$

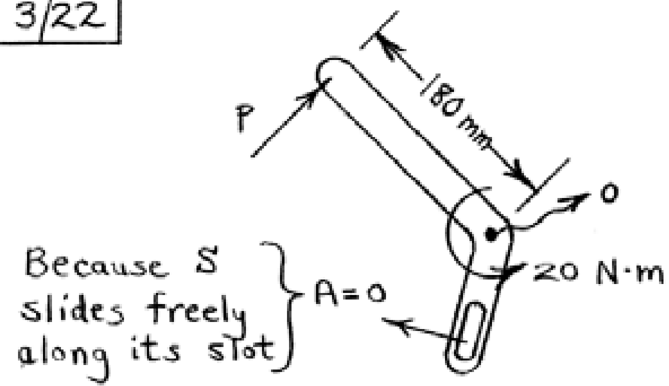
$$\sqrt{r^2-(r-h)^2} = \sqrt{2rh-h^2}$$

$$\sin\alpha = \sqrt{2rh-h^2}/r$$

$$+\circlearrowleft \Sigma M_P = 0: \quad P(r-h) - mg\,r\sin\alpha = 0$$

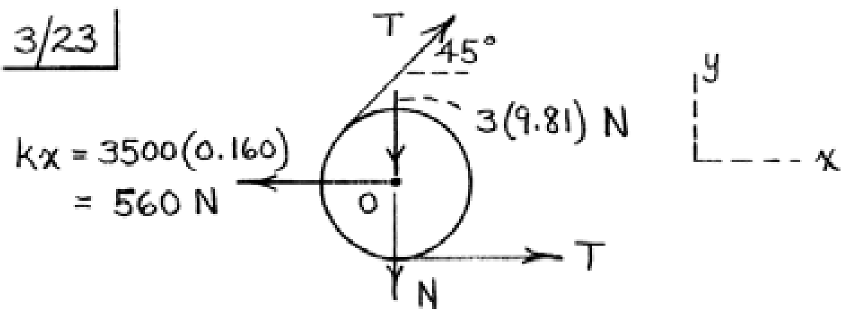
$$\Rightarrow \quad P = \frac{mg\sqrt{2rh-h^2}}{r-h}$$

3/22



$\circlearrowleft + \sum M_O = 0: \; 20 - P(0.180) = 0, \quad \underline{P = 111.1 \text{ N}}$

3/23



$\Sigma F_x = 0: \; T(1 + \cos 45°) - 560 = 0$

$T = 328 \text{ N}$

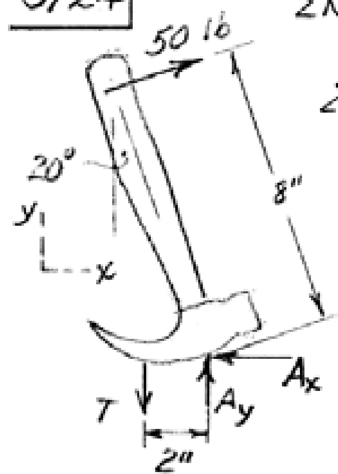
$\Sigma F_y = 0: \; 328(\sin 45°) - 3(9.81) - N = 0$

$N = 203 \text{ N}$ (down)

The force on the guide is then

$N = 203 \text{ N}$ up

3/24



$\Sigma M_A = 0;\ 50(8) - 2T = 0,\ \underline{T = 200\ lb}$

$\Sigma F_x = 0;\ 50 \cos 20° - A_x = 0$

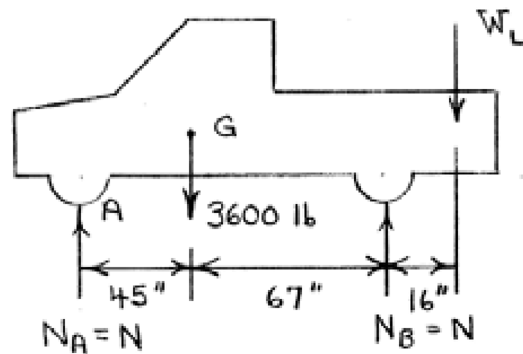
$A_x = 46.98\ lb$

$\Sigma F_y = 0;\ A_y + 50 \sin 20° - 200 = 0$

$A_y = 182.9\ lb$

$A = \sqrt{(46.98)^2 + (182.9)^2} = \underline{188.8\ lb}$

3/25



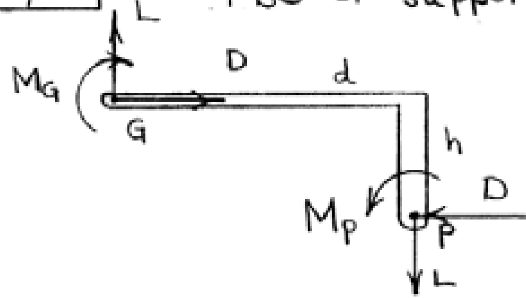
$+\circlearrowleft \Sigma M_A = 0: \ 3600(45) - N(112) + W_L(128) = 0$

$+\uparrow \Sigma F = 0: \ 2N - 3600 - W_L = 0$

Solve to obtain $N = 2075$ lb

$\underline{W_L = 550 \text{ lb}}$

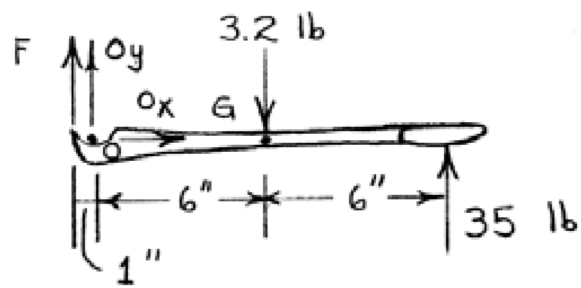
3/26 FBD of support & model:



$\circlearrowleft + \; \Sigma M_P = 0: \quad M_P - M_G - Ld - Dh = 0$

$\underline{M_G = M_P - Ld - Dh}$

3/27

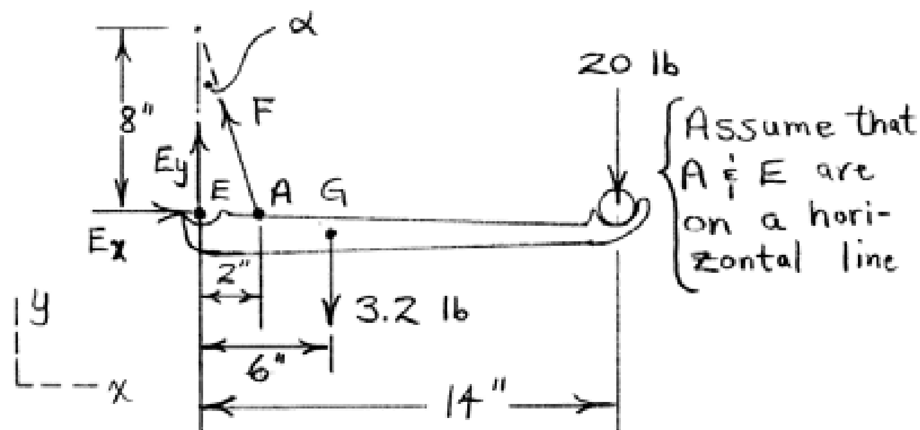


$$\circlearrowleft + \ \Sigma M_O = 0: \ -F(1) - 3.2(6) + 35(12) = 0$$

$$\underline{F = 401 \text{ lb}}$$

3/28

$\alpha = \tan^{-1}\frac{2}{8} = 14.04°$



$\curvearrowleft + \Sigma M_E = 0$: $F\cos 14.04°(2) - 3.2(6) - 20(14) = 0$

$F = 154.2 \text{ lb}$

$\Sigma F_x = 0$: $-154.2(\sin 14.04°) + E_x = 0$

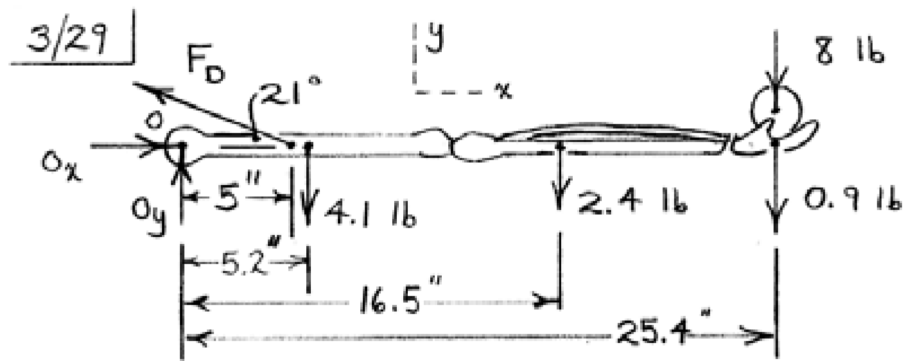
$E_x = 37.4 \text{ lb}$

$\Sigma F_y = 0$: $154.2\cos 14.04 - 3.2 - 20 + E_y = 0$

$E_y = -126.4 \text{ lb}$

$E = \sqrt{E_x^2 + E_y^2} = \sqrt{37.4^2 + 126.4^2} = 131.8 \text{ lb}$

3/29



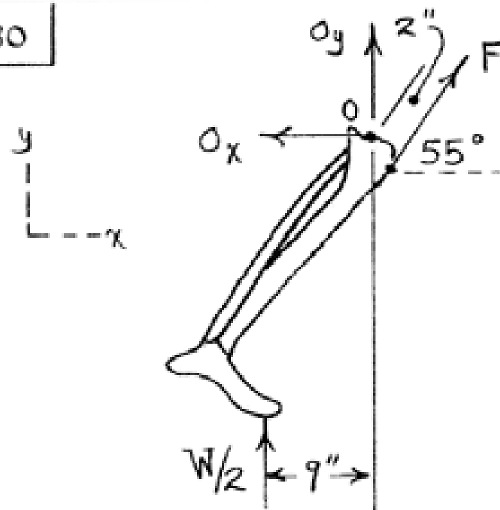
$$\circlearrowleft + \ \Sigma M_O = 0: \ (F_D \sin 21°)(5) - 4.1(5.2) - 2.4(16.5) - (8+0.9)(25.4) = 0, \quad \underline{F_D = 160.2 \text{ lb}}$$

$$\overset{+}{\rightarrow} \ \Sigma F_x = 0: \quad O_x - 160.2 \cos 21° = 0$$

$$\underline{O_x = 149.5 \text{ lb}}$$

$$+\uparrow \ \Sigma F_y = 0: \ O_y + 160.2 \sin 21° - 4.1 - 2.4 - 8.9 = 0, \quad \underline{O_y = -42.2 \text{ lb}}$$

3/30



$$\circlearrowleft + \Sigma M_O = 0: \quad F(2) - \frac{W}{2}(9) = 0, \quad \underline{F = 2.25W}$$

$$\Sigma F_x = 0: \quad -O_x + 2.25W\cos 55° = 0, \quad O_x = 1.291W$$

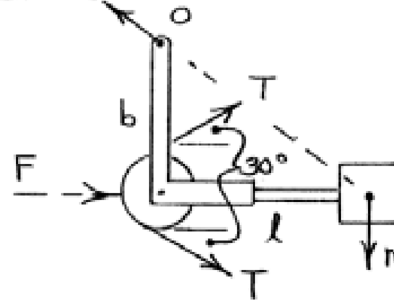
$$\Sigma F_y = 0: \quad \frac{W}{2} + O_y + 2.25W\sin 55° = 0$$

$$O_y = -2.34W$$

$$O = \sqrt{O_x^2 + O_y^2} = \sqrt{(1.291W)^2 + (2.34W)^2}$$

$$= \underline{2.67W}$$

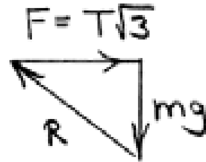
3/31



Replace 2 T's by $F = 2T\cos 30° = T\sqrt{3}$

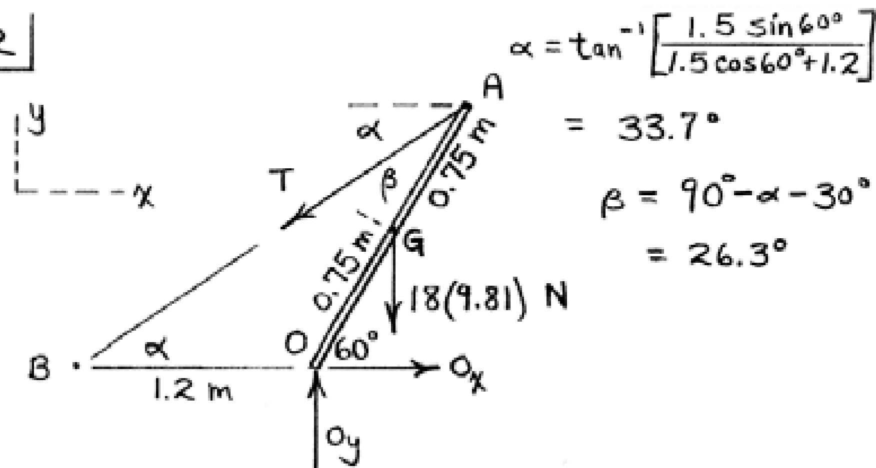
$\circlearrowleft \Sigma M_O = 0: \; mgl - T\sqrt{3}\, b = 0$

$$l = \frac{T\sqrt{3}}{mg}\, b$$



$$R = \sqrt{(T\sqrt{3})^2 + (mg)^2} = \sqrt{3T^2 + m^2g^2}$$

3/32



$$\alpha = \tan^{-1}\left[\frac{1.5\sin 60^\circ}{1.5\cos 60^\circ + 1.2}\right] = 33.7^\circ$$

$$\beta = 90^\circ - \alpha - 30^\circ = 26.3^\circ$$

$$\circlearrowleft + \Sigma M_O = 0: \; T\sin 33.7^\circ(1.2) - 18(9.81)(0.75)\cos 60^\circ = 0$$

$$\underline{T = 99.5 \text{ N}}$$

$$\Sigma F_x = 0: \; -99.5\cos 33.7^\circ + O_x = 0$$

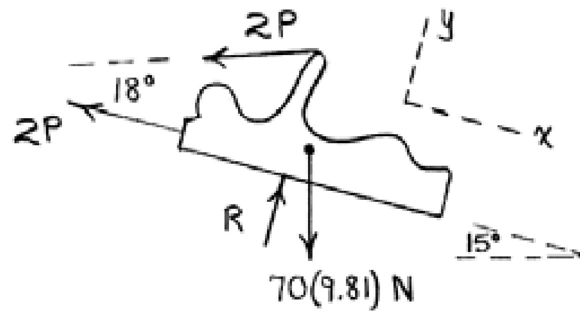
$$O_x = 82.8 \text{ N}$$

$$\Sigma F_y = 0: \; -99.5\sin 33.7^\circ - 18(9.81) + O_y = 0$$

$$O_y = 232 \text{ N}$$

So $O = 246$ N @ 70.3° CCW from +x-axis

3/33



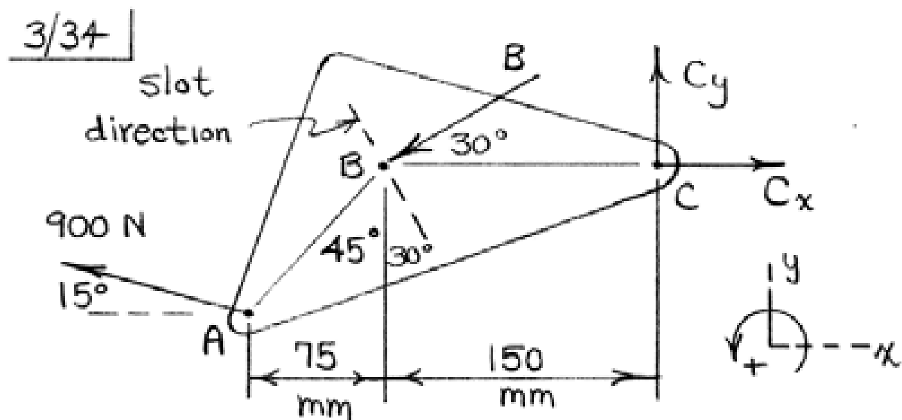
$\Sigma F_x = 0: \; 70(9.81)\sin 15° - 2P - 2P\cos 18° = 0$

$\underline{P = 45.5 \text{ N}}$

$\Sigma F_y = 0: \; R - 70(9.81)\cos 15° - 2(45.5)\sin 18° = 0$

$\underline{R = 691 \text{ N}}$

3/34



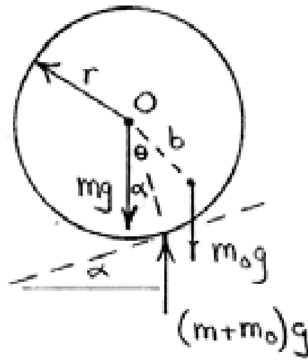
$$\begin{cases} \Sigma F_x = 0: \; C_x - B\cos 30° - 900\cos 15° = 0 \\ \Sigma F_y = 0: \; C_y - B\sin 30° + 900\sin 15° = 0 \\ \Sigma M_C = 0: \; B\sin 30°(150) - 900\cos 15°(75) \\ \qquad - 900\sin 15°(225) = 0 \end{cases}$$

Solution : $B = 1568$ N

$C_x = 2230$ N, $C_y = 551$ N $\Rightarrow$ $\underline{C = 2290 \text{ N}}$

(where $C = \sqrt{C_x^2 + C_y^2}$)

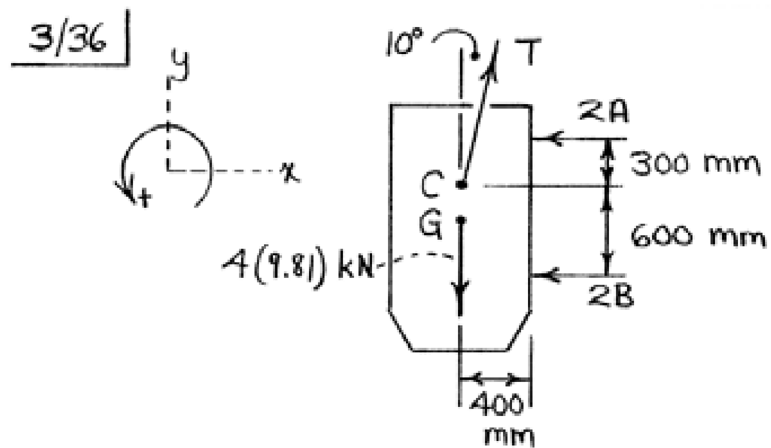
3/35



$$\circlearrowleft + \sum M_O = 0 : \ (m+m_o)g\, r\sin\alpha - m_o g\, b\sin\theta = 0$$

$$\Rightarrow \quad \theta = \sin^{-1}\left\{ \frac{r}{b}\left(1+\frac{m}{m_o}\right)\sin\alpha \right\}$$

3/36



$\Sigma F_y = 0: \; T\cos 10^\circ - 4(9.81) = 0, \quad T = 39.8 \text{ kN}$

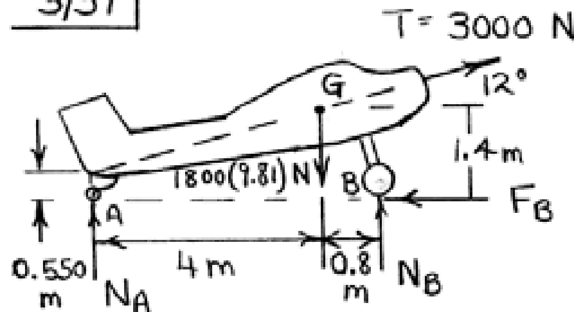
$\Sigma M_C = 0: \; 2A(300) - 2B(600) = 0, \quad A = 2B$

$\Sigma F_x = 0: \; 39.8 \sin 10^\circ - 2(2B) - 2B = 0$

$\underline{B = 1.153 \text{ kN}}$

$A = 2B = 2(1.153) = \underline{2.31 \text{ kN}}$

3/37



Engine off : $T = 0$, $F_B = 0$

$$\begin{cases} \Sigma M_A = 0: & 1800(9.81)4 - N_B(4.8) = 0 \qquad N_B = 14\,720 \text{ N} \\ \Sigma F_y = 0: & N_A + 14\,720 - 1800(9.81) = 0, \qquad N_A = 2940 \text{ N} \end{cases}$$

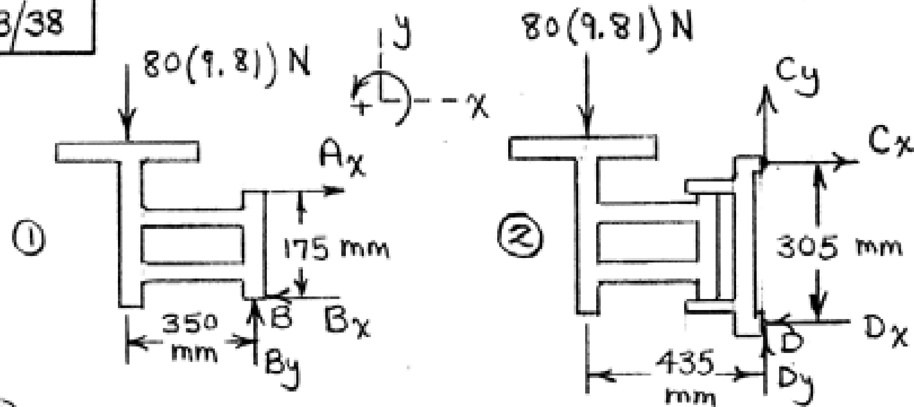
$\Sigma M_A = 0: \quad 1800(9.81)4 - N_B'(4.8) + 3000 \cos 12° (0.550) = 0$

$N_B' = 15{,}050$ N

$\Sigma F_y = 0: \quad N_A' + 15{,}050 - 1800(9.81) + 3000 \sin 12° = 0, \; N_A' = 1983$ N

$n_A = \dfrac{N_A' - N_A}{N_A}(100) = \underline{-32.6\%}, \quad n_B = \dfrac{N_B' - N_B}{N_B} = \underline{2.28\%}$

3/38



①

$\Sigma F_y = 0$: $B_y = 80(9.81) = \underline{785 \text{ N}}$

$\Sigma M_B = 0$: $-A_x(175) + 80(9.81)(350) = 0$

$\underline{A_x = B_x = 1570 \text{ N}}$

②

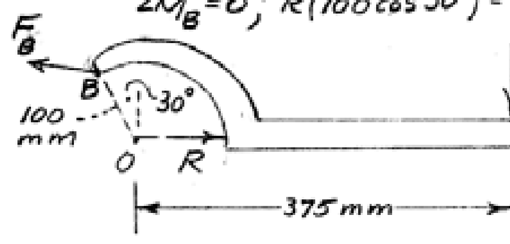
$\Sigma M_D = 0$: $-C_x(305) + 80(9.81)(435) = 0$

$\underline{C_x = D_x = 1119 \text{ N}}$

(All results due to L only)

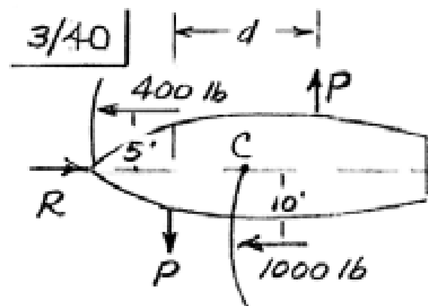
3/39 $M = Fd$; $80 = P(0.375)$, $P = 213$ N

$\Sigma M_B = 0$; $R(100 \cos 30°) - (375 + 100 \sin 30°)(213) = 0$



$R = 1047$ N

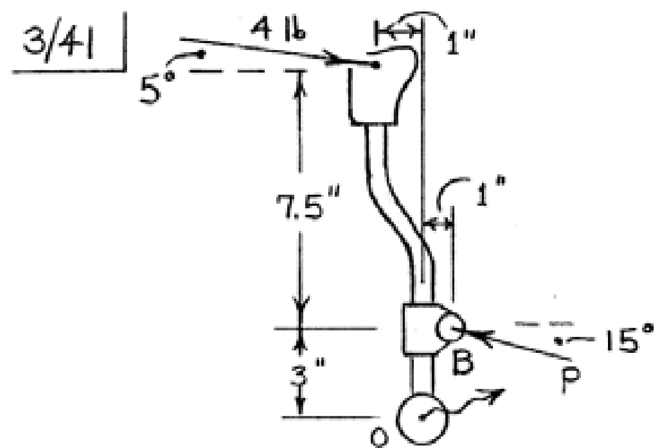
3/40



$\Sigma M_C = 0;\ Pd + 400(5) - 1000(10) = 0$

Resultant of lateral forces is couple

$M = Pd = \underline{8000 \text{ lb-ft}}$

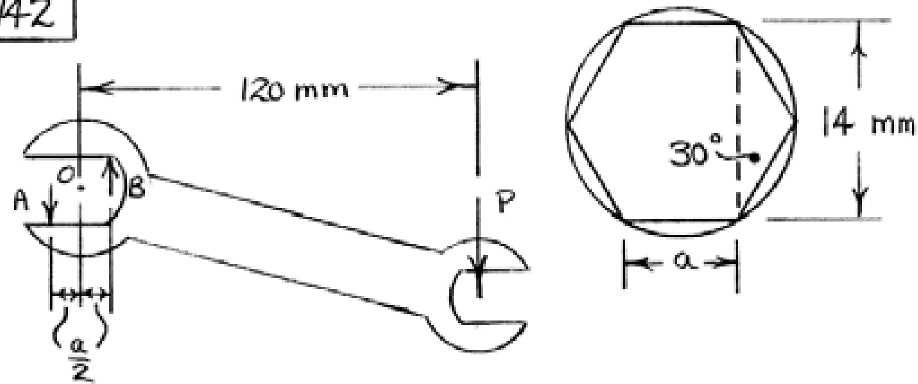


$\curvearrowleft_{+} \Sigma M_O = 0: \; -4\cos 5°(10.5) + 4\sin 5°(1)$

$+ \; P\cos 15°(3) + P\sin 15°(1) = 0$

$\underline{P = 13.14 \text{ lb}}$

3/42



$2a\cos 30° = 14$, $\frac{a}{2} = 4.04$ mm

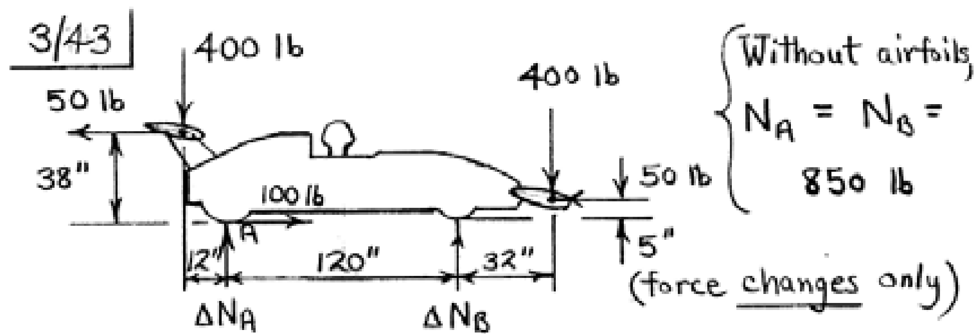
$\circlearrowleft + \Sigma M_O = 0: \; 0.120P - 24 = 0$, $\underline{P = 200 \text{ N}}$

(for wrench and bolt)

For wrench alone,

$\circlearrowleft + \Sigma M_A = 0: \; 200(0.120 + 0.00404) - B(2 \cdot 0.00404) = 0$, $\underline{B = 3070 \text{ N}}$

$+\uparrow \Sigma F = 0: \; -A + 3070 - 200 = 0$, $\underline{A = 2870 \text{ N}}$



With airfoils,

$+\uparrow \Sigma F_y = 0 : \Delta N_A + \Delta N_B - 2(400) = 0$

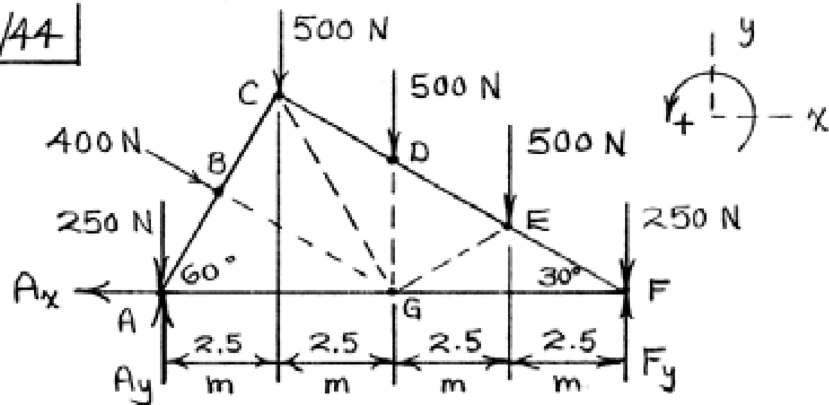
$\circlearrowleft\!+ \Sigma M_A = 0 : 50(38) + 400(12) + \Delta N_B(120)$

$+50(5) - 400(152) = 0$

$\left.\begin{array}{l}\Delta N_A = 351 \text{ lb} \\ \Delta N_B = 449 \text{ lb}\end{array}\right\} \Rightarrow \begin{array}{l}N_A = 850 + 351 = 1201 \text{ lb} \quad (48.0\%) \\ N_B = 850 + 449 = 1299 \text{ lb} \quad (52.0\%)\end{array}$

Note that a 100-lb propulsive force has been added (at A) to maintain equilibrium.

3/44



$\Sigma F_x = 0: \; -A_x + 400 \cos 30° = 0, \quad \underline{A_x = 346 \text{ N}}$

$\Sigma M_A = 0: \; 400\left(\frac{10}{4}\right) + 500\,(2.5) + 500\,(5)$

$+ 500\,(7.5) + 250\,(10) - 10 F_y = 0$

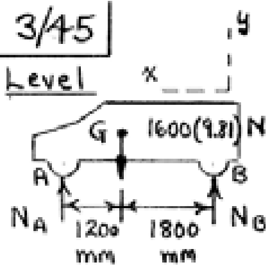
$\underline{F_y = 1100 \text{ N}}$

$\Sigma F_y = 0: -250 - 400 \sin 30° - 500(3) - 250$

$+ 1100 + A_y = 0, \quad \underline{A_y = 1100 \text{ N}}$

3/45

Level



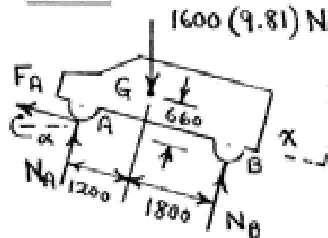
$$+\circlearrowleft \Sigma M_A = 0: \quad 1600(9.81)(1.2) - N_B(3) = 0$$

$$\underline{N_B = 6280 \text{ N}}$$

$$\Sigma F_y = 0: \quad N_A + N_B - 1600(9.81) = 0$$

$$\underline{N_A = 9420 \text{ N}}$$

Climb



$$+\circlearrowleft \Sigma M_A = 0: \quad 1600(9.81)(1.2 \cos\alpha)$$

$$+ 1600(9.81)(0.66)\sin\alpha - 3N_B = 0$$

$$\underline{N_B = 6590 \text{ N}}$$

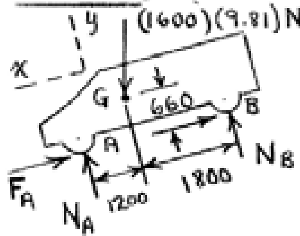
$$\Sigma F_y = 0: \quad N_A + N_B - 1600(9.81)\cos\alpha = 0$$

$$\underline{N_A = 9030 \text{ N}}$$

$\alpha = \tan^{-1}(0.1) = 5.71°$

$$n_A = \frac{9030 - 9420}{9420} = \underline{-4.14\%}, \quad n_B = \frac{6590 - 6278}{6278} = \underline{4.98\%}$$

Descend



$$+\circlearrowleft \Sigma M_A = 0: \quad 1600(9.81)(1.2 \cos\alpha)$$

$$- 1600(9.81)(0.66)\sin\alpha - 3N_B = 0$$

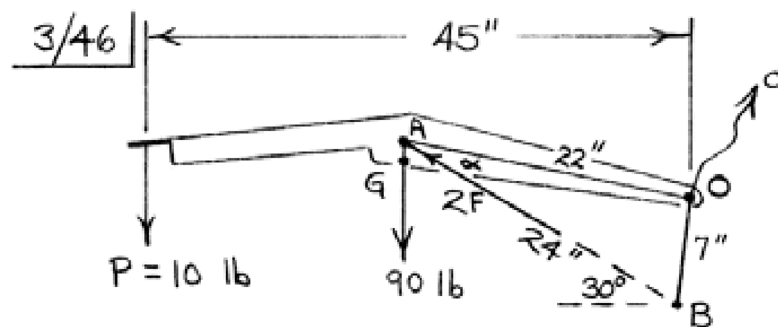
$$\underline{N_B = 5900 \text{ N}}$$

$$\Sigma F_y = 0: \quad N_A + N_B - 1600(9.81)\cos\alpha = 0$$

$$\underline{N_A = 9710 \text{ N}}$$

$$n_A = \frac{9710 - 9420}{9420} = \underline{3.15\%}, \quad n_B = \frac{5900 - 6280}{6280} = \underline{-5.97\%}$$

3/46



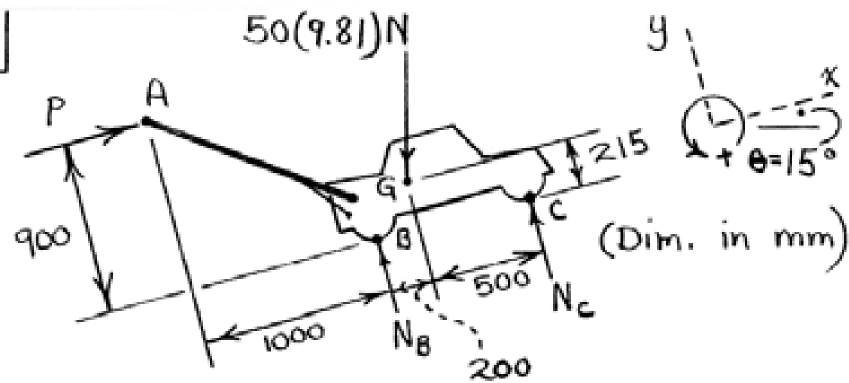
Law of cosines: $7^2 = 22^2 + 24^2 - 2(22)(24)\cos\alpha$

$\alpha = 16.79°$

$\circlearrowleft + \Sigma M_O = 0: \quad 10(45) - 2F(22\sin\alpha) + 90(22\cos(30° - \alpha))$

$= 0, \quad \underline{\underline{F = 187.1 \text{ lb}}}$

3/47



$\Sigma F_x = 0: \quad P - 50(9.81)\sin 15° = 0 \qquad (1)$

$\Sigma F_y = 0: \quad N_B + N_C - 50(9.81)\cos 15° = 0 \qquad (2)$

$\Sigma M_C = 0: \quad -P(900) - N_B(700) + 50(9.81)[500 \cos 15° + 215 \sin 15°] = 0 \qquad (3)$

Solution to Eqs. (1)-(3):

$P = 127.0$ N

$N_B = 214$ N

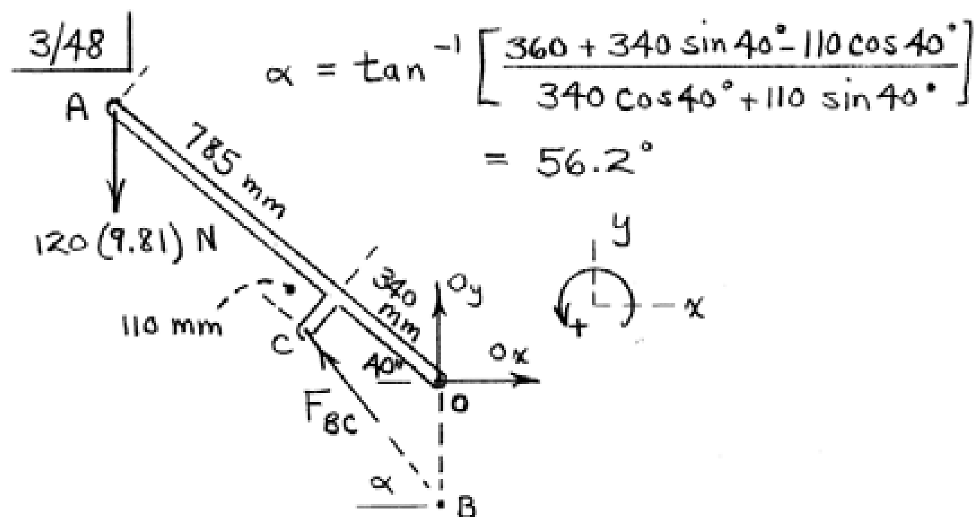
$N_C = 260$ N

With $\theta = P = 0$:

$P = 0$

$N_B = 350$ N

$N_C = 140.1$ N



$$\alpha = \tan^{-1}\left[\frac{360 + 340\sin 40° - 110\cos 40°}{340\cos 40° + 110\sin 40°}\right] = 56.2°$$

$\Sigma M_O = 0: \; 120(9.81)(785+340)\cos 40°$

$\qquad - F_{BC}\cos\alpha\,(360) = 0, \quad F_{BC} = 5060 \text{ N}$

$\Sigma F_x = 0: \; O_x - 5060\cos\alpha = 0, \quad O_x = 2820 \text{ N}$

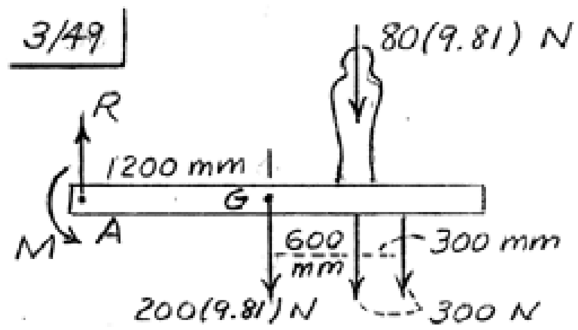
$\Sigma F_y = 0: \; O_y - 120(9.81) + 5060\sin\alpha = 0$

$\qquad O_y = -3030 \text{ N}$

$O = \sqrt{O_x^2 + O_y^2} = \underline{4140 \text{ N}}$

$p = \dfrac{F_{BC}}{\pi d^2/4} = \dfrac{5060}{\pi\, 50^2/4} = 2.58 \dfrac{\text{N}}{\text{mm}^2} \left(2.58(10^6)\text{ Pa}\right)$

3/49

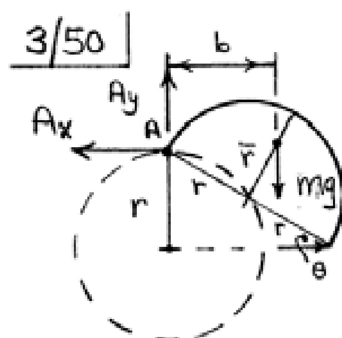


$\Sigma M_A = 0;\quad 80(9.81)(1800) + 200(9.81)(1200)$

$+ 300(1800 + 2100) - M = 0$

$M = 4.94(10^6)$ N·mm or $\underline{M = 4.94 \text{ kN}\cdot\text{m}}$

3/50



From Table D/3, $\bar{r} = 2r/\pi$

$b = r\cos\theta + \bar{r}\sin\theta$

Here, $\sin\theta = \frac{1}{2}$, $\cos\theta = \sqrt{3}/2$

So $b = \frac{r}{2}\left(\sqrt{3} + \frac{2}{\pi}\right)$

$$\circlearrowleft + \ \Sigma M_A = 0: \quad mg\frac{r}{2}\left(\sqrt{3} + \frac{2}{\pi}\right) - Cr = 0$$

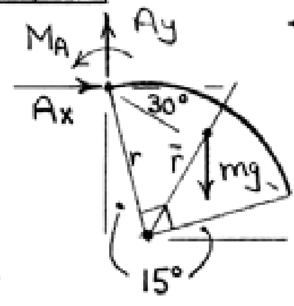
$$\underline{C = \frac{mg}{2}\left(\sqrt{3} + \frac{2}{\pi}\right)}$$

$\Sigma F_x = 0: \quad A_x = C$

$\Sigma F_y = 0: \quad A_y = mg$

$$F_A = \sqrt{C^2 + (mg)^2} = \underline{1.550\, mg}$$

3/51



From Table D/3, $\bar{r} = \dfrac{r \sin\alpha}{\alpha}$

For $\alpha = \pi/4$, $\bar{r} = 2\sqrt{2}\, r/\pi$

$\curvearrowleft + \sum M_A = 0$:

$M_A - mg\left(r \sin 15^\circ + \bar{r} \cos 60^\circ\right) = 0$

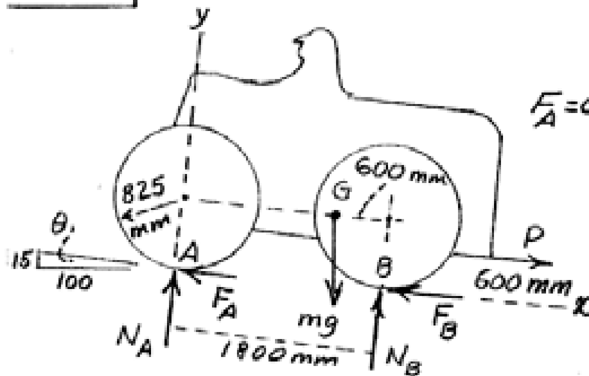
$\underline{M_A = 0.709\ mgr}$

3/52 $mg = 13.5(9.81) = 132.4$ kN; $\tan\theta = 0.15$

$\sin\theta = 0.148$

$\cos\theta = 0.989$

$F_A = 0.8N_A$, $F_B = 0.8N_B$



$\Sigma M_A = 0$; $-P(0.6) - 132.4(0.148)(0.825) - 132.4(0.989)(1.8 - 0.6) + 1.8N_B = 0$

$\Sigma F_x = 0$; $P + 132.4(0.148) - 0.8(N_A + N_B) = 0$

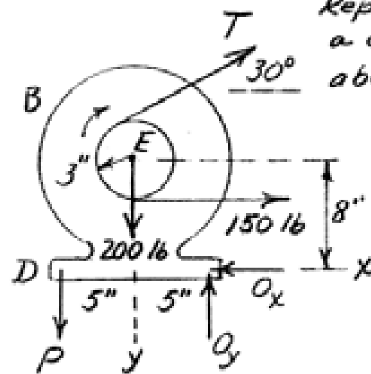
$\Sigma F_y = 0$; $N_A + N_B - 132.4(0.989) = 0$

Solve & get $N_A + N_B = 131.0$ kN

$\underline{P = 85.1 \text{ kN}}$, $\underline{N_B = 124.7 \text{ kN}}$

3/53 Torque $M = 900 = (150 - T)9$, $T = 50$ lb

Replace T by force at E and a couple, so that moment of T about D is



$(50 \cos 30°)8 - (50 \sin 30°)5 + 50(3) = 371.4$ lb-in. CW

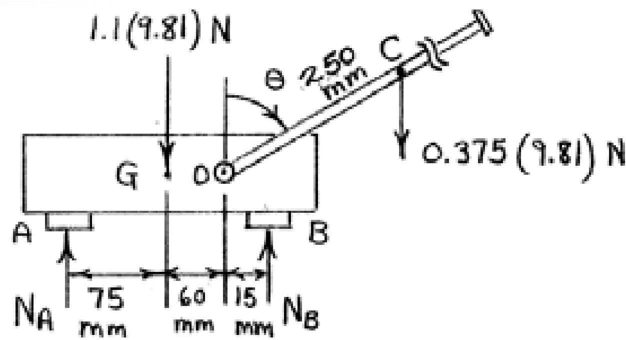
$\Sigma M_D = 0;$

$371.4 + 150(8-3) + 200(5) - 10\,O_y = 0, \quad O_y = 212.1$ lb

$\Sigma F_x = 0; \quad 150 + 50 \cos 30° - O_x = 0; \quad O_x = 193.3$ lb

$R = \sqrt{O_x^2 + O_y^2} = \sqrt{(193.3)^2 + (212.1)^2} = \underline{287 \text{ lb}}$

3/54



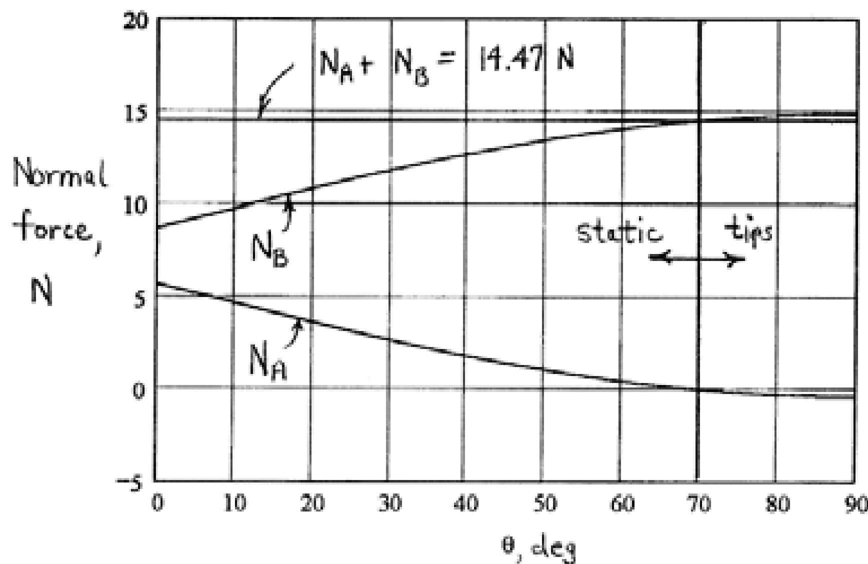
$+\uparrow \Sigma F = 0: \; N_A + N_B - (1.1 + 0.375)(9.81) = 0 \quad (1)$

$\Sigma M_B = 0: \; -N_A(150) + 1.1(9.81)(75)$
$\quad + (15 - 250 \sin\theta)(0.375)(9.81) = 0 \quad (2)$

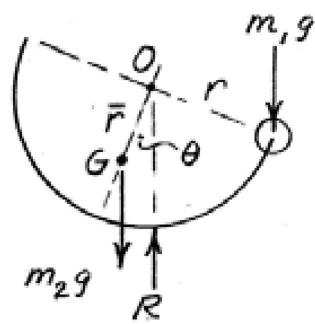
(2): $N_A = 5.76 - 6.13 \sin\theta$
(1): $N_B = 8.71 + 6.13 \sin\theta$

N_A, N_B in newtons

Note that N_A goes to zero at $\theta = 70.1°$, meaning that the receiver would tip if θ exceeds this value.



3/55



$\Sigma M_O = 0$;

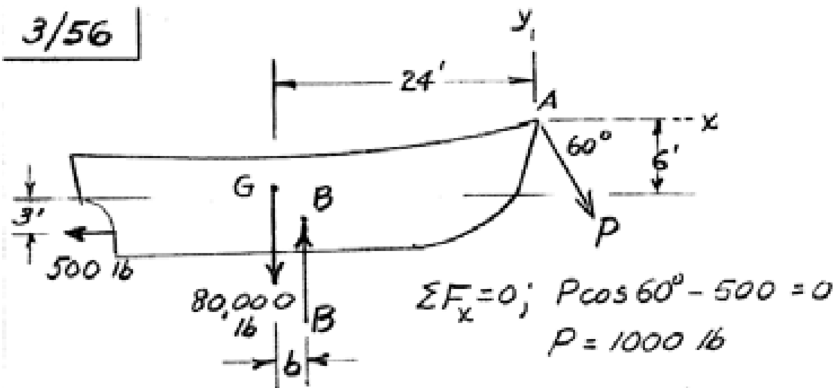
$m_1 g\, r\cos\theta - m_2 g\, \bar{r}\sin\theta = 0$

where $\bar{r} = 2r/\pi$

so $\tan\theta = \dfrac{m_1 r}{m_2 \bar{r}} = \dfrac{m_1 \pi}{2m_2}$

$\theta = \tan^{-1} \dfrac{\pi m_1}{2m_2}$

3/56



$\Sigma F_x = 0;\ P\cos 60° - 500 = 0$

$P = 1000\text{ lb}$

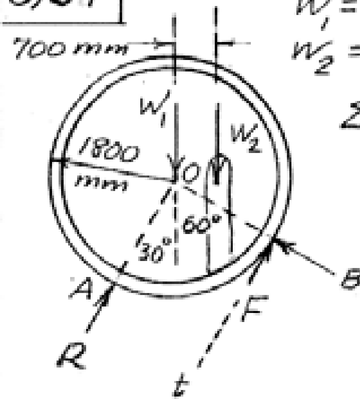
$\Sigma F_y = 0;\ B - 80{,}000 - 1000\sin 60° = 0,\ B = 80{,}866\text{ lb}$

$\Sigma M_A = 0;\ 80{,}000(24) - 80{,}866(24-6) - 500(6+3) = 0$

$1{,}920{,}000 - 1{,}940{,}784 + 80{,}866\,b - 4500 = 0$

$b = \dfrac{25284}{80{,}866} = 0.3127\text{ ft}$ or $\underline{b = 3.75\text{ in.}}$

3/57



$$W_1 = m_1 g = 400(9.81) = 3924 \text{ N}$$

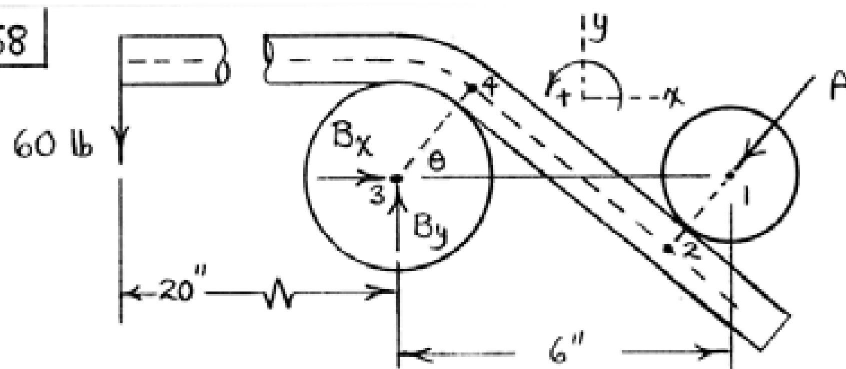
$$W_2 = m_2 g = 80(9.81) = 785 \text{ N}$$

$$\Sigma M_O = 0; \quad 785(0.7) - 1.8F = 0$$

$$\underline{F = 305 \text{ N}}$$

$$\Sigma F_t = 0; \quad R + 305 - (3924 + 785)\cos 30^\circ = 0$$

$$\underline{R = 3770 \text{ N}}$$



$\overline{1\text{-}3} = 6"$; $\overline{1\text{-}2} = 1 + \frac{1.050}{2} = 1.525"$; $\overline{3\text{-}4} = 1.75 + \frac{1.050}{2} = 2.275"$

$\theta = \cos^{-1}\left(\frac{1.525 + 2.275}{6}\right) = 50.7°$

$\overline{2\text{-}4} = 6 \sin 50.7° = 4.64"$

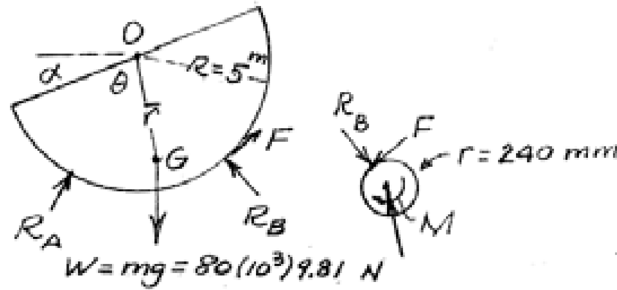
$\Sigma M_3 = 0$: $60(20) - 4.64A = 0$, $\underline{A = 258 \text{ lb}}$

$\Sigma F_x = 0$: $B_x - 258 \cos 50.7° = 0$, $B_x = 163.7 \text{ lb}$

$\Sigma F_y = 0$: $B_y - 60 - 258 \sin 50.7° = 0$, $B_y = 260 \text{ lb}$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{163.7^2 + 260^2} = \underline{307 \text{ lb}}$$

▶3/59 For gear $Fr = M$, $F = \frac{M}{r} = \frac{M}{0.24}$



$W = mg = 80(10^3)9.81$ N

$\alpha = 0$, $\theta < 90°$, $M = 2460$ N·m

$\Sigma M_O = 0$; $\frac{2460}{r}R = mg\bar{r}\cos\theta$ - - - - - (a)

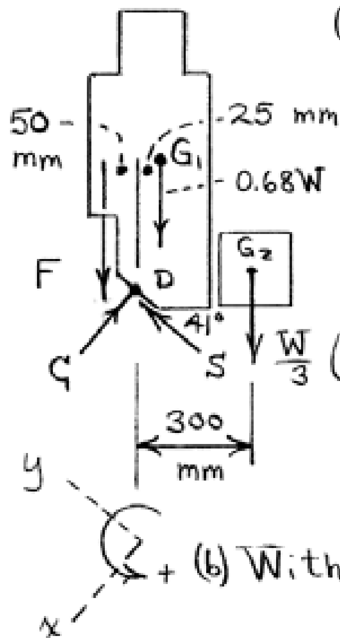
$\alpha = 30°$, $\theta + \alpha > 90°$, $M = 4680$ N·m

$\Sigma M_O = 0$; $\frac{4680}{r}R = mg\bar{r}\sin(\theta + 30° - 90°)$ - - - (b)

Divide (a) by (b) $\frac{2460}{4680} = \frac{\cos\theta}{\sin(\theta - 60°)}$, Solve & get $\theta = \tan^{-1}5.54$ $= 79.8°$

From (a) $\bar{r} = \frac{2460(5)}{(0.24)80(10^3)(9.81)\cos 79.8°} = 0.367$ m or 367 mm

▸3/60 Consider a FBD of the upper torso



(a) $\Sigma M_D = 0$: $F(50) - 0.68W(25) = 0$

$F = 0.34W$

$\Sigma F_y = 0$: $S - 0.68W \sin 41° - F \sin 41° = 0$, $\underline{S = 0.669W}$

$\Sigma F_x = 0$: $-C - 0.68W \cos 41° - F \cos 41° = 0$, $\underline{C = 0.770W}$

(b) With weight $\frac{W}{3}$:

$\Sigma M_D = 0$: $F(50) - 0.68W(25) - \frac{W}{3}(300) = 0$

$F = 2.34W$

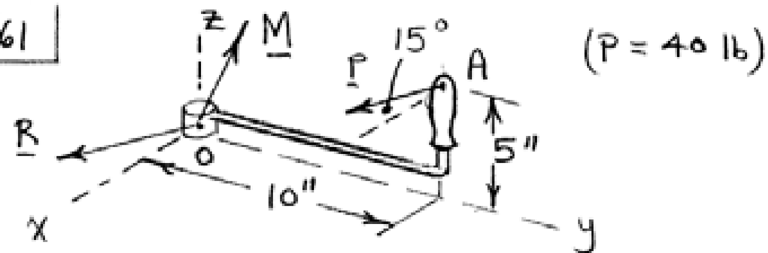
$\Sigma F_y = 0$: $S - 0.68W \sin 41° - F \sin 41° - \frac{W}{3} \sin 41° = 0$

$\underline{S = 2.20W}$

$\Sigma F_x = 0$: $-C + 0.68W \cos 41° + F \cos 41° + \frac{W}{3} \cos 41° = 0$

$\underline{C = 2.53W}$

3/61



($P = 40$ lb)

$\Sigma \underline{F} = \underline{0}: \quad \underline{R} + \underline{P} = \underline{0}$

$\underline{R} = -\underline{P} = -40 \left[\cos 15° \underline{i} + \sin 15° \underline{k} \right]$

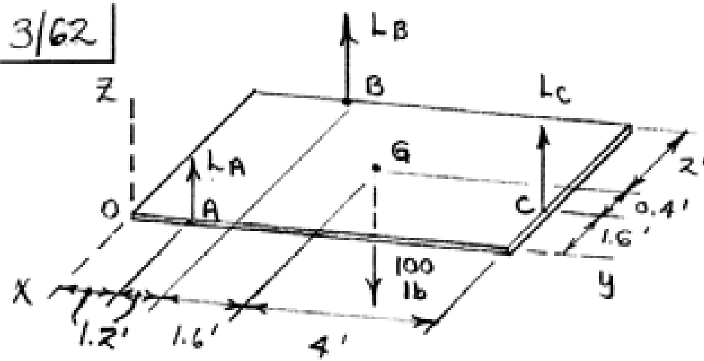
$= \underline{-38.6 \, \underline{i} - 10.35 \, \underline{k} \ \text{lb}}$

$\Sigma \underline{M} = \underline{0}: \quad \underline{M} + \underline{r} \times \underline{P} = \underline{0}$

$\underline{M} = -\underline{r} \times \underline{P} = -(10\underline{j} + 5\underline{k}) \times 40 \left[\cos 15° \underline{i} + \sin 15° \underline{k} \right]$

$= \underline{-103.5 \, \underline{i} - 193.2 \, \underline{j} + 386 \, \underline{k} \ \text{lb-in.}}$

3/62



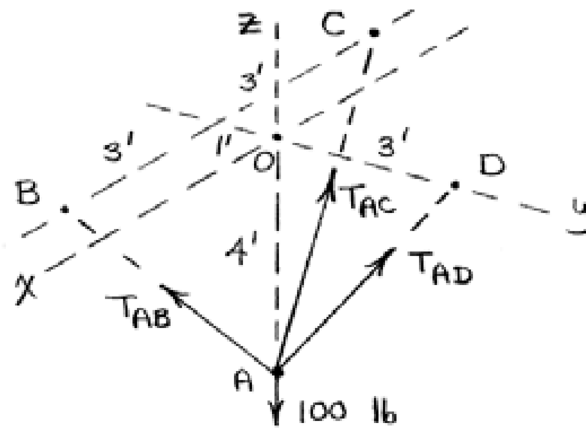
$\Sigma F_z = 0: \quad L_A + L_B + L_C - 100 = 0$

$\Sigma M_{O_x} = 0: \quad L_A(1.2) + L_B(2.4) + L_C(8) - 100(4) = 0$

$\Sigma M_{O_y} = 0: \quad L_B(4) + L_C(1.6) - 100(2) = 0$

Solution :
$$\begin{cases} L_A = 29.1 \text{ lb} \\ L_B = 36.1 \text{ lb} \\ L_C = 34.8 \text{ lb} \end{cases}$$

3/63



$$\underline{T}_{AB} = T_{AB}\left[\frac{3\underline{i} - \underline{j} + 4\underline{k}}{\sqrt{3^2+1^2+4^2}}\right] = T_{AB}\left[0.588\underline{i} - 0.1961\underline{j} + 0.784\underline{k}\right]$$

$$\underline{T}_{AC} = T_{AC}\left[\frac{-3\underline{i} - \underline{j} + 4\underline{k}}{\sqrt{3^2+1^2+4^2}}\right] = T_{AC}\left[-0.588\underline{i} - 0.1961\underline{j} + 0.784\underline{k}\right]$$

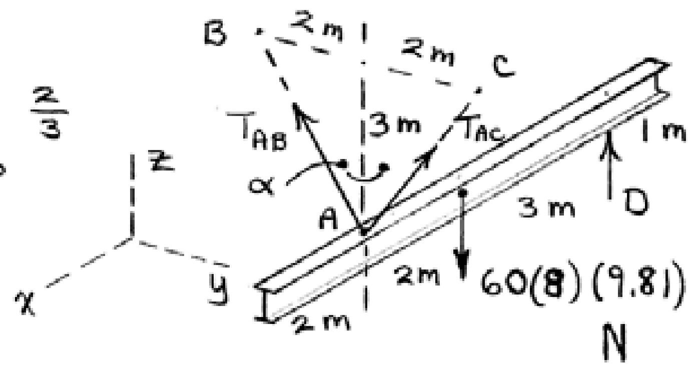
$$\underline{T}_{AD} = T_{AD}\left[\frac{3\underline{j} + 4\underline{k}}{\sqrt{3^2+4^2}}\right] = T_{AD}\left[0.6\underline{j} + 0.8\underline{k}\right]$$

$$\begin{cases} \Sigma F_x = 0: \quad 0.588T_{AB} - 0.588T_{AC} = 0 \quad (1) \\ \Sigma F_y = 0: \quad -0.1962T_{AB} - 0.1962T_{AC} + 0.6\,T_{AD} = 0 \quad (2) \\ \Sigma F_z = 0: \quad 0.784T_{AB} + 0.784T_{AC} + 0.8T_{AD} - 100 = 0 \quad (3) \end{cases}$$

Solution :
$$\begin{cases} T_{AB} = 47.8 \text{ lb} \\ T_{AC} = 47.8 \text{ lb} \\ T_{AD} = 31.2 \text{ lb} \end{cases}$$

3/64

$\alpha = \tan^{-1} \frac{2}{3}$
$= 33.7°$



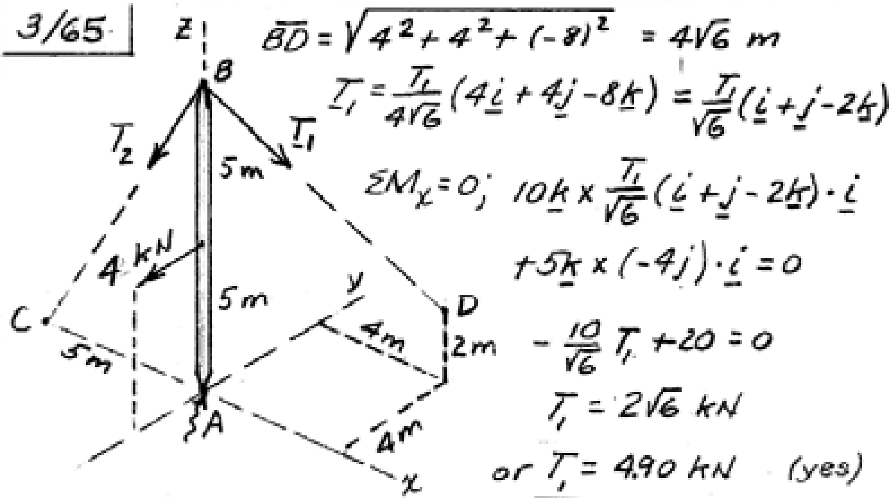
From $\Sigma F_y = 0$, $T_{AB} = T_{AC} = T$

$\Sigma M_{A_y} = 0 : -60(8)(9.81)(2) + D(5) = 0$

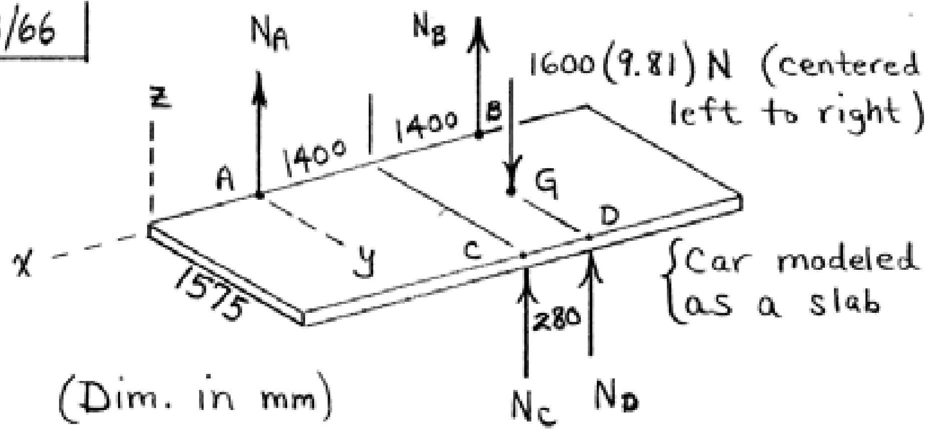
$D = 1884 \text{ N}$

$\Sigma F_z = 0 : 2T\cos\alpha + D - 60(8)(9.81) = 0$

$T = 1698 \text{ N} = T_{AB} = T_{AC}$


3/65
z
B
T2
T1
5m
4 kN
5m
y
C
D
5m
4m
2m
A
4m
x
BD = √(4² + 4² + (-8)²) = 4√6 m
T1 = T1/(4√6) (4i + 4j - 8k) = T1/√6 (i + j - 2k)
ΣMx = 0; 10k × T1/√6 (i + j - 2k) · i
+ 5k × (-4j) · i = 0
- 10/√6 T1 + 20 = 0
T1 = 2√6 kN
or T1 = 4.90 kN (yes)

3/66



Jacking at C ($N_D = 0$):

$$\Sigma M_x = 0: \ -1600(9.81)\left(\frac{1575}{2}\right) + N_C(1575) = 0$$

$$\underline{N_C = 7850 \text{ N}}$$

$$\Sigma M_y = 0: -1600(9.81)(1680) + N_B(2800) + N_C(1400) = 0$$

$$\Sigma F_z = 0: \ N_A + N_B + N_C - 1600(9.81) = 0$$

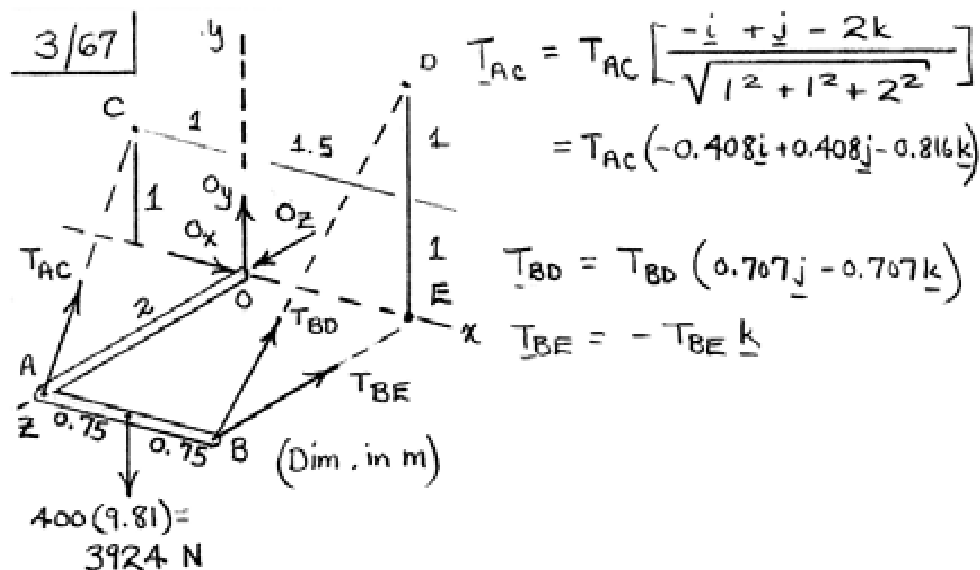
$$\Rightarrow \underline{N_A = 2350 \text{ N}}, \quad \underline{N_B = 5490 \text{ N}}$$

Jacking at D ($N_C = 0$): $\underline{N_D = 7850 \text{ N}}$ {Same as for N_C

$$\Sigma M_y = 0: \ -1600(9.81)(1680) + N_B(2800) + N_D(1680) = 0$$

$$\Sigma F_z = 0: \ N_A + N_B + N_D - 1600(9.81) = 0$$

$$\Rightarrow \underline{N_A = 3140 \text{ N}}, \quad \underline{N_B = 4710 \text{ N}}$$



$$\mathbf{T}_{AC} = T_{AC}\left[\frac{-\mathbf{i} + \mathbf{j} - 2\mathbf{k}}{\sqrt{1^2+1^2+2^2}}\right]$$

$$= T_{AC}(-0.408\mathbf{i} + 0.408\mathbf{j} - 0.816\mathbf{k})$$

$$\mathbf{T}_{BD} = T_{BD}(0.707\mathbf{j} - 0.707\mathbf{k})$$

$$\mathbf{T}_{BE} = -T_{BE}\,\mathbf{k}$$

$\Sigma F_x = 0$: $\quad O_x - 0.408\,T_{AC} = 0$

$\Sigma F_y = 0$: $\quad O_y + 0.408\,T_{AC} + 0.707\,T_{BD} - 3924 = 0$

$\Sigma F_z = 0$: $\quad O_z - 0.816\,T_{AC} - 0.707\,T_{BD} - T_{BE} = 0$

$\Sigma M_{O_x} = 0$: $\quad -0.408\,T_{AC}(2) - 0.707\,T_{BD}(2) + 3924(2) = 0$

$\Sigma M_{O_y} = 0$: $\quad -0.408\,T_{AC}(2) + 0.707\,T_{BD}(1.5) + T_{BE}(1.5) = 0$

$\Sigma M_{O_z} = 0$: $\quad -3924(0.75) + 0.707\,T_{BD}(1.5) = 0$

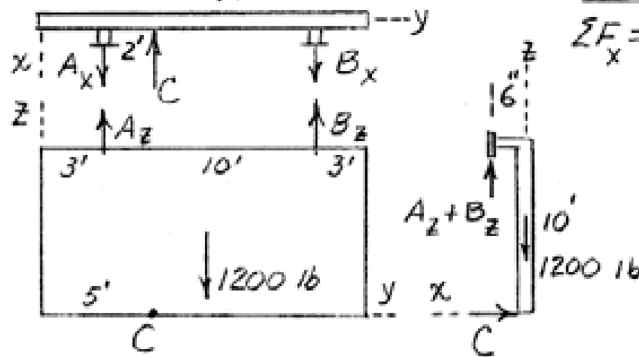
Solution: $\quad O_x = 1962$ N $\qquad T_{AC} = 4810$ N

$O_y = 0$ (Note $\Sigma M_{AB} = 0$) $\qquad T_{BD} = 2770$ N

$O_z = 6540$ N $\qquad T_{BE} = 654$ N

3/68 x-z; $\Sigma M_{AB} = 0$; $10C - 1200(6/12) = 0$, $C = 60$ lb

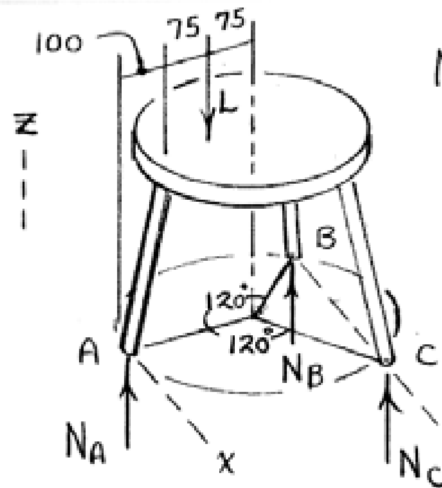
x-y; $\Sigma M_A = 0$; $2(60) - 10B_x = 0$; $\underline{B_x = 12 \text{ lb}}$



$\Sigma F_x = 0$; $A_x + 12 - 60 = 0$

$\underline{A_x = 48 \text{ lb}}$

3/69



Note: x-axis is ∥ BC.

$N_B = N_C = N$, by symmetry.

$\Sigma M_x = 0: \quad -L(175) + 2N(250 + 250\cos 60°) = 0$

$N = \underline{0.233L = N_B = N_C}$

$\Sigma F_z = 0: \quad N_A + 2(0.233L) - L = 0$

$\underline{N_A = 0.533L}$

3/70

$3/(2\cos 30°) = \sqrt{3}$

P, 30°, $T\sin\theta$, 30°, P, y, x

$y\text{-}z;\ \Sigma F_z = 0;\ T\cos 23.4° - 280 = 0$

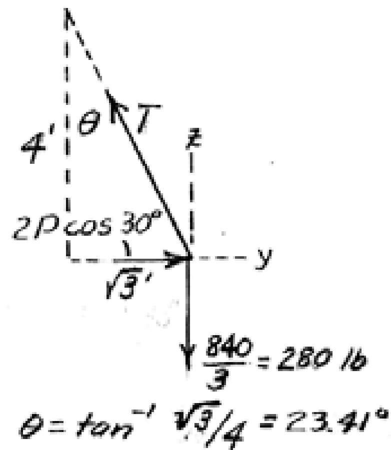
$T = 280/0.9177$

$= 305$ lb

$y\text{-}z;\ \Sigma F_y = 0;\ 305 \sin 23.4° - 2P\cos 30° = 0$

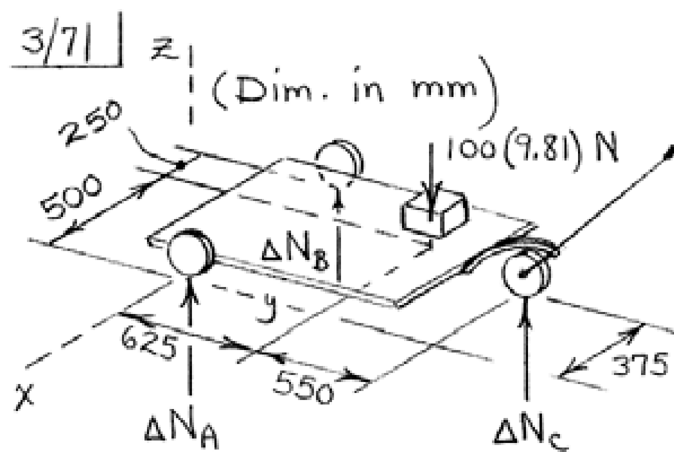
$$P = \frac{305(0.3973)}{2(0.8660)}$$

$= \underline{70.0 \text{ lb}}$



$\theta = \tan^{-1} \sqrt{3}/4 = 23.41°$

3/71



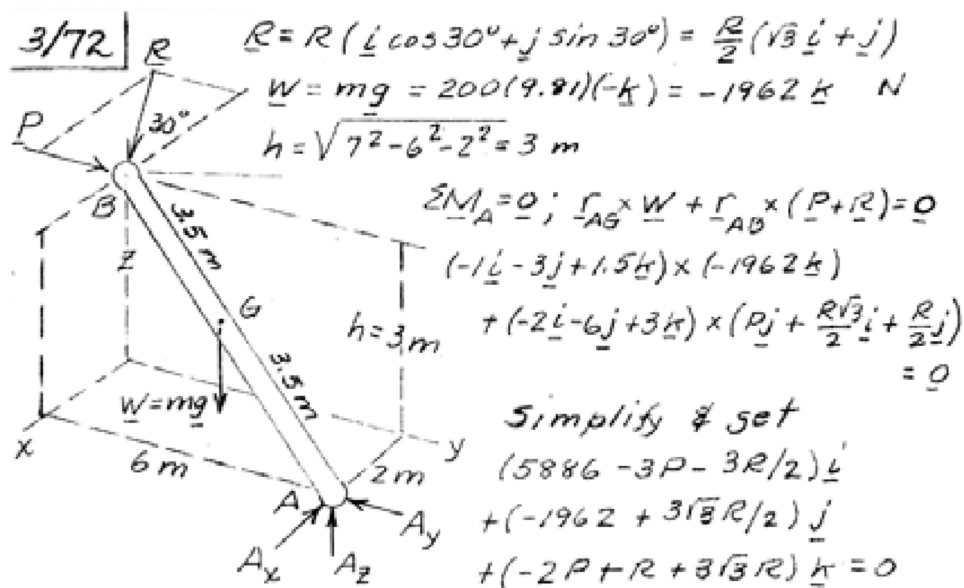
$\Sigma M_x = 0: \; -100(9.81)(625) + \Delta N_C(1175) = 0$

$\underline{\Delta N_C = 522 \text{ N}}$

$\Sigma M_y = 0: \; \Delta N_B(750) - 100(9.81)(500)$
$+ 522(375) = 0, \quad \underline{\Delta N_B = 393 \text{ N}}$

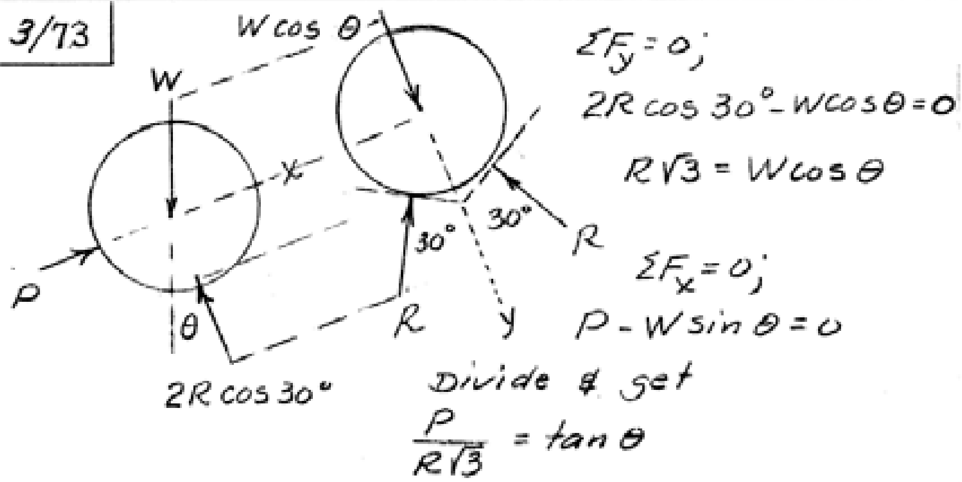
$\Sigma F_z = 0: \; 522 + 393 + \Delta N_A - 100(9.81) = 0$

$\underline{\Delta N_A = 66.1 \text{ N}}$



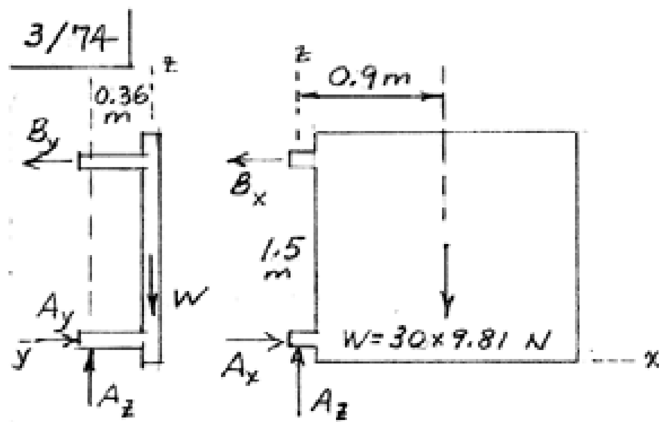
$R = \dfrac{2(1962)}{3\sqrt{3}} = \underline{755 \text{ N}}$

$3P = 5886 - \frac{3}{2}755$, $P = \underline{1584 \text{ N}}$



So with $P = R$, $\theta = \tan^{-1} 1/\sqrt{3} = \underline{30^\circ}$

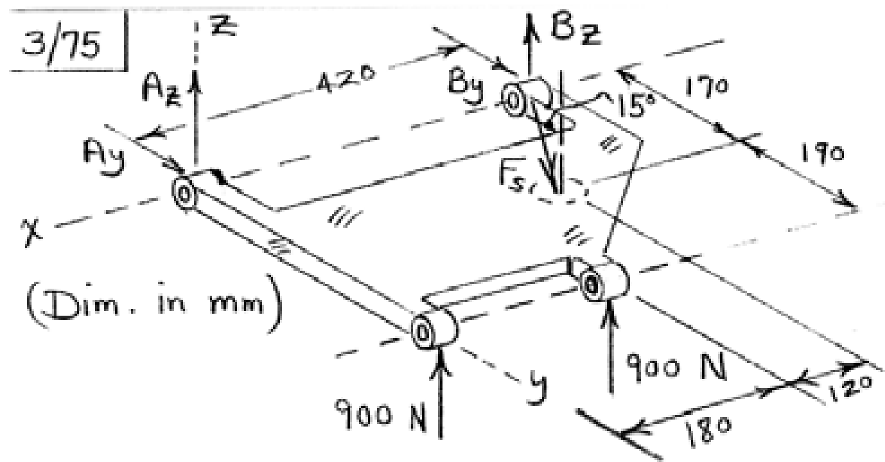
3/74



x-z; $\Sigma M_A = 0$; $1.5 B_x - 0.9(30)(9.81) = 0$, $B_x = 176.6$ N

y-z; $\Sigma M_A = 0$; $1.5 B_y - 0.36(30)(9.81) = 0$, $B_y = 70.6$ N

$$B = \sqrt{176.6^2 + 70.6^2} = \underline{190.2 \text{ N}}$$



$$\Sigma M_x = 0: \ 2(900)(360) - F_S \cos 15°(170) = 0, \ \underline{F_S = 3950 \text{ N}}$$

$$\Sigma M_y = 0: \ B_z(420) + 900(180) - 3950 \cos 15°(300) = 0$$

$$B_z = 2340 \text{ N}$$

$$\Sigma M_z = 0: -B_y(420) - 3950 \sin 15°(300) = 0, \ B_y = -730 \text{ N}$$

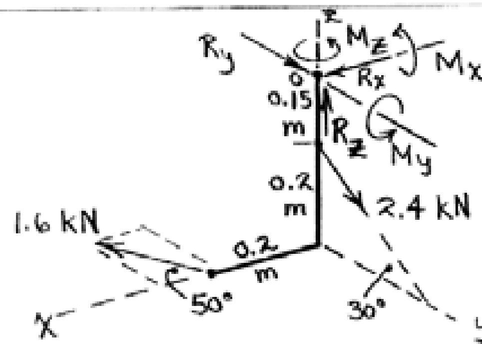
$$\Sigma F_y = 0: \ A_y - 730 + 3950 \sin 15° = 0, \quad A_y = -292 \text{ N}$$

$$\Sigma F_z = 0: \ A_z + 2340 - 3950 \cos 15° + 1800 = 0, \ A_z = -325 \text{ N}$$

$$F_A = \sqrt{A_y^2 + A_z^2} = \underline{437 \text{ N}}$$

$$F_B = \sqrt{B_y^2 + B_z^2} = \underline{2450 \text{ N}}$$

3/76



$\Sigma F_x = 0: \; R_x + 1.6 \cos 50°, \qquad R_x = -1.028 \text{ kN}$

$\Sigma F_y = 0: \; R_y + 2.4 \cos 30° - 1.6 \sin 50° = 0, \quad R_y = -0.853 \text{ kN}$

$\Sigma F_z = 0: \; R_z - 2.4 \sin 30° = 0, \qquad R_z = 1.2 \text{ kN}$

$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \underline{1.796 \text{ kN}}$

$\Sigma M_{o_x} = 0: \; M_x + 2.4 \cos 30° (0.15) - 1.6 \sin 50° (0.35) = 0$

$M_x = 0.1172 \text{ kN·m}$

$\Sigma M_{o_y} = 0: \; M_y - 1.6 \cos 50° (0.35) = 0, \quad M_y = 0.360 \text{ kN·m}$

$\Sigma M_{o_z} = 0: \; M_z - 1.6 \sin 50° (0.2) = 0, \quad M_z = 0.245 \text{ kN·m}$

$M = \sqrt{M_x^2 + M_y^2 + M_z^2} = \underline{0.451 \text{ kN·m}}$

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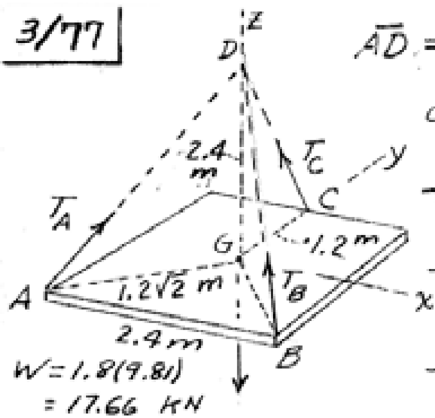


$$\overline{AD} = \overline{BD} = \sqrt{\overline{2.4}^2 + \overline{1.2\sqrt{2}}^2} = 1.2\sqrt{6} \text{ m}$$

$$\overline{CD} = \sqrt{\overline{1.2}^2 + \overline{2.4}^2} = 1.2\sqrt{5} \text{ m}$$

$$\underline{T}_A = \frac{T_A}{1.2\sqrt{6}}(1.2\underline{i} + 1.2\underline{j} + 2.4\underline{k})$$

$$\underline{T}_B = \frac{T_B}{1.2\sqrt{6}}(-1.2\underline{i} + 1.2\underline{j} + 2.4\underline{k})$$

$$\underline{T}_C = \frac{T_C}{1.2\sqrt{5}}(-1.2\underline{j} + 2.4\underline{k})$$

$\Sigma\underline{F} = \underline{0}$; $\underline{T}_A + \underline{T}_B + \underline{T}_C + \underline{W} = \underline{0}$

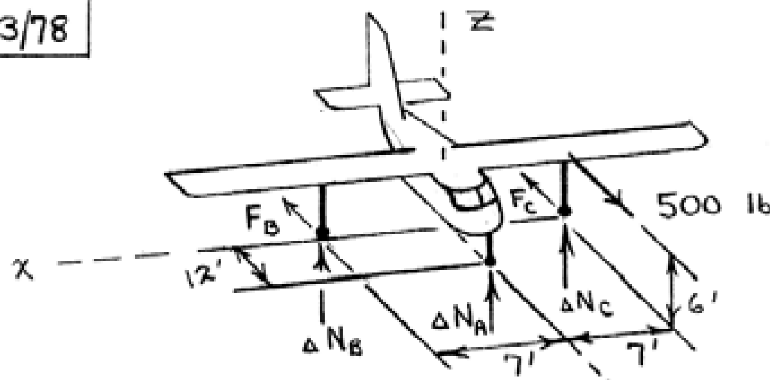
$$\underline{i}\left(\frac{T_A}{\sqrt{6}} - \frac{T_B}{\sqrt{6}}\right) + \underline{j}\left(\frac{T_A}{\sqrt{6}} + \frac{T_B}{\sqrt{6}} - \frac{T_C}{\sqrt{5}}\right) + \underline{k}\left(\frac{2T_A}{\sqrt{6}} + \frac{2T_B}{\sqrt{6}} + \frac{2T_C}{\sqrt{5}} - 17.66\right) = \underline{0}$$

$T_A = T_B$; $4T_A/\sqrt{6} + 2T_C/\sqrt{5} = 17.66$; $2T_A/\sqrt{6} = T_C/\sqrt{5}$

Solve & get $\underline{T_A = T_B = 5.41 \text{ kN}}$

$\underline{T_C = 9.87 \text{ kN}}$

3/78



$\Sigma M_x = 0: \quad \Delta N_A(12) - 500(6) = 0, \qquad \underline{\Delta N_A = 250 \text{ lb}}$

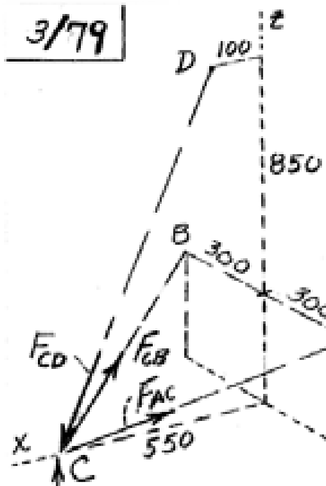
$\Sigma F_z = 0: \quad \Delta N_A + \Delta N_B + \Delta N_C = 0$

$\Sigma M_y = 0: \quad \Delta N_C(7) - \Delta N_B(7) = 0$

$\Delta N_B = \Delta N_C = \underline{-125 \text{ lb}}$

More information would be required to determine F_B and F_C. x-components of friction at B and C are possible.

3/79



$R = \frac{1}{3} mg$; From Table C2 for Mars $g = 3.73\ m/s^2$

$R = \frac{1}{3} 600(3.73) = 746\ N$

$\overline{AC} = \sqrt{(550)^2 + (300)^2 + (350)^2} = 718\ mm$

$\overline{CD} = \sqrt{(450)^2 + (1200)^2} = 1282\ mm$

$\underline{F}_{AC} = \frac{F_{AC}}{718}(-550\underline{i} + 300\underline{j} + 350\underline{k})$

$\underline{F}_{CB} = \frac{F_{CB}}{718}(-550\underline{i} - 300\underline{j} + 350\underline{k})$

$\underline{F}_{CD} = \frac{F_{CD}}{1282}(450\underline{i} - 1200\underline{k})$

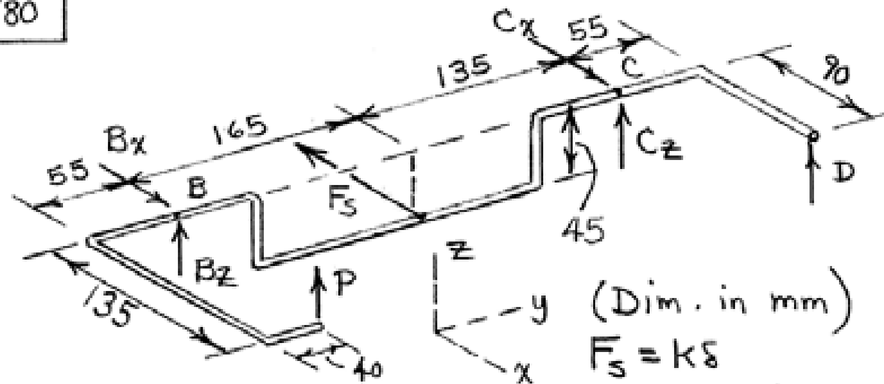
$\Sigma \underline{F} = \underline{0}$ with $F_{CB} = F_{AC}$ gives upon collecting terms

$$\left(-\frac{2F_{AC}}{718}550 + \frac{F_{CD}}{1282}450\right)\underline{i} + \left(746 + \frac{2F_{AC}}{718}350 - \frac{F_{CD}}{1282}1200\right)\underline{k} = \underline{0}$$

Equate coefficients to zero & solve simultaneously to get $F_{CD} = \underline{1046\ N}$ compression

$F_{AC} = F_{CB} = \underline{240\ N\ \text{tension}}$

3/80



$F_s = k\delta$

$= 900(0.060) = 54$ N

D=0 :

$\Sigma M_{BC} = 0: \; -P(135) + 54(45) = 0, \quad \underline{P_{min} = 18 \text{ N}}$

$\Sigma M_{B_x} = 0: \; -18(15) + C_z(300) = 0, \quad C_z = 0.9 \text{ N}$

$\Sigma M_{B_z} = 0: \; 54(165) - C_x(300) = 0, \quad C_x = 29.7 \text{ N}$

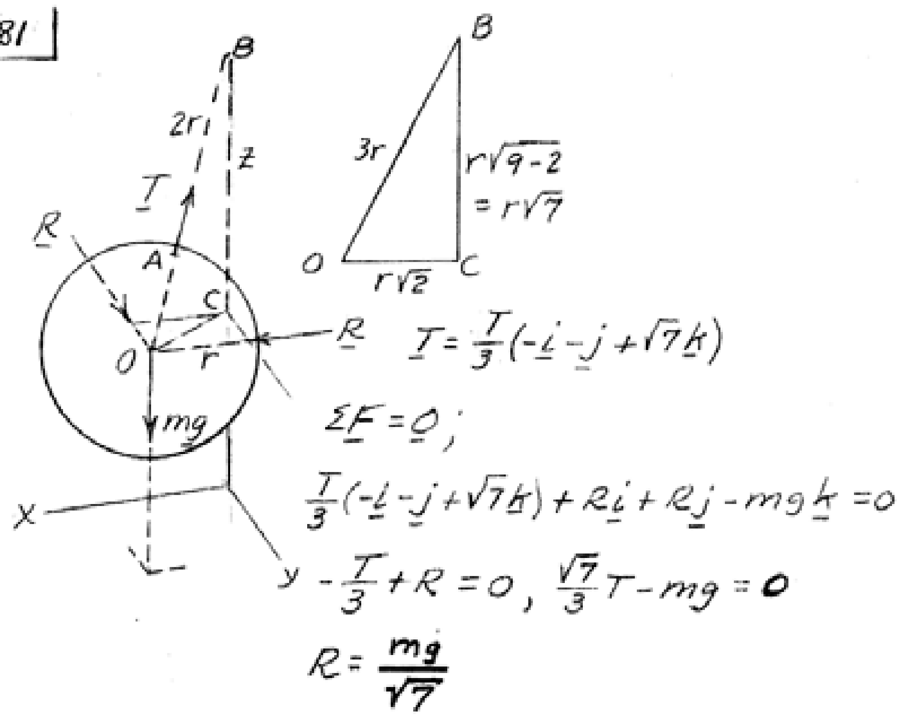
$\Sigma F_x = 0: \; 29.7 + B_x - 54 = 0, \quad B_x = 24.3 \text{ N}$

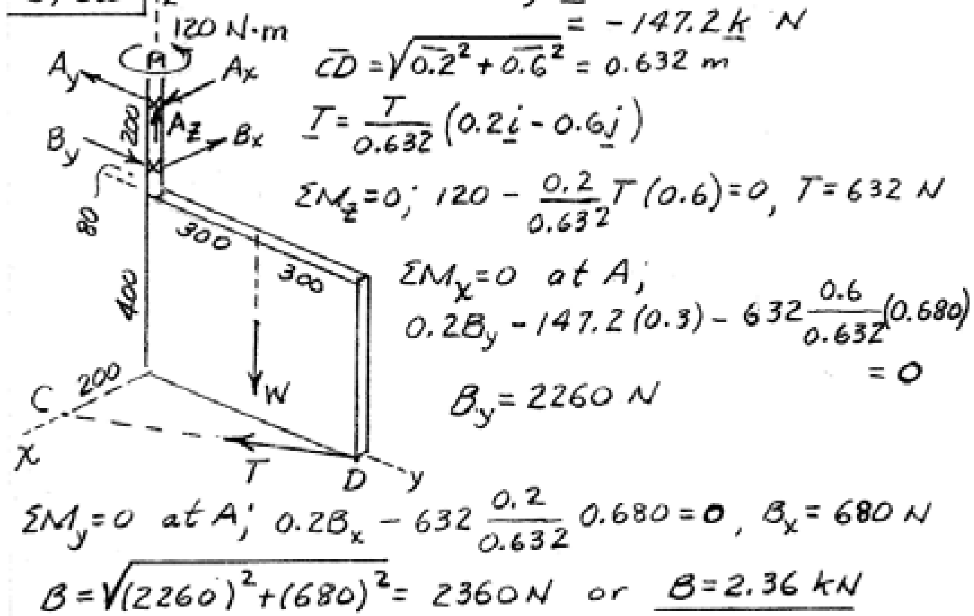
$\Sigma F_z = 0: \; 0.9 + B_z + 18 = 0, \quad B_z = -18.9 \text{ N}$

$B = \sqrt{B_x^2 + B_z^2} = \underline{30.8 \text{ N}}, \; C = \sqrt{C_x^2 + C_z^2} = \underline{29.7 \text{ N}}$

If $P = P_{min}/2 = 18/2 = 9$ N, $(D \neq 0)$:

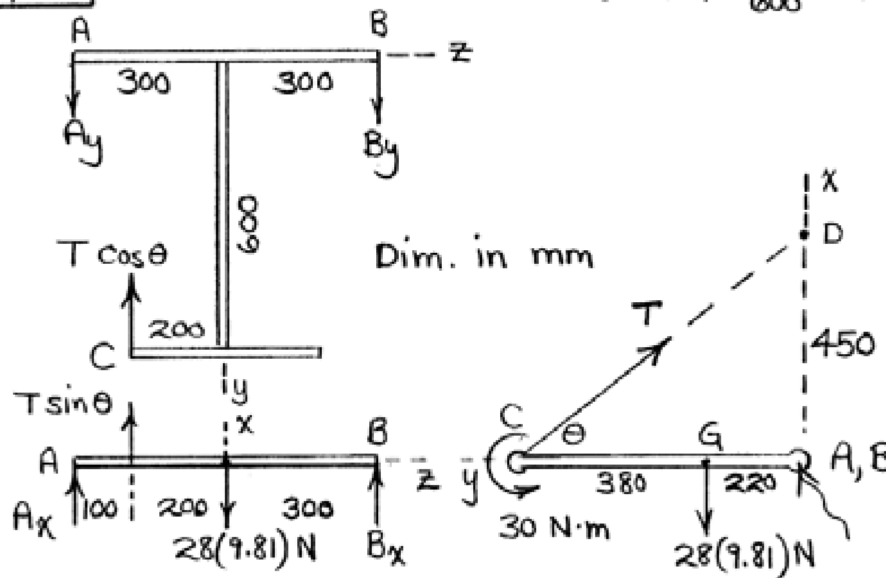
$\Sigma M_{BC} = 0: \; -9(135) + 54(45) - D(90) = 0, \; \underline{D = 13.5 \text{ N}}$


3/81
B
2r
z
T
R
A
C
O
r
R
mg
x
y
B
3r
$r\sqrt{9-2}$
$= r\sqrt{7}$
O
C
$r\sqrt{2}$
$\underline{T} = \frac{T}{3}(-\underline{i} - \underline{j} + \sqrt{7}\underline{k})$
$\Sigma \underline{F} = \underline{0};$
$\frac{T}{3}(-\underline{i} - \underline{j} + \sqrt{7}\underline{k}) + R\underline{i} + R\underline{j} - mg\underline{k} = 0$
$-\frac{T}{3} + R = 0, \quad \frac{\sqrt{7}}{3}T - mg = 0$
$R = \frac{mg}{\sqrt{7}}$


3/82
Dimensions in mm; W = 15(9.81)(-k) N
= -147.2k N
CD = √(0.2² + 0.6²) = 0.632 m
T = T/0.632 (0.2i - 0.6j)
ΣMz = 0; 120 - (0.2/0.632) T (0.6) = 0, T = 632 N
ΣMx = 0 at A;
0.2By - 147.2(0.3) - 632 (0.6/0.632)(0.680) = 0
By = 2260 N
ΣMy = 0 at A; 0.2Bx - 632 (0.2/0.632) 0.680 = 0, Bx = 680 N
B = √((2260)² + (680)²) = 2360 N or B = 2.36 kN
120 N·m
A
Ay
Ax
Az
By
Bx
z
200
80
300
300
400
200
W
C
x
T
D
y

3/83

$\theta = \tan^{-1}\frac{450}{600} = 36.9°$



(x-y) $\Sigma M_z = 0: 28(9.81)(0.220) - T\sin 36.9°(0.600) + 30 = 0, \quad T = 251\text{ N}$

(x-z) $\Sigma M_B = 0: 28(9.81)(0.300) - 251\sin 36.9°(0.500) - 0.600A_x = 0, \quad A_x = 11.74\text{ N}$

$\Sigma F_x = 0: 11.74 + 251\sin 36.9° - 28(9.81) + B_x = 0, \quad B_x = 112.2\text{ N}$

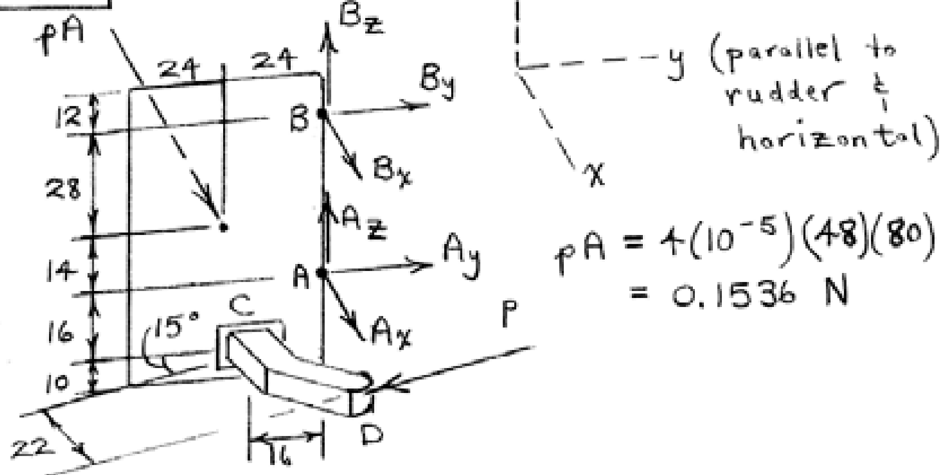
(y-z) $\Sigma M_B = 0: 251\cos 36.9°(0.500) - 0.6A_y = 0, \quad A_y = 167.5\text{ N}$

$\Sigma F_y = 0: 167.5 + B_y - 251\cos 36.9° = 0, \quad B_y = 33.5\text{ N}$

$A = \sqrt{11.74^2 + 167.5^2} = \underline{167.9\text{ N}}$

$B = \sqrt{112.2^2 + 33.5^2} = \underline{117.1\text{ N}}$

Couple may be applied at any place on rigid body with the same external effect.



$pA = 4(10^{-5})(48)(80)$
$= 0.1536$ N

$\Sigma M_{AB} = 0: \; -P(22 - 16 \sin 15°) + 0.1536(24) = 0$

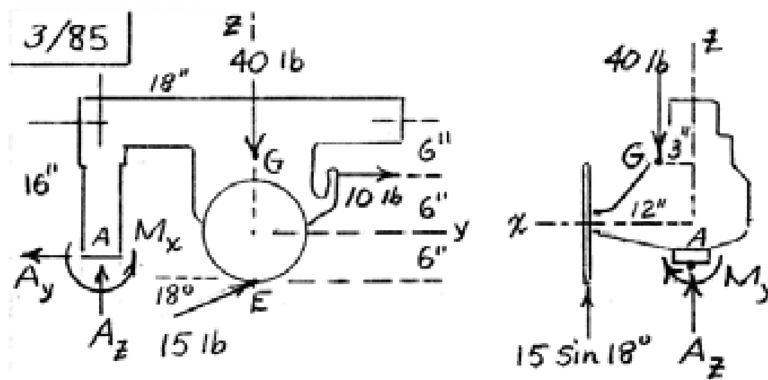
$\underline{P = 0.206 \text{ N}}$

$\Sigma M_{B_x} = 0: \; A_y(42) - 0.206 \cos 15°(58) = 0$

$\underline{A_y = 0.275 \text{ N}}$

$\Sigma M_{A_x} = 0: \; -B_y(42) - 0.206 \cos 15°(16) = 0$

$\underline{B_y = -0.0760 \text{ N}}$



y-z; $\Sigma M_A = 0$; $40(18) + 10(16-6) - 15\sin 18°(18) - 15\cos 18°(2) - M_x = 0$

$M_x = 708$ lb-in.

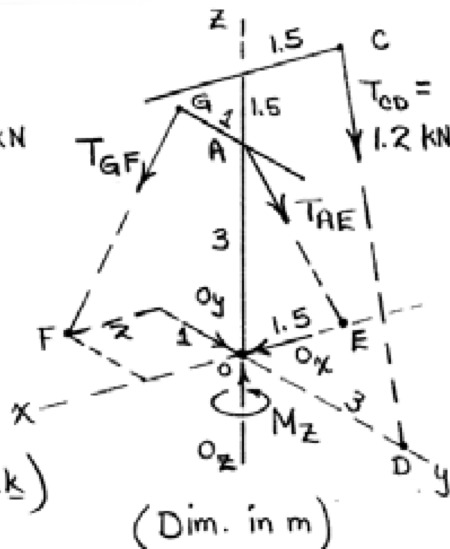
x-z; $\Sigma M_A = 0$; $15\sin 18°(12) + M_y - 40(3) = 0$

$M_y = 64.4$ lb-in.

$$M = \sqrt{M_x^2 + M_y^2} = \sqrt{(708)^2 + (64.4)^2} = \underline{711 \text{ lb-in.}}$$

Acceleration of mass center negligible, so equilibrium equations may be used.

3/86 From Prob. 2/95,



$\underline{T}_{CD} = 0.321\underline{i} + 0.641\underline{j} - 0.962\underline{k}$ kN

$\underline{T}_{AE} = T_{AE} \dfrac{-1.5\underline{i} - 3\underline{k}}{\sqrt{1.5^2 + 3^2}}$

$= T_{AE}\left(-0.447\underline{i} - 0.894\underline{k}\right)$

$\underline{T}_{GF} = T_{GF} \dfrac{2\underline{i} - 3\underline{k}}{\sqrt{2^2 + 3^2}}$

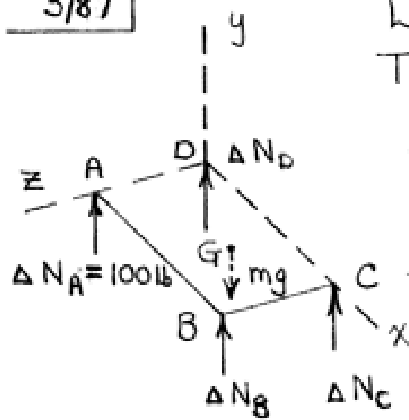
$= T_{GF}\left(0.555\underline{i} - 0.832\underline{k}\right)$

$\Sigma \underline{M}_o = \underline{0}: \quad \underline{OC} \times \underline{T}_{CD} + \underline{OA} \times \underline{T}_{AE} + \underline{OG} \times \underline{T}_{GF} + M_z\underline{k} = \underline{0}$

$(-1.5\underline{i} + 4.5\underline{k}) \times (0.321\underline{i} + 0.641\underline{j} - 0.962\underline{k}) + 3\underline{k} \times$
$T_{AE}(-0.447\underline{i} - 0.894\underline{k}) + (-\underline{j} + 3\underline{k}) \times T_{GF}(0.555\underline{i} - 0.832\underline{k})$
$+ M_z\underline{k} = \underline{0} \Rightarrow \begin{cases} 0.832T_{GF} - 2.89 = 0 \\ 1.664\, T_{GF} - 1.342T_{AE} = 0 \\ 0.555\, T_{GF} - 0.962 + M_z = 0 \end{cases}$

Solve to obtain $\underline{T_{GF} = 3.47 \text{ kN}}$, $\underline{T_{AE} = 4.30 \text{ kN}}$,
$\underline{M_z = -0.962 \text{ kN}\cdot\text{m}}$

3/87



Location of G does not change. Thus,

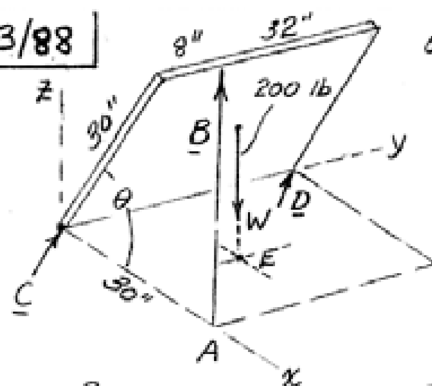
$\underline{\Delta N_D = -100 \text{ lb}}$ (preserves total rear-axle loading)

$\underline{\Delta N_B = -100 \text{ lb}}$ (preserves total right-side loading)

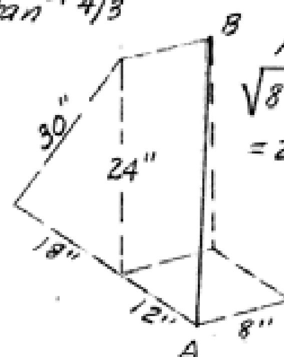
$\underline{\Delta N_C = 100 \text{ lb}}$ (preserves total normal force; preserves total front-axle loading)

(Note: The results for ΔN_B & ΔN_C hold only if the track (distance between tire centers) at the front is equal to that at the rear.)

3/88



$\theta = \tan^{-1} 4/3$



$\overline{AB} = \sqrt{8^2 + 12^2 + 24^2} = 28 \text{ in.}$

$\underline{B} = \frac{B}{28}(-12\underline{i} + 8\underline{j} + 24\underline{k}) = \frac{B}{7}(-3\underline{i} + 2\underline{j} + 6\underline{k})$

$\underline{W} = -200\underline{k}$ lb

$\Sigma M_{CD} = 0$; $(\underline{r}_{CA} \times \underline{B} + \underline{r}_{CE} \times \underline{W}) \cdot \underline{j} = 0$; moment of $\underline{D} = 0$

$$\left[30\underline{i} \times \frac{B}{7}(-3\underline{i} + 2\underline{j} + 6\underline{k}) + (9\underline{i} + 20\underline{j}) \times (-200\underline{k})\right] \cdot \underline{j} = 0$$

$$(-4000\underline{i} + 180(10 - B/7)\underline{j} + 60B/7\,\underline{k}) \cdot \underline{j} = 0$$

$B/7 - 10 = 0$, $\underline{B = F_B = 70 \text{ lb}}$

$\Sigma M_x = 0$; $40D_z - 200(20) = 0$, $D_z = 100$ lb

$\Sigma M_z = 0$; $30B_y - 40D_x = 0$, $D_x = \frac{3}{4}20 = 15$ lb

$D_{normal} = D_n = \sqrt{100^2 + 15^2} = \underline{101.1 \text{ lb}}$

3/89

$\overline{AC} = \overline{AD} = \sqrt{4^2+5^2+10^2} = 11.87$ m ; $|\underline{P}| = 20$ kN

$\underline{T}_1 = \frac{T_1}{11.87}(-4\underline{i}-5\underline{j}-10\underline{k})$; $\sin 20° = 0.3420$

$\cos 20° = 0.9397$

$\underline{T}_2 = \frac{T_2}{11.87}(4\underline{i}-5\underline{j}-10\underline{k})$

$\Sigma M_x = 0$; $\underline{r}_{EC} \times \underline{T}_2 \cdot \underline{i} + \underline{r}_{FD} \times \underline{T}_1 \cdot \underline{i} + \underline{r}_{BM} \times \underline{P} \cdot \underline{i} = 0$

$-2\underline{j} \times \frac{T_2}{11.87}(4\underline{i}-5\underline{j}-10\underline{k}) \cdot \underline{i}$

$-2\underline{j} \times \frac{T_1}{11.87}(-4\underline{i}-5\underline{j}-10\underline{k}) \cdot \underline{i} + (1.5\underline{j}+5\underline{k}) \times 20(0.342\underline{i} + 0.9397\underline{j}) \cdot \underline{i} = 0$

Simplify & get $T_1 + T_2 = 55.79$ ---- (1)

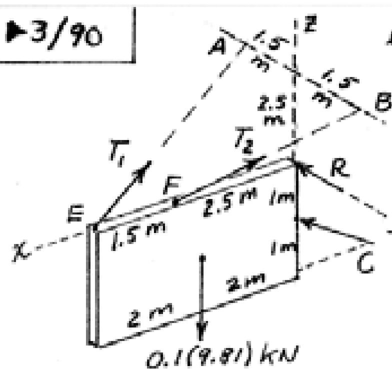
$\Sigma M_z = 0$; $\underline{r}_{BA} \times (\underline{T}_1 + \underline{T}_2) \cdot \underline{k} + \underline{r}_{BM} \times \underline{P} \cdot \underline{k} = 0$

$(3\underline{j}+10\underline{k}) \times \frac{1}{11.87}([-4T_1+4T_2]\underline{i} + [-5T_1-5T_2]\underline{j} - [10T_1+10T_2]\underline{k}) \cdot \underline{k} - 20(0.3420)(1.5) = 0$

Simplify to $T_1 - T_2 = 10.14$ ---- (2)

Solve (1) & (2) & get $\underline{T_1 = 33.0 \text{ kN}}$, $\underline{T_2 = 22.8 \text{ kN}}$

▸3/90



$l_1 = \overline{AE} = \sqrt{4^2 + 1.5^2 + 2.5^2} = 4.95$ m

$l_2 = \overline{BF} = \sqrt{2.5^2 + 1.5^2 + 2.5^2} = 3.84$ m

$\underline{T}_1 = \frac{T_1}{l_1}(-4\underline{i} - 1.5\underline{j} + 2.5\underline{k})$

$\underline{T}_2 = \frac{T_2}{l_2}(-2.5\underline{i} + 1.5\underline{j} + 2.5\underline{k})$

Note: $\underline{T}_1$, $\underline{T}_2$, $\underline{R}$, and weight all pass through x-axis, so $\underline{C}$ must also. Thus $C_y = 0$

$\Sigma M_{AB} = 0$; $C_x(3.5) - 9.81(2) = 0$, $C_x = 0.561$ kN

$\Sigma M_z = 0$; $4\underline{i} \times \frac{T_1}{l_1}(-1.5\underline{j}) + 2.5\underline{i} \times \frac{T_2}{l_2}(1.5\underline{j}) = \underline{0}$, $8T_1/l_1 = 5T_2/l_2$

$\Sigma F_x = 0$; $-\frac{T_1}{l_1}(4) - \frac{T_2}{l_2}(2.5) + 0.561 = 0$, $8T_1/l_1 + 5T_2/l_2 = 1.121$ kN

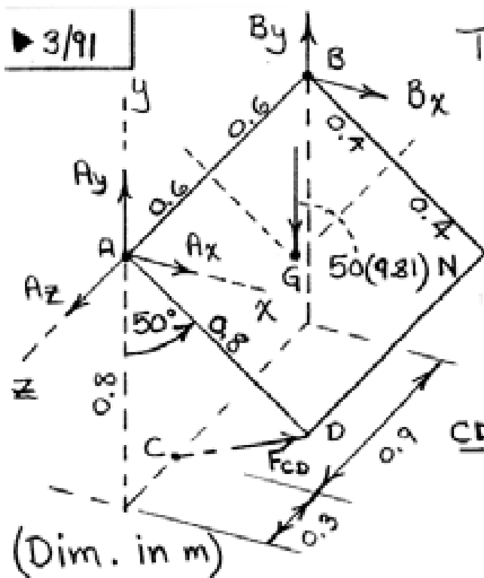
solve & get $T_1 = 1.121\, l_1/16 = \underline{0.347 \text{ kN}}$, $T_2 = \frac{1.121}{10} l_2 = \underline{0.431 \text{ kN}}$

$\Sigma F_y = 0$; $\frac{1.121}{16} l_1 \frac{1.5}{l_1} - \frac{1.121}{10} l_2 \frac{1.5}{l_2} + R = 0$, $\underline{R = 0.0631 \text{ kN}}$

$\Sigma F_z = 0$; $\frac{1.121}{16} l_1 \frac{2.5}{l_1} + \frac{1.121}{10} l_2 \frac{2.5}{l_2} + C_z - 0.981 = 0$, $C_z = 0.526$ kN

Thus $C = \sqrt{(0.561)^2 + (0.526)^2} = \underline{\mathbf{0.768 \text{ kN}}}$

▶ 3/91



(Dim. in m)

The coordinates of point D are $(0.8 \sin 50°, -0.8 \cos 50°, 0)$
$= (0.613, -0.514, 0)$

So $\underline{F}_{CD}$ can be written as $\underline{F}_{CD} = F_{CD}\, \underline{n}_{CD}$,

where $\underline{n}_{CD} = \dfrac{\underline{CD}}{CD}$

$\underline{CD} = 0.613\underline{i} + (0.8 - 0.514)\underline{j} + 0.3\underline{k}$

$= 0.613\underline{i} + 0.286\underline{j} + 0.3\underline{k}$ m

So $\underline{F}_{CD} = F_{CD}\left[0.828\underline{i} + 0.386\underline{j} + 0.406\underline{k}\right]$

$\Sigma F_x = 0: \quad A_x + B_x + 0.828 F_{CD} = 0$ (1)

$\Sigma F_y = 0: \quad A_y + B_y + 0.386 F_{CD} - 50(9.81) = 0$ (2)

$\Sigma F_z = 0: \quad A_z + 0.406 F_{CD} = 0$ (3)

Now sum moments about x, y, z axes through A (note change in origin from text illustration):

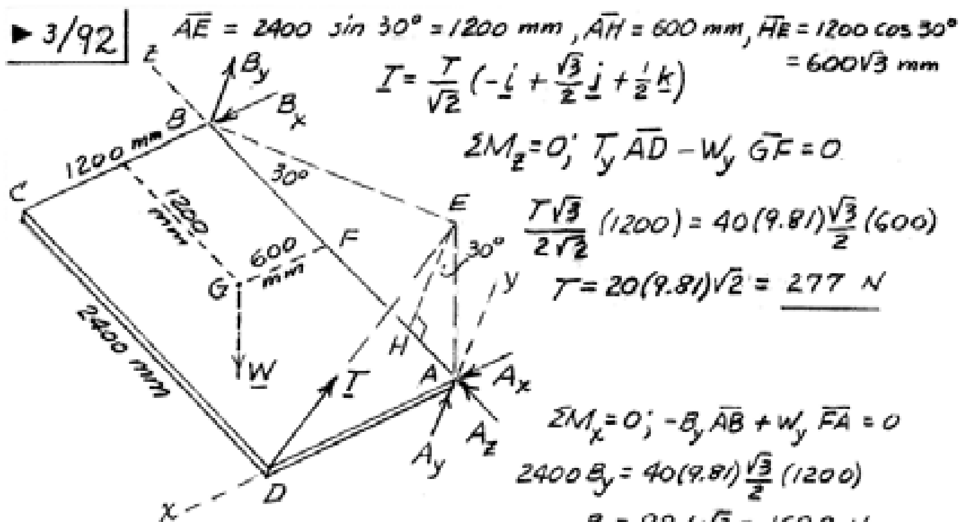
$\Sigma M_{A_x} = 0: \; B_y(1.2) - 50(9.81)(0.6) - 0.406 F_{CD}(0.8)$
$+ 0.386 F_{CD}(0.3) = 0$ (4)

$\Sigma M_{A_y} = 0: \; -B_x(1.2) - 0.828 F_{CD}(0.3) = 0$ (5)

$\Sigma M_{A_z} = 0: \; -50(9.81)(0.4 \sin 50°) + 0.828 F_{CD}(0.8) = 0$ (6)

(note: $\underline{F}_{CD}$ considered as acting at C)

Solution: $A_x = -140.9$ N $\qquad B_x = -47.0$ N
$A_y = 118.2$ N $\qquad B_y = 285$ N
$A_z = -92.0$ N $\qquad F_{CD} = 227$ N



$\overline{AE} = 2400 \sin 30° = 1200$ mm, $\overline{AH} = 600$ mm, $\overline{HE} = 1200 \cos 30° = 600\sqrt{3}$ mm

$$\underline{T} = \frac{T}{\sqrt{2}}\left(-\underline{i} + \frac{\sqrt{3}}{2}\underline{j} + \frac{1}{2}\underline{k}\right)$$

$\Sigma M_z = 0;\ T_y \overline{AD} - W_y \overline{GF} = 0$

$$\frac{T\sqrt{3}}{2\sqrt{2}}(1200) = 40(9.81)\frac{\sqrt{3}}{2}(600)$$

$$T = 20(9.81)\sqrt{2} = \underline{277\ N}$$

$\Sigma M_x = 0;\ -B_y \overline{AB} + W_y \overline{FA} = 0$

$$2400 B_y = 40(9.81)\frac{\sqrt{3}}{2}(1200)$$

$$B_y = 98.1\sqrt{3} = 169.9\ N$$

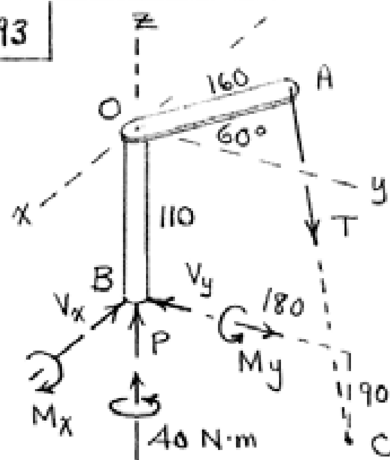
$\Sigma M_y = 0;\ B_x \overline{AB} + W_z \overline{GF} - T_z \overline{AD} = 0$

$$2400 B_x + 40(9.81)0.5(600) - \frac{277}{2\sqrt{2}}(1200) = 0$$

$$B_x = 0$$

$B = B_y = \underline{169.9\ N}$ ($B_x = 0$ can be obtained by inspection by noting $\Sigma M_{AE} = 0$ eliminates all terms except $B_x \overline{BE}$ so $B_x = 0$)

▸3/93



$(\overline{AC})_x = 0.16 \sin 60° = 0.1386$ m

$(\overline{AC})_y = (0.18 - 0.16 \cos 60°)$

$= 0.10$ m

$(\overline{AC})_z = 0.2$ m

$\overline{AC} = \sqrt{0.1386^2 + 0.1^2 + 0.2^2}$

$= 0.263$

$\overrightarrow{BC} = 0.18\underline{j} - 0.090\underline{k}$ m

$$\underline{T} = \frac{T}{0.263}\left(0.1386\,\underline{i} + 0.1\underline{j} - 0.2\underline{k}\right)$$

$\Sigma \underline{M}_B = \underline{0}: \quad \overrightarrow{BC} \times \underline{T} - M_x \underline{i} + M_y \underline{j} + 40\,\underline{k} = \underline{0}$

$\Rightarrow (-0.1026T - M_x)\underline{i} + (-0.0474T + M_y)\underline{j} + (-0.0948T + 40)\underline{k} = 0$

$\Rightarrow T = 422$ N, $M_x = -43.3$ N·m, $M_y = 20.0$ N·m

Bending moment $M = \sqrt{M_x^2 + M_y^2} = \underline{47.7 \text{ N·m}}$

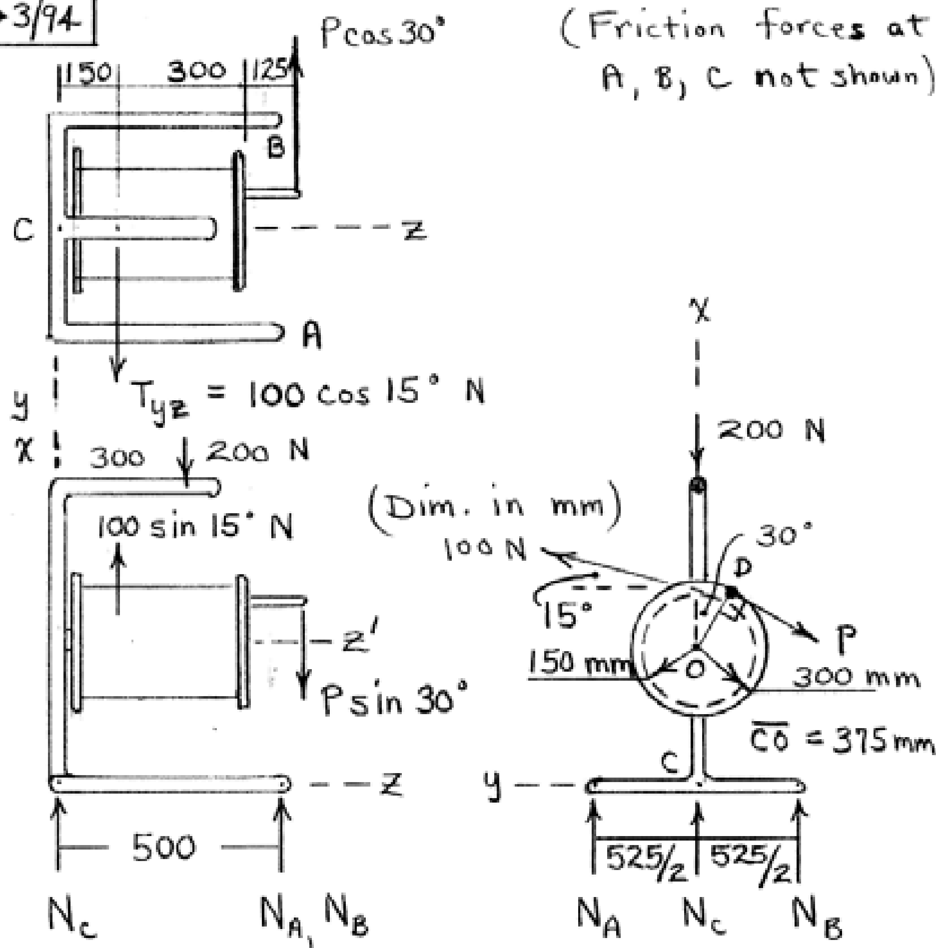
$\Sigma F_x = 0: \quad 422\,\frac{0.1386}{0.263} - V_x = 0, \quad V_x = 222$ N

$\Sigma F_y = 0: \quad 422\,\frac{0.1}{0.263} - V_y = 0, \quad V_y = 160$ N

$V = \sqrt{V_x^2 + V_y^2} = \underline{274 \text{ N}}$

$\Sigma F_z = 0: \quad -422\left(\frac{0.2}{0.263}\right) + P = 0, \quad \underline{P = 320 \text{ N}}$

▶3/94



$\Sigma M_{z'} = 0$ for reel alone:

$100\,(150) - P\,(300) = 0,\qquad \underline{P = 50\text{ N}}$

$\Sigma M_{AB} = 0:\ N_C\,(500) - 200\,(200) + 100 \sin 15°(350)$

$+ 50 \sin 30°\,(75) = 0,\quad \underline{N_C = 58.1\text{ N}}$

$\Sigma F_x = 0:\ N_A + N_B + N_C - 200 - 50 \sin 30°$

$+ 100 \sin 15° = 0 \qquad (1)$

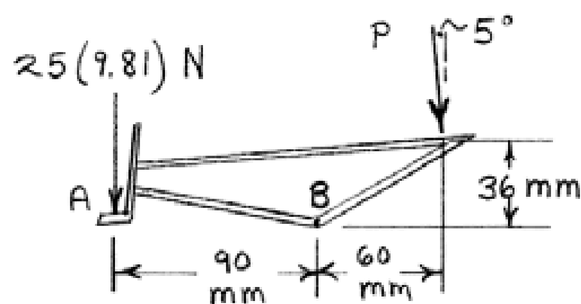
$\Sigma M_{C_z} = 0:\ (N_B - N_A)\dfrac{525}{2} + 100\,(150) - 50\,(300)$

$+ 100 \cos 15°\,(375) - 50 \cos 30°\,(375) = 0 \quad (2)$

Solve Eqs. (1) & (2) to obtain

$\begin{cases} \underline{N_A = 108.6\text{ N}} \\ \underline{N_B = 32.4\text{ N}} \end{cases}$

3/95

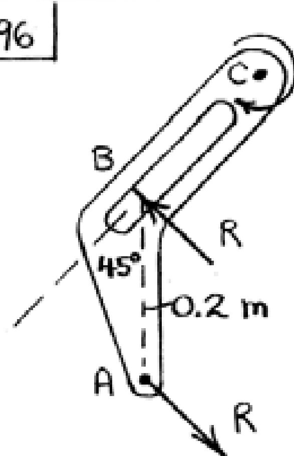


$\circlearrowleft + \sum M_B = 0: \; 25(9.81)(90) - P\cos 5^\circ (60)$
$- P\sin 5^\circ (36) = 0$

$P = 351 \text{ N}$

Assumptions: The weight of the panel acts at the 90-mm dimension as shown above; panel does not slip.

3/96



Forces at A and B must constitute a couple.

$\circlearrowleft + \Sigma M = 0: \ 80 - R(0.2\cos 45°) = 0$

$\underline{R = 566 \text{ N}}$

3/97 Isolate wheel of unicycle:

50(9.81) N

T T

α α

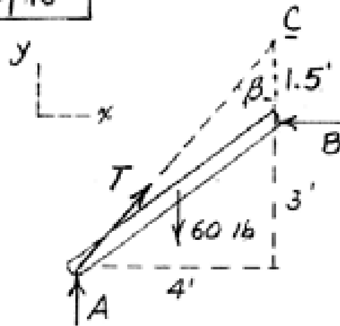
$$\alpha = \tan^{-1}\left(\frac{0.075}{9}\right) = 0.477^\circ$$

$$+\uparrow \Sigma F = 0: \quad 2T\sin\alpha - 50(9.81) = 0$$

$$T = 29\,400 \text{ N}$$

or $\underline{T = 29.4 \text{ kN}}$

3/98



$\Sigma M_A = 0;\ 60(2) - 3B = 0$

$\underline{B = 40 \text{ lb}}$

$\Sigma F_x = 0;\ T\sin\beta - 40 = 0$

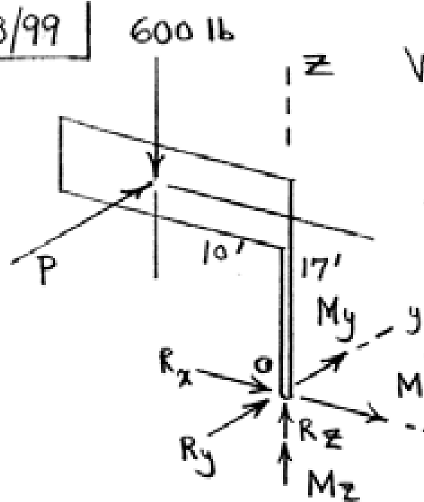
$T = 40\dfrac{\sqrt{4^2 + 4.5^2}}{4} = \underline{60.2 \text{ lb}}$

$\Sigma F_y = 0;\ T\cos\beta + A - 60 = 0$

$A = 60 - 60.2\dfrac{4.5}{\sqrt{4^2 + 4.5^2}}$

$\underline{A = 15 \text{ lb}}$

3/99



Wind force $P = pA$

$= 17.5(6)(12) = 1260$ lb

$\Sigma F_x = 0 \Rightarrow R_x = 0$

$\Sigma F_y = 0 \Rightarrow R_y = -1260$ lb

$\Sigma F_z = 0 \Rightarrow R_z = 600$ lb

$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$

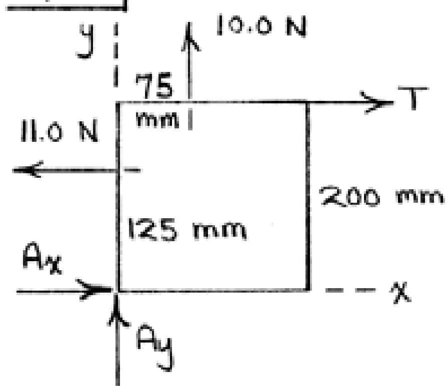
$= 1396$ lb

$\Sigma \underline{M}_o = \underline{0}: \ \underline{M} + (-10\underline{i} + 17\underline{k}) \times (1260\underline{j} - 600\underline{k}) = \underline{0}$

$\Rightarrow \underline{M} = 21{,}400\,\underline{i} + 6000\,\underline{j} + 12{,}600\,\underline{k}$ lb-ft

$M = 25{,}600$ lb-ft

3/100

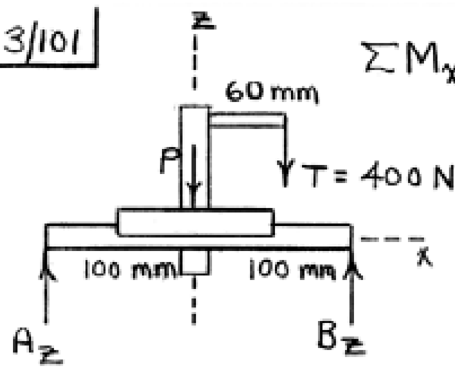


Isolate entire system of plate, pulleys, & tape.

$\overset{+}{\circlearrowleft} \sum M_A = 0: \quad T(200) - 10.0(75) - 11.0(125) = 0$

$\underline{T = 10.62 \text{ N}}$

3/101



$\Sigma M_x = 0: \; P(200) - 400(120 \cos 30°) = 0$

$P = 208 \text{ N}$

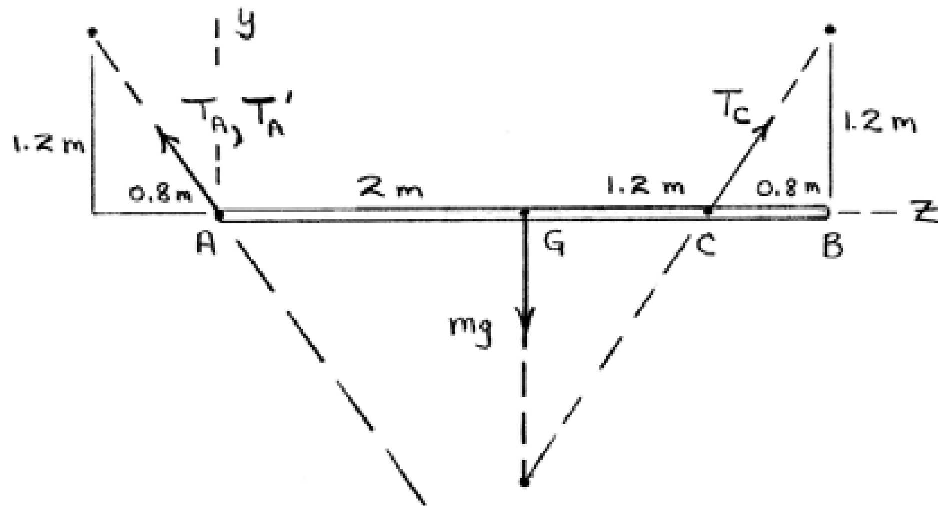
$\Sigma M_{A_y} = 0: \; 208(100) + 400(160) - B_z(200) = 0$

$B_z = 424 \text{ N}$

$\Sigma F_z = 0: \; A_z + 424 - 208 - 400 = 0, \; A_z = 183.9 \text{ N}$

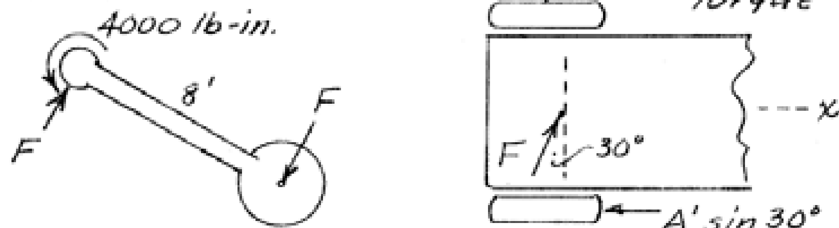
Because $A_y = B_y = 0$, $\underline{A = A_z = 183.9 \text{ N}}$, $\underline{B = B_z = 424 \text{ N}}$

3/102 y-z view :



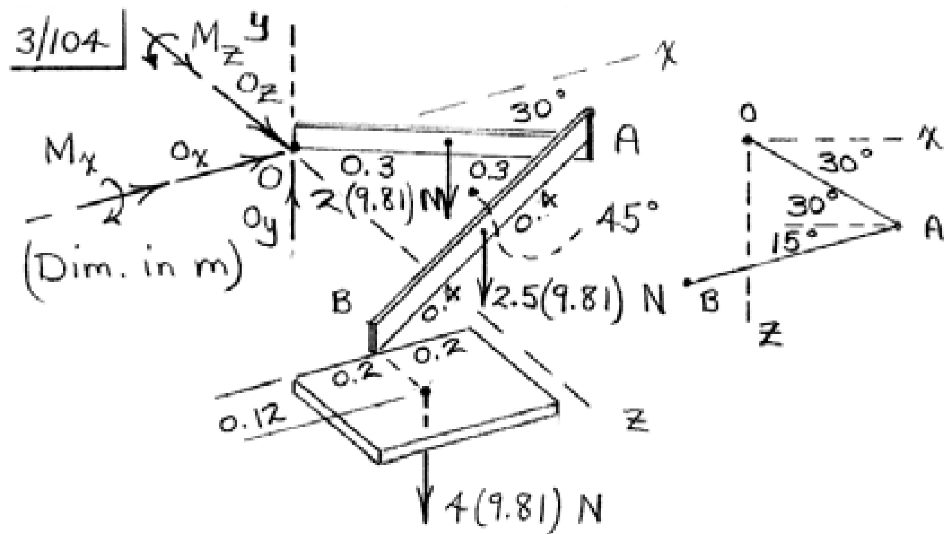
Forces are not concurrent in the y-z plane. Therefore the indicated position is not one of static equilibrium.

3/103 Torque on auger is opposite to applied torque



Arm: $\Sigma M = 0$; $8(12)F - 4000 = 0$, $F = 41.7$ lb

Truck: $\Sigma F_x = 0$; $A' \sin 30° - 41.7 \sin 30°$, $\underline{A' = 41.7 \text{ lb}}$



$\Sigma F_x = 0: \quad \underline{O_x = 0} \qquad \Sigma F_z = 0: \quad \underline{O_z = 0}$

$\Sigma F_y = 0: \quad O_y - 8.5(9.81) = 0, \qquad \underline{O_y = 83.4\ \text{N}}$

$\Sigma M_{O_x} = 0: \quad M_x + 2(9.81)(0.3 \sin 30°) + 2.5(9.81) \times (0.6 \sin 30° + 0.4 \sin 15°) + 4(9.81)(0.6 \sin 30° + 0.8 \sin 15° + 0.12) = 0, \qquad \underline{M_x = -37.4\ \text{N}\cdot\text{m}}$

$\Sigma M_{O_z} = 0: \quad M_z - 2(9.81)(0.3 \cos 30°) - 2.5(9.81) \times (0.6 \cos 30° - 0.4 \cos 15°) - 4(9.81)(0.6 \cos 30° - 0.8 \cos 15°) = 0, \qquad \underline{M_z = -1.567\ \text{N}\cdot\text{m}}$

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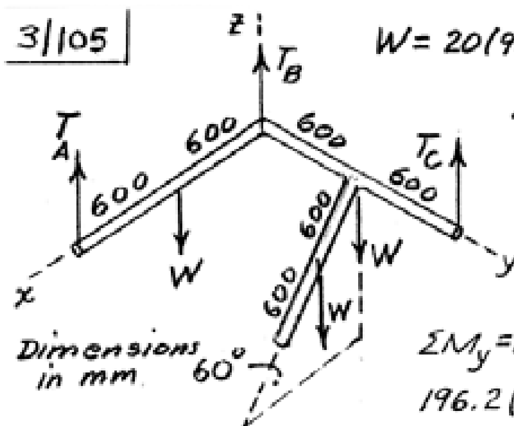


$W = 20(9.81) = 196.2$ N

$\Sigma M_x = 0;$

$1200T_C - 2(196.2)600 = 0$

$\underline{T_C = 196.2 \text{ N}}$

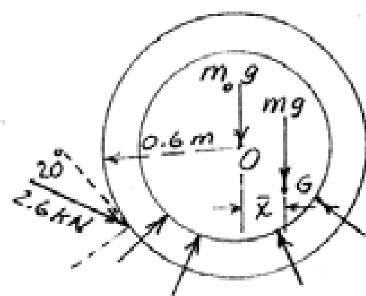
$\Sigma M_y = 0;$

$196.2(600 + 600\cos 60°) - 1200T_A = 0$

$\underline{T_A = 147.2 \text{ N}}$

$\Sigma F_z = 0;\ T_B + 147.2 + 196.2 - 3(196.2) = 0,\ \underline{T_B = 245 \text{ N}}$

3/106



$mg = 750(9.81)(10^{-3}) = 7.358$ kN

$\Sigma M_O = 0$; $2.6 \cos 20° (600)$

$- 7.358\,\bar{x} = 0$

$\bar{x} = 199.2$ mm

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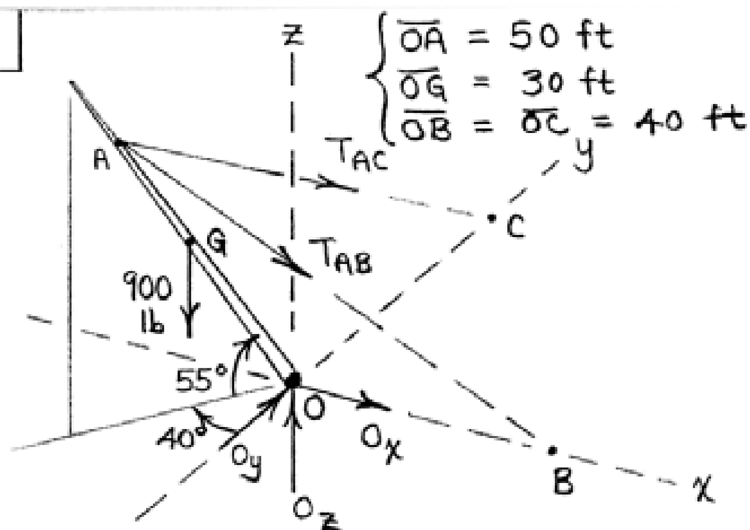

Coordinates of A: $50(-\cos 55^\circ \sin 40^\circ, -\cos 55^\circ \cos 40^\circ, \sin 55^\circ) = (-18.43, -22.0, 41.0)$ ft

Coordinates of G: $30(-\cos 55^\circ \sin 40^\circ, -\cos 55^\circ \cos 40^\circ, \sin 55^\circ) = (-11.06, -13.18, 24.6)$ ft

$$\underline{T}_{AB} = T_{AB}\left[\frac{(18.43+40)\underline{i} + 22.0\underline{j} - 41.0\underline{k}}{\sqrt{(18.43+40)^2 + 22.0^2 + 41.0^2}}\right]$$

$$= T_{AB}\left[0.783\underline{i} + 0.294\underline{j} - 0.549\underline{k}\right]$$

$$T_{AC} = T_{AC}\left[\frac{18.43\underline{i} + (22.0+40)\underline{j} - 41.0\underline{k}}{\sqrt{18.43^2 + (22.0+40)^2 + 41.0^2}}\right]$$

$$= T_{AC}\left[0.241\underline{i} + 0.810\underline{j} - 0.535\underline{k}\right]$$

$\Sigma F_x = 0: \quad 0.783 T_{AB} + 0.241 T_{AC} + O_x = 0$ (1)

$\Sigma F_y = 0: \quad 0.294 T_{AB} + 0.810 T_{AC} + O_y = 0$ (2)

$\Sigma F_z = 0: \quad -0.549 T_{AB} - 0.535 T_{AC} + O_z - 900 = 0$ (3)

$\Sigma M_{BC} = 0: \quad \Sigma \underline{M}_B \cdot \underline{n}_{BC} = 0:$

$$\left\{-40\underline{i} \times O_z\underline{k} + \left[(-40 - 11.06)\underline{i} - 13.18\underline{j}\right] \times \left[-900\underline{k}\right]\right\} \cdot \left(-\frac{\sqrt{2}}{2}\underline{i} + \frac{\sqrt{2}}{2}\underline{j}\right) = 0$$

or $-(40 O_z - 46{,}000)\frac{\sqrt{2}}{2} + 11{,}860 \frac{\sqrt{2}}{2} = 0$ (4)

$\Sigma M_{O_y} = 0: \quad 0.549 T_{AB}(40) - 900(11.06) = 0$ (5)

Solve Eqs. (1)–(5) in reverse order to obtain

$O_x = -489$ lb $\qquad T_{AB} = 454$ lb

$O_y = -582$ lb $\qquad T_{AC} = 554$ lb

$O_z = 1445$ lb

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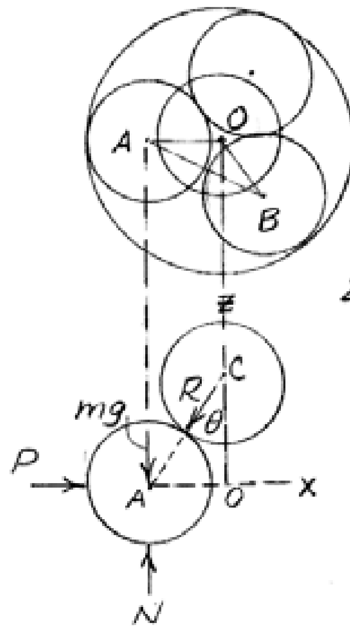


∠s OAB & ABO are 30°

also $\overline{AB} = 2r$ where

r = radius of sphere. Thus

$\overline{AO} = r/\cos 30° = 2r/\sqrt{3}$

$\overline{AC} = 2r$

so $\overline{OC} = r\sqrt{2^2 - (2/\sqrt{3})^2} = 2r\sqrt{2/3}$

Equil. of top ball

$\Sigma F_z = 0$; $3R\cos\theta - mg = 0$

$$3R\,\frac{2r\sqrt{2/3}}{2r} = mg$$

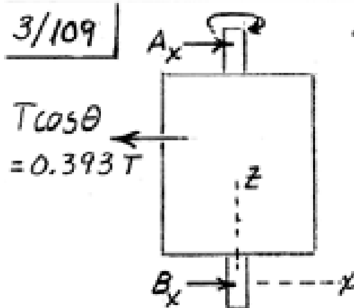
$$R = mg/\sqrt{6}$$

Lower ball

$\Sigma F_x = 0$; $P - R\sin\theta = 0$

$$P = \frac{mg}{\sqrt{6}}\,\frac{2r/\sqrt{3}}{2r} = \frac{mg}{3\sqrt{2}}$$

3/109



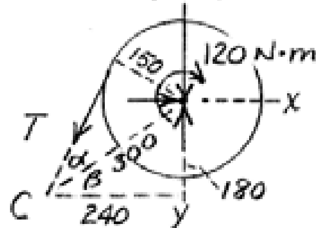
$\Sigma M_z = 0$; $120 - 0.150T = 0$, $T = 800$ N

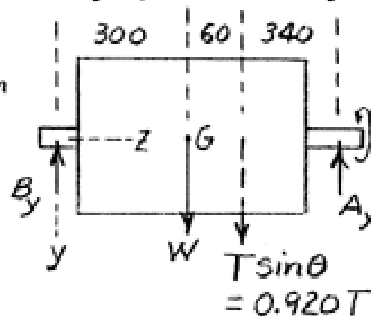
x-z; $\Sigma M_B = 0$; $0.393(800)(0.360) - 0.700A_x = 0$; $A_x = 161.6$ N

$\Sigma F_x = 0$; $B_x + 161.6 - 0.393(800) = 0$
$B_x = 152.6$ N

y-z; $\Sigma M_B = 0$; $0.7A_y - 0.920(800)(0.360) - 50(9.81)(0.300) = 0$; $A_y = 588.6$ N

$\Sigma F_y = 0$; $W + T\sin\theta - A_y - B_y = 0$,
$B_y = 637.6$ N





$\beta = \tan^{-1}\dfrac{180}{240} = 36.9°$

$\alpha = \sin^{-1}\dfrac{150}{300} = 30°$

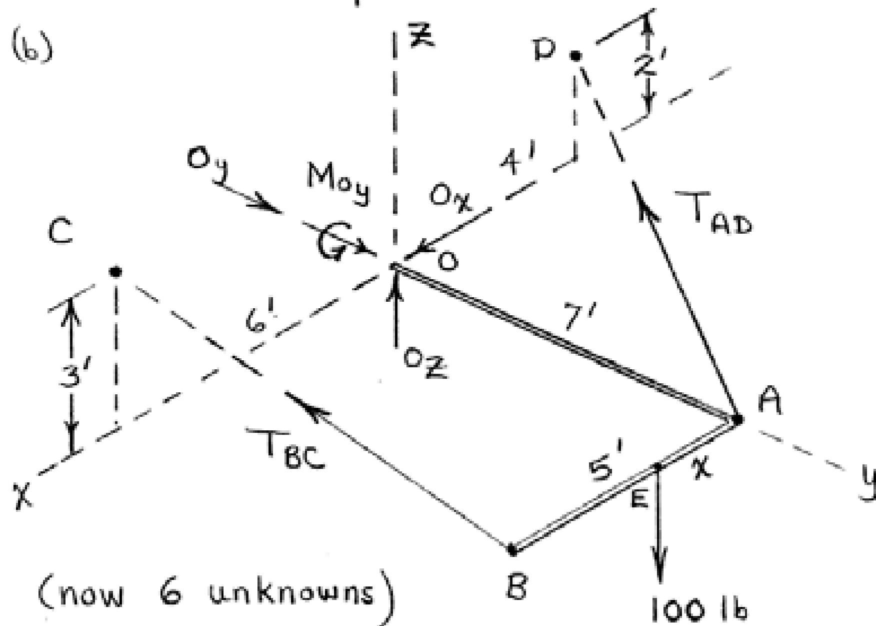
$\theta = \alpha + \beta = 66.9°$

$A = \sqrt{(161.6)^2 + (588.6)^2} = \underline{610 \text{ N}}$

$B = \sqrt{(152.6)^2 + (637.6)^2} = \underline{656 \text{ N}}$

3/110 (a) There are 5 unknown constraint forces. The bar is free to rotate about a line which passes through point O and through which the lines of action of both tension forces pass.

(b)



(now 6 unknowns)

$$\underline{T}_{AD} = T_{AD}\left[\frac{-4\underline{i} - 7\underline{j} + 2\underline{k}}{\sqrt{69}}\right]$$

$$\underline{T}_{BC} = T_{BC}\left[\frac{\underline{i} - 7\underline{j} + 3\underline{k}}{\sqrt{59}}\right]$$

$$\Sigma F_x = 0: \; O_x - \frac{4}{\sqrt{69}} T_{AD} + \frac{1}{\sqrt{59}} T_{BC} = 0 \qquad (1)$$

$$\Sigma F_y = 0: \; O_y - \frac{7}{\sqrt{69}} T_{AD} - \frac{7}{\sqrt{59}} T_{BC} = 0 \qquad (2)$$

$$\Sigma F_z = 0: \; O_z + \frac{2}{\sqrt{69}} T_{AD} + \frac{3}{\sqrt{59}} T_{BC} - 100 = 0 \qquad (3)$$

$$\Sigma M_{O_x} = 0: \; 7\left(\frac{2}{\sqrt{69}} T_{AD}\right) + 7\left(\frac{3}{\sqrt{59}} T_{BC}\right) - 7(100) = 0 \qquad (4)$$

$$\Sigma M_{O_y} = 0: \; -5\left(\frac{3}{\sqrt{59}} T_{BC}\right) + M_{O_y} + 100\,x = 0 \qquad (5)$$

$$\Sigma M_{O_z} = 0: \; 7\left(\frac{4}{\sqrt{69}} T_{AD}\right) - 7\left(\frac{1}{\sqrt{59}} T_{BC}\right) - 5\left(\frac{7}{\sqrt{59}} T_{BC}\right) = 0 \qquad (6)$$

Solve Eqs. (1)–(6) over $0.5 \le x \le 4.5$ ft and discover that three of the requested quantities are constant:

$$\begin{cases} O_x = 83.3 \text{ lb} \\ O_y = 292 \text{ lb} \\ O_z = 0 \end{cases}$$

$\underline{T_{AD} = 208 \text{ lb} = \text{constant}}$

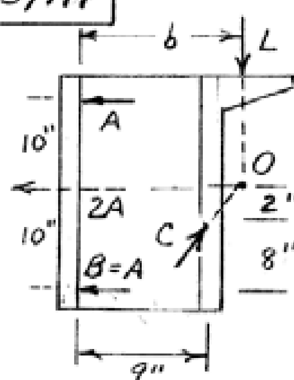
$\underline{T_{BC} = 128.0 \text{ lb} = \text{constant}}$

$\underline{O = \sqrt{O_x^2 + O_y^2 + O_z^2} = 303 \text{ lb} = \text{constant}}$

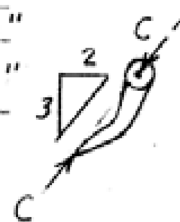
and $\underline{M_{O_y} = -100x + 250}$ (in lb-ft if x in ft)

(Note that O_y could have been obtained from $\Sigma M_{CD} = 0$ & O_z from $\Sigma M_{AB} = 0$)

3/111



Reduce to three forces concurrent at O.



$b = 9 + \frac{2}{3}(2) = \underline{10.33 \text{ in.}}$

Alternative sol.

$\Sigma M_C = 0;\quad L(b-9) - A(12) + A(8) = 0$

$L(b-9) = 4A \quad \text{------}(1)$

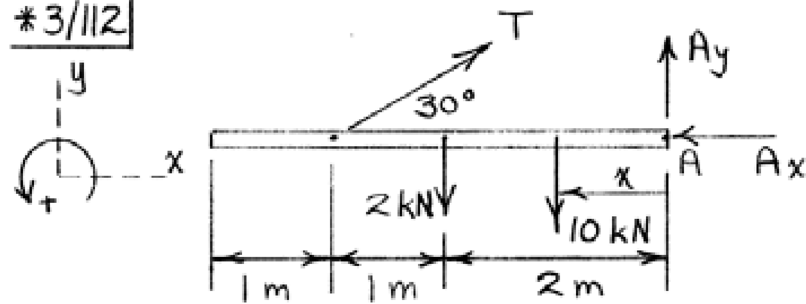
$\Sigma F_y = 0;\quad \frac{3}{\sqrt{13}} C = L$

$\Sigma F_x = 0;\quad \frac{2}{\sqrt{13}} C = 2A$

$\Big\} \quad L = 3A \quad \text{--}(2)$

(1) & (2) give $\quad b - 9 = \frac{4}{3},\quad b = 10.33 \text{ in.}$

*3/112



(Weight of beam = 200(10)/1000 = 2 kN)

$\Sigma M_A = 0: \quad 10x + 2(2) - T\sin 30°(3) = 0$

$T = \frac{2}{3}(10x + 4) \quad$ (in kN)

$\Sigma F_x = 0: \quad T\cos 30° - A_x = 0$

$A_x = \frac{1}{\sqrt{3}}(10x + 4)$

$\Sigma F_y = 0: \quad T\sin 30° - 2 - 10 + A_y = 0$

$A_y = \frac{1}{3}(-10x + 32)$

$R = \{A_x^2 + A_y^2\}^{1/2} = \frac{1}{3}\{400x^2 - 400x + 1072\}^{1/2}$

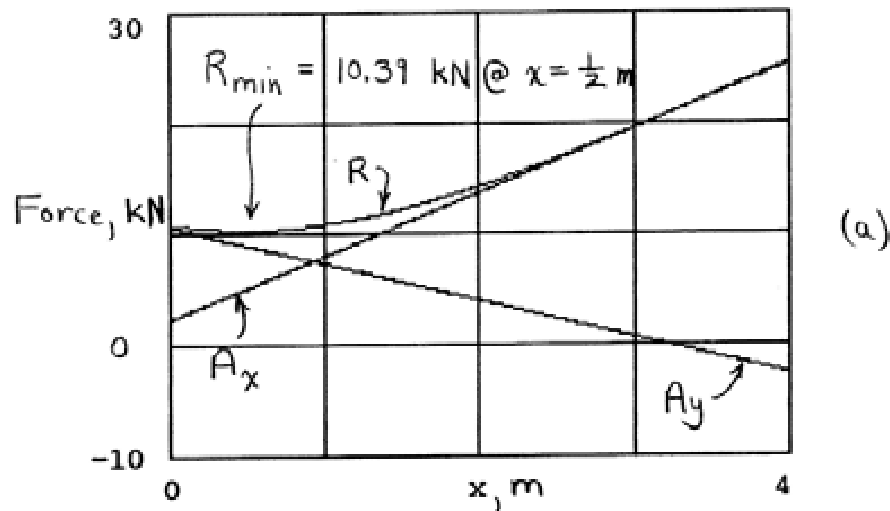
Set $\frac{dR^2}{dx} = 0$: $800x - 400 = 0$

$$x = \tfrac{1}{2}\ \text{m}$$

$$R_{min} = \tfrac{1}{3}\left\{400\left(\tfrac{1}{2}\right)^2 - 400\left(\tfrac{1}{2}\right) + 1072\right\}^{1/2}$$

$$= 10.39\ \text{kN} \quad \text{(b)}$$

Plot of A_x, A_y, and R:



(a)

(c)

$R_{max} = 24.3$ kN @ $x = 3.8$ m is the value of R which must be used for the design of the pin at A.

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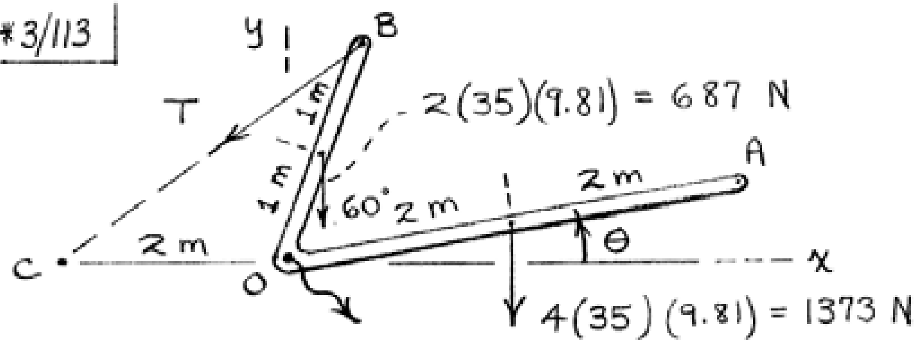


$$\underline{BC} = [-2 - 2\cos(\theta+60°)]\underline{i} - [2\sin(\theta+60°)]\underline{j}$$

$$\overline{BC}^2 = \{4 + 8\cos(\theta+60°) + 4\cos^2(\theta+60°) + 4\sin^2(\theta+60°)\}$$

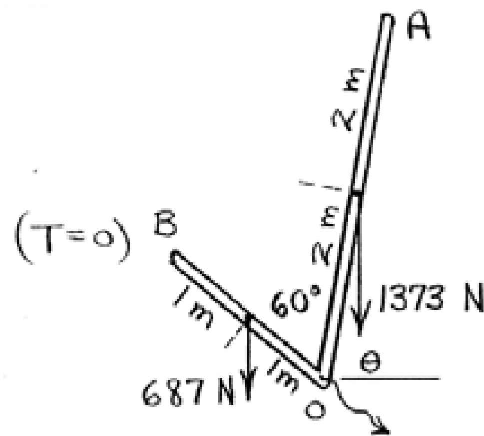
$$\overline{BC} = \{8 + 8\cos(\theta+60°)\}^{1/2}$$

$$\underline{n}_{BC} = \frac{[-2-2\cos(\theta+60°)]\underline{i} - [2\sin(\theta+60°)]\underline{j}}{\{8+8\cos(\theta+60°)\}^{1/2}}$$

Then $\underline{T} = T\underline{n}_{BC}$

$$\curvearrowleft\!+ \sum M_O = 0: -1373(2\cos\theta) - 687(1\cos(\theta+60°)) + \frac{2T\sin(\theta+60°)}{\{8+8\cos(\theta+60°)\}^{1/2}}(2) = 0$$

$$T = \left[\frac{2750\cos\theta + 687\cos(\theta+60°)}{4\sin(\theta+60°)}\right]\left[8+8\cos(\theta+60°)\right]^{\frac{1}{2}}$$



$$\Sigma M_O = 0 \Rightarrow 687(1\cos(120°-\theta)) = 1373(2\cos\theta)$$

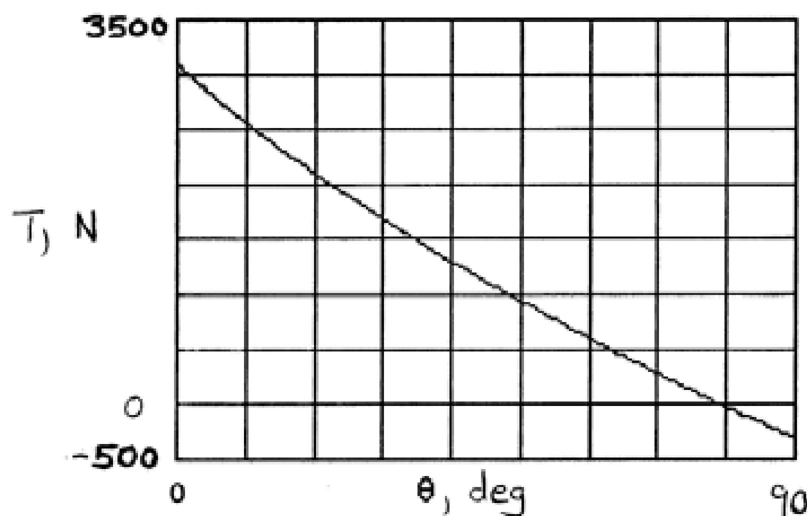
or $\cos(120°-\theta) = 4\cos\theta$

$$\cos 120° \cos\theta + \sin 120° \sin\theta = 4\cos\theta$$

$$-\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta = 4\cos\theta$$

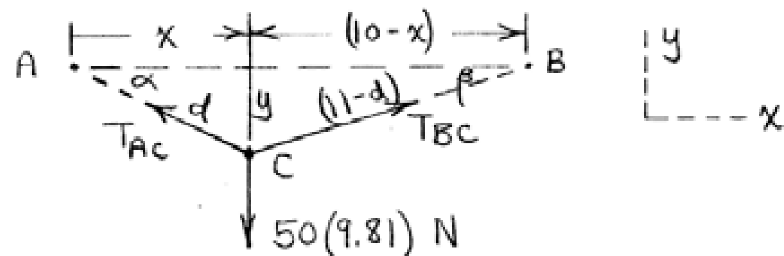
$$\tan\theta = \frac{4.5(2)}{\sqrt{3}} \Rightarrow \underline{\theta_{max} = 79.1°}$$

Plot of T vs. θ:



(T goes negative above $\theta = \theta_{max} = 79.1°$)

*3/114



Geometry: $y^2 = d^2 - x^2 = (11-d)^2 - (10-x)^2$

Simplify to obtain $d = \frac{10}{11}x + \frac{21}{22}$

Then $y = \sqrt{d^2 - x^2} = \left[-\frac{21}{121}x^2 + \frac{210}{121}x + \frac{441}{484}\right]^{1/2}$

$\tan\alpha = \frac{y}{x} = \frac{\left[-\frac{21}{121}x^2 + \frac{210}{121}x + \frac{441}{484}\right]^{1/2}}{x}$

$\tan\beta = \frac{y}{10-x} = \frac{\left[-\frac{21}{121}x^2 + \frac{210}{121}x + \frac{441}{484}\right]^{1/2}}{10-x}$

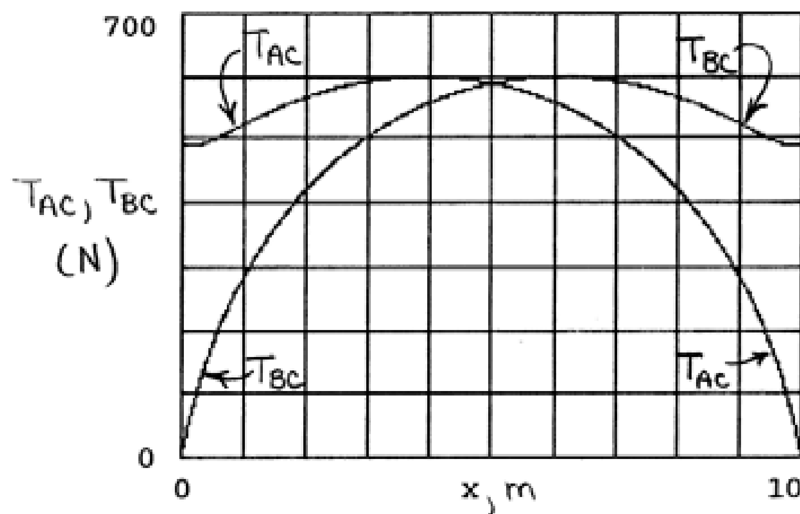
Equilibrium:

$\Sigma F_x = 0: \; -T_{AC}\cos\alpha + T_{BC}\cos\beta = 0$

$\Sigma F_y = 0: \; T_{AC}\sin\alpha + T_{BC}\cos\beta - 50(9.81) = 0$

Solve the above equations over $0 \le x \le 10$ m

to obtain the following plot:



The maxima are:

$(T_{AC})_{max}$ = 600 N @ x = 3.91 m

$(T_{BC})_{max}$ = 600 N @ x = 6.09 m

As a matter of interest:

$(T_{AC})_{min}$ = 488 N @ x = 0.1287 m

$(T_{BC})_{min}$ = 488 N @ x = 9.87 m

(These are local, not global, minima; the two global minima are both zero.)

Note that $x = 0$ does not represent a unique clamping point. For $x = 0$, the clamp could be anywhere from zero to $\frac{21}{22}$ m along the cable from A.

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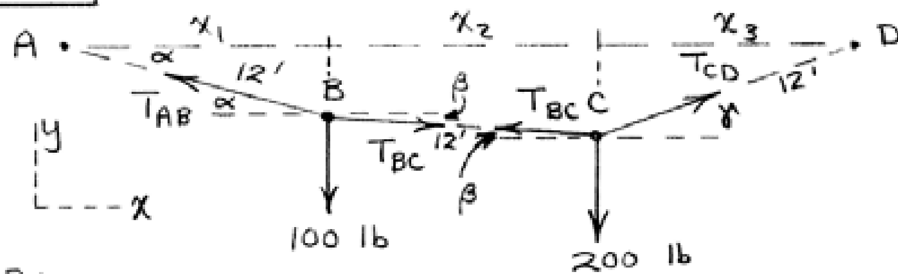


B:

$$\Sigma F_x = 0: \quad -T_{AB}\cos\alpha + T_{BC}\cos\beta = 0 \qquad (1)$$

$$\Sigma F_y = 0: \quad T_{AB}\sin\alpha - T_{BC}\sin\beta - 100 = 0 \qquad (2)$$

C:

$$\Sigma F_x = 0: \quad -T_{BC}\cos\beta + T_{CD}\cos\gamma = 0 \qquad (3)$$

$$\Sigma F_y = 0: \quad T_{BC}\sin\beta + T_{CD}\sin\gamma - 200 = 0 \qquad (4)$$

$$\cos\alpha = \frac{x_1}{12}, \quad \cos\beta = \frac{x_2}{12}, \quad \cos\gamma = \frac{x_3}{12}$$

So $\quad 12\cos\alpha + 12\cos\beta + 12\cos\gamma = 35 \qquad (5)$

$\sin\alpha + \sin\beta = \sin\gamma$ (from figure) (6)

Solve numerically:

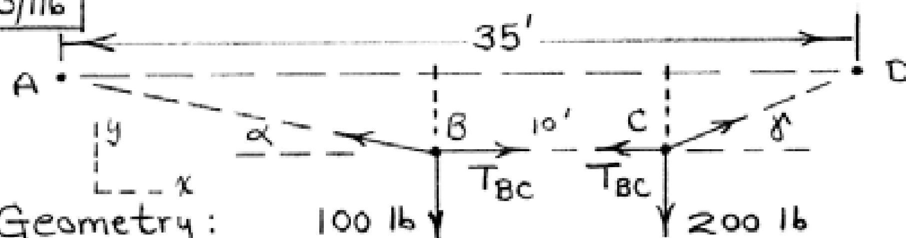
$$\begin{cases} \alpha = 14.44° \\ \beta = 3.57° \\ \gamma = 18.16° \end{cases}$$

$T_{AB} = 529$ lb

$T_{BC} = 513$ lb

$T_{CD} = 539$ lb

*3/116



Geometry:

$\overline{AB} + \overline{BC} + \overline{CD} = 36$ ft (1)

$\overline{AB}\cos\alpha + \overline{BC} + \overline{CD}\cos\gamma = 35$ ft (2)

$\overline{AB}\sin\alpha = \overline{CD}\sin\gamma$ (3)

Equilibrium:

Ⓑ $\Sigma F_x = 0: \; -T_{AB}\cos\alpha + T_{BC} = 0$ (4)

$\Sigma F_y = 0: \; T_{AB}\sin\alpha - 100 = 0$ (5)

Ⓒ $\Sigma F_x = 0: \; -T_{BC} + T_{CD}\cos\gamma = 0$ (6)

$\Sigma F_y = 0: \; T_{CD}\sin\gamma - 200 = 0$ (7)

With $\overline{BC}$ set to 10 ft, solve 7 equations in 7 unknowns & obtain

$\overline{AB} = 17.01$ ft	$\alpha = 11.47°$	$T_{AB} = 503$ lb
$\overline{CD} = 8.99$ ft	$\gamma = 22.1°$	$T_{BC} = 493$ lb
		$T_{CD} = 532$ lb

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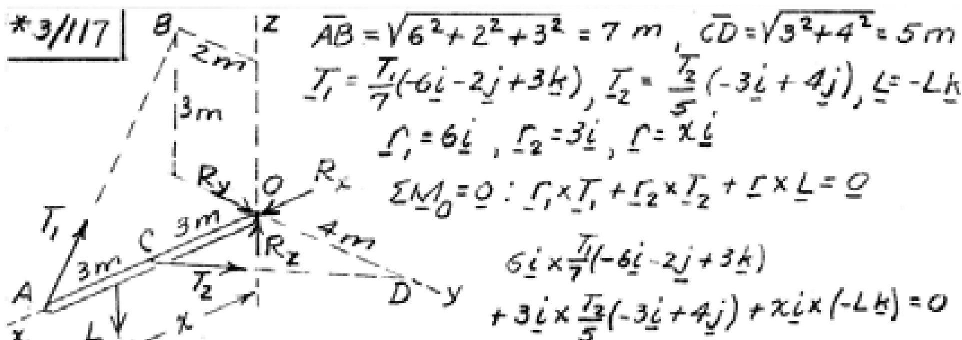


$\overline{AB} = \sqrt{6^2+2^2+3^2} = 7\text{ m}$, $\overline{CD} = \sqrt{3^2+4^2} = 5\text{ m}$

$\underline{T}_1 = \frac{T_1}{7}(-6\underline{i} - 2\underline{j} + 3\underline{k})$, $\underline{T}_2 = \frac{T_2}{5}(-3\underline{i} + 4\underline{j})$, $\underline{L} = -L\underline{k}$

$\underline{r}_1 = 6\underline{i}$, $\underline{r}_2 = 3\underline{i}$, $\underline{r} = x\underline{i}$

$\Sigma \underline{M}_O = \underline{0}: \underline{r}_1 \times \underline{T}_1 + \underline{r}_2 \times \underline{T}_2 + \underline{r} \times \underline{L} = \underline{0}$

$$6\underline{i} \times \frac{T_1}{7}(-6\underline{i} - 2\underline{j} + 3\underline{k}) + 3\underline{i} \times \frac{T_2}{5}(-3\underline{i} + 4\underline{j}) + x\underline{i} \times (-L\underline{k}) = 0$$

Expand: $\frac{6}{7}T_1(-2\underline{k} - 3\underline{j}) + \frac{3}{5}T_2(4\underline{k}) + Lx\underline{j} = \underline{0}$

$-\frac{12}{7}T_1 + \frac{12}{5}T_2 = 0$, $-\frac{18}{7}T_1 + Lx = 0$, so $T_1 = \frac{7}{18}Lx$, $T_2 = \frac{5}{18}Lx$

$\Sigma F_x = 0$; $R_x - \frac{3}{5}T_2 - \frac{6}{7}T_1 = 0$, $R_x = \frac{3}{5}\frac{5}{18}Lx + \frac{6}{7}\frac{7}{18}Lx = \frac{1}{2}Lx$

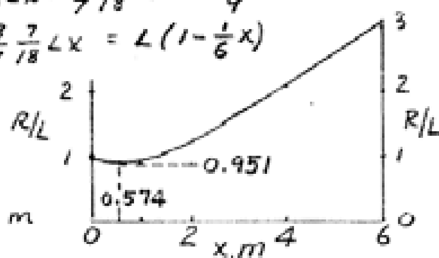
$\Sigma F_y = 0$; $R_y + \frac{4}{5}T_2 - \frac{2}{7}T_1 = 0$, $R_y = -\frac{4}{5}\frac{5}{18}Lx + \frac{2}{7}\frac{7}{18}Lx = -\frac{1}{9}Lx$

$\Sigma F_z = 0$; $R_z - L + \frac{3}{7}T_1 = 0$, $R_z = L - \frac{3}{7}\frac{7}{18}Lx = L(1 - \frac{1}{6}x)$

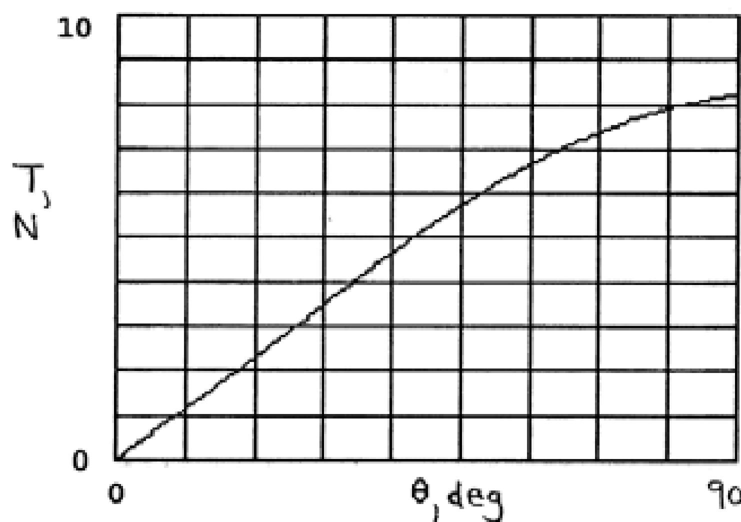
$R^2 = L^2\left(\frac{x^2}{4} + \frac{x^2}{81} + 1 - \frac{x}{3} + \frac{x^2}{36}\right)$

$R/L = \sqrt{(47x^2/162) - x/3 + 1}$

$\frac{dR^2/L^2}{dx} = \frac{47x}{81} - \frac{1}{3} = 0$ for min.; $x = 0.574$ m



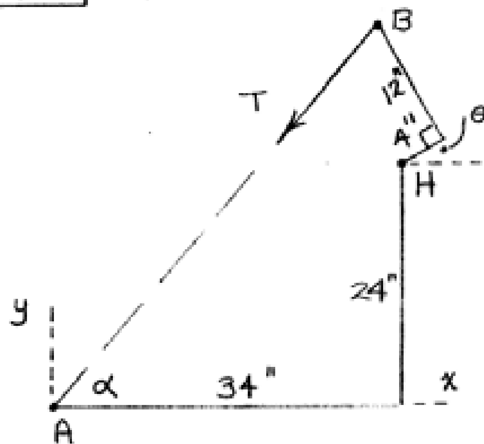
With $k = 25$ N/m, δ given by (1), α given by (2), $\overline{OA} = 0.48$ m, $m = 1.5$ kg, $g = 9.81\text{ m/s}^2$, $\overline{OG} = 0.16$ m, $\overline{OD} = 0.48$ m, and $\beta = \frac{90° + \theta}{2}$, we obtain the following plot:

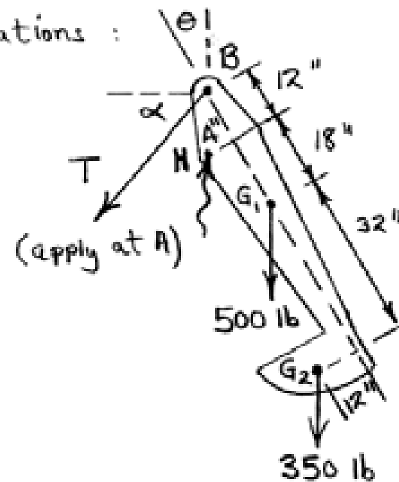


When $\theta = 45°$, $T = 5.23$ N

When $\theta = 90°$, $T = 8.22$ N

*3/118 Geometrical Considerations :





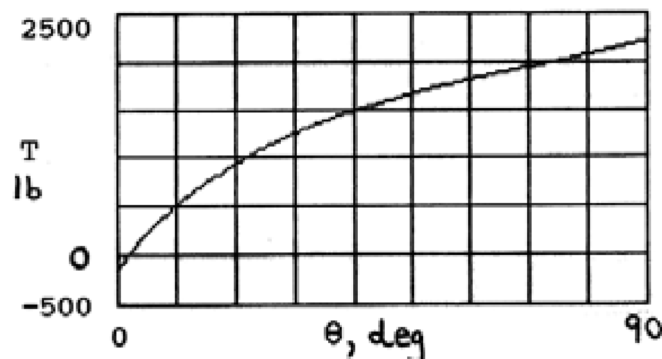
Coordinates of B : $\begin{cases} x = 34 + 4\cos\theta - 12\sin\theta \\ y = 24 + 4\sin\theta + 12\cos\theta \end{cases}$ (in.) (1)

$\therefore\ \alpha = \tan^{-1}\dfrac{y}{x}$ (aims T) (2)

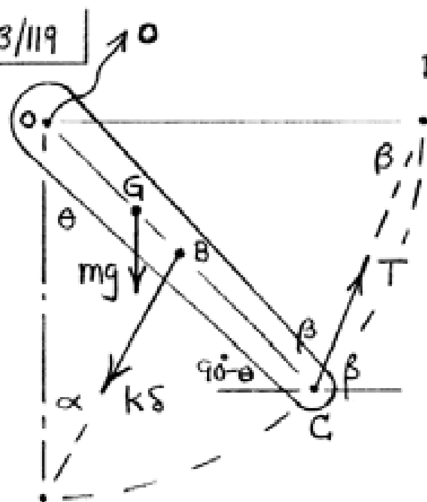
$\circlearrowleft \Sigma M_H = 0 : -(T\cos\alpha)(24) + (T\sin\alpha)(34) - 500(18\sin\theta + 4\cos\theta) - 350(4\cos\theta + (18+32)\sin\theta - 12\cos\theta) = 0$

$\Rightarrow\ T = \dfrac{26{,}500\sin\theta - 800\cos\theta}{34\sin\alpha - 24\cos\alpha}$ (lb; positive is tension in AB)

Solve (1), (2), & (3) & plot : (Note T=0 @ θ = 1.729°)



*3/119



$2\beta + (90° - \theta) = 180°$

$\beta = \dfrac{90° + \theta}{2}$

By law of cosines, $\overline{AB} = \sqrt{\overline{OA}^2 + \overline{OB}^2 - 2\cdot\overline{OA}\cdot\overline{OB}\cdot\cos\theta}$;

then spring deflection

$\delta = \overline{AB} - \overline{BC}$ (1)

Law of sines : $\dfrac{\sin\alpha}{\overline{OB}} = \dfrac{\sin\theta}{\overline{AB}}$

Consider $k\delta$ applied at A:

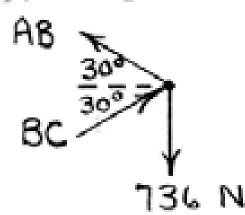
$\alpha = \sin^{-1}\left[\dfrac{\overline{OB}}{\overline{AB}}\sin\theta\right]$ (2)

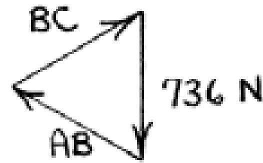
$\circlearrowleft + \Sigma M_O = 0 : -(k\delta\sin\alpha)(\overline{OA}) - mg(\overline{OG}\sin\theta) + T\sin\beta(\overline{OD}) = 0$

$T = \dfrac{(k\delta\sin\alpha)(\overline{OA}) + mg(\overline{OG}\sin\theta)}{\overline{OD}\sin\beta}$ (3)

4/1 Load = $75(9.81) = 736$ N

Joint B:

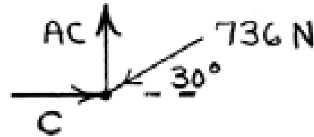


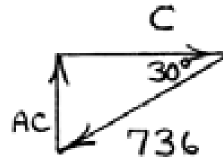


$\underline{AB = 736 \text{ N } T}$

$\underline{BC = 736 \text{ N } C}$

Joint C:





$AC = 736\left(\frac{1}{2}\right)$

$= \underline{368 \text{ N } T}$

4/2

$\alpha = \tan^{-1} \frac{2.5}{6} = 22.6°$ $\left(\cos\alpha = \frac{12}{13},\ \sin\alpha = \frac{5}{13}\right)$

Joint A:

AB, AC, 100 lb, α, y, x

$\Sigma F_y = 0$: $AC \sin\alpha - 100 = 0$

$AC = 260$ lb C

$\Sigma F_x = 0$: $AB - 260 \cos\alpha = 0$

$AB = 240$ lb T

Joint C:

260 lb, BC, x′, α, C, 45°

$\Sigma F_x = 0$: $260\left(\frac{12}{13}\right) - C\frac{\sqrt{2}}{2} = 0$

$C = 339$ lb (reaction)

$\Sigma F_y = 0$: $-260\frac{5}{13} - BC + 339\frac{\sqrt{2}}{2} = 0$

$BC = 140$ lb C

Could use $\Sigma F_{x'}$ to find BC without involving calculation of C. Nonetheless, observe that changing the 45° support angle would affect BC, but not AB or AC!

4/3

Joint B:

$\Sigma F_y = 0: \quad BC\frac{\sqrt{2}}{2} - 5 = 0$

$BC = 5\sqrt{2}$ kN C

$\Sigma F_x = 0: -AB + 5\sqrt{2}\frac{\sqrt{2}}{2} = 0$, $AB = 5$ kN T

Joint C:

$\Sigma F_y = 0: -5\sqrt{2}\frac{\sqrt{2}}{2} + AC\frac{2}{2\sqrt{5}} = 0$

$AC = 5\sqrt{5}$ kN T

$\Sigma F_x = 0: \quad CD - 5\sqrt{5}\frac{4}{2\sqrt{5}} - 5\sqrt{2}\frac{\sqrt{2}}{2} = 0$

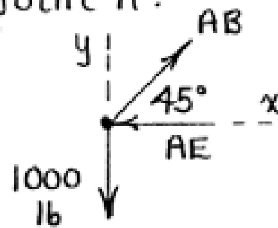
$CD = 15$ kN C

Joint D:

From $\Sigma F_y = 0$, $AD = 0$

4/4

Joint A:



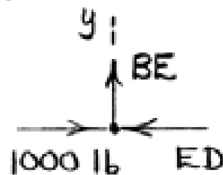
$\Sigma F_y = 0$: $AB \sin 45° - 1000 = 0$

$AB = 1414$ lb T

$\Sigma F_x = 0$: $1414 \cos 45° - AE = 0$

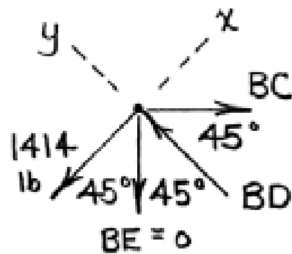
$AE = 1000$ lb C

Joint E:



$\Sigma F_y = 0$: $BE = 0$

Joint B:



$\Sigma F_x = 0$: $BC \cos 45° - 1414 = 0$

$BC = 2000$ lb T

$\Sigma F_y = 0$: $BD - 2000 \cos 45° = 0$

$BD = 1414$ lb C

4/5

y, x

Joint C:

BC, 30°, CD, 3 kN

$$\Sigma F_y = 0: \; CD\left(\tfrac{1}{2}\right) - 3 = 0, \quad \underline{CD = 6 \text{ kN } C}$$

$$\Sigma F_x = 0: \; -BC + 6\frac{\sqrt{3}}{2} = 0, \quad \underline{BC = 5.20 \text{ kN } T}$$

Joint D:

DE, 60°, BD, 60°, 6 kN

$$\Sigma F_x = 0 \Rightarrow \underline{DE = 6 \text{ kN } C}$$

$$\Sigma F_y = 0: \; BD - 2(6)\left(\tfrac{1}{2}\right) = 0$$

$$\underline{BD = 6 \text{ kN } T}$$

Joint B:

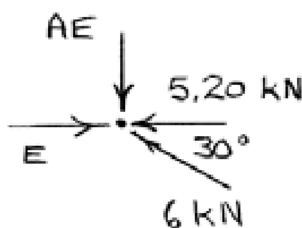
AB, 30°, BE, 5.20 kN, 6 kN

$$\Sigma F_y = 0: \; AB\left(\tfrac{1}{2}\right) - 6 = 0, \quad \underline{AB = 12 \text{ kN } T}$$

$$\Sigma F_x = 0: \; BE - 12\frac{\sqrt{3}}{2} + 5.20 = 0$$

$$\underline{BE = 5.20 \text{ kN } C}$$

Joint E:

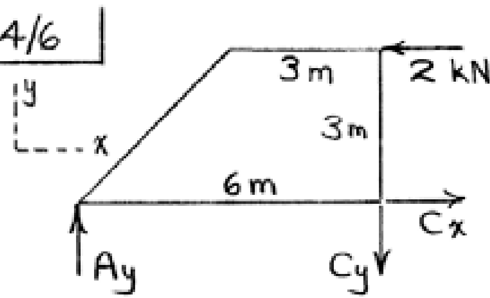


$$\Sigma F_y = 0: \; 6\left(\tfrac{1}{2}\right) - AE = 0$$

$$\underline{AE = 3 \text{ kN } C}$$

(Joint A checks after external reactions are determined from the truss as a whole.)

4/6



$+\circlearrowleft \Sigma M_C = 0: 6A_y - 2(3) = 0$

$A_y = 1$ kN

$C_x = 2$ kN, $C_y = 1$ kN

Joint A:

$\Sigma F_y = 0: 1 - AE \sin 45° = 0$

$AE = 1.414$ kN C

$\Sigma F_x = 0: AB - 1.414 \cos 45° = 0$

$AB = 1$ kN T

Joint E:

$\Sigma F_x = 0: 1.414 \sin 45° - DE = 0$

$DE = 1$ kN C

$\Sigma F_y = 0: 1.414 \cos 45° - BE = 0$

$BE = 1$ kN T

Joint B:

$\Sigma F_y = 0: 1 - BD \sin 45° = 0$

$BD = 1.414$ kN C

$\Sigma F_x = 0: BC - 1.414 \cos 45° - 1 = 0$

$BC = 2$ kN T

Joint C:

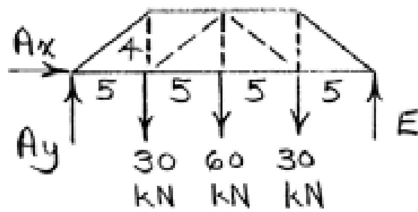
$\Sigma F_y = 0: CD - 1 = 0$

$CD = 1$ kN T

(Joint D checks)

4/7 As a whole: $\Sigma F_x = 0 \Rightarrow A_x = 0$

(Dim. in m) $A_y = E = 60$ kN by $\Sigma F_y = 0$ and symmetry.



Joint A: $\left(\theta = \tan^{-1}\left(\frac{4}{5}\right) = 38.7°\right)$

AB, θ, AH, 60 kN

$\Sigma F_y = 0: 60 - AB\sin\theta = 0$, $\underline{AB = 96.0 \text{ kN C}}$

$\Sigma F_x = 0: AH - 96.0\cos\theta = 0$, $\underline{AH = 75 \text{ kN T}}$

Joint B:

51.3°, BC, BH, 96.0 kN

$\Sigma F_x = 0: -BC + 96.0\sin 51.3° = 0$, $\underline{BC = 75 \text{ kN C}}$

$\Sigma F_y = 0: -BH + 96.0\cos 51.3° = 0$, $\underline{BH = 60 \text{ kN T}}$

Joint H:

60 kN, CH, θ, 75 kN, GH, 30 kN

$\Sigma F_y = 0: -CH\sin\theta + 30 = 0$, $\underline{CH = 48.0 \text{ kN C}}$

$\Sigma F_x = 0: -48.0\cos\theta + GH - 75 = 0$

$\underline{GH = 112.5 \text{ kN T}}$

Joint G:

CG, FG, 112.5 kN, 60 kN

$\Sigma F_y = 0 \Rightarrow \underline{CG = 60 \text{ kN T}}$

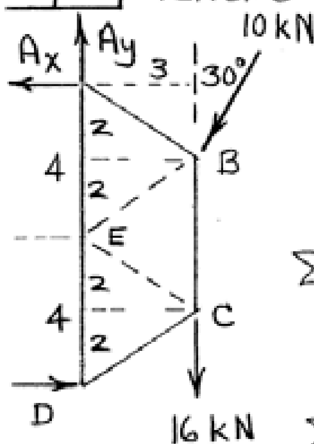
By symmetry:

FG = 112.5 kN T, CF 48.0 kN C

CD = 75 kN C, DF = 60 kN T

EF = 75 kN T, DE = 96.0 kN C

4/8 Entire truss:



$\curvearrowleft + \Sigma M_A = 0: -10\cos 30°(3) - 10\sin 30°(2) - 16(3) + D(8) = 0$

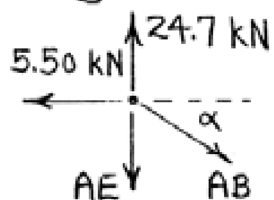
$D = 10.50$ kN

$\Sigma F_x = 0: -A_x - 10\left(\frac{1}{2}\right) + 10.50 = 0$

$A_x = 5.50$ kN

$\Sigma F_y = 0: -10\frac{\sqrt{3}}{2} - 16 + A_y = 0, \; A_y = 24.7$ kN

Joint A: $\left(\alpha = \tan^{-1}\left(\frac{2}{3}\right) = 33.7°\right)$



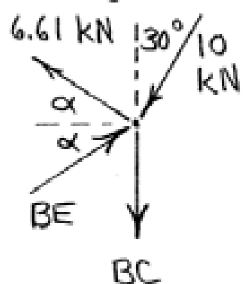
$\Sigma F_x = 0: AB\cos 33.7° - 5.50 = 0$

$\underline{AB = 6.61 \text{ kN } T}$

$\Sigma F_y = 0: 24.7 - AE - 6.61\sin 33.7° = 0$

$\underline{AE = 21.0 \text{ kN } T}$

Joint B:

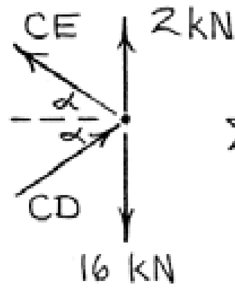


$\Sigma F_x = 0: -6.61\cos 33.7° - 10\sin 30° + BE\cos 33.7° = 0, \; \underline{BE = 12.62 \text{ kN } C}$

$\Sigma F_y = 0: 6.61\sin 33.7° - 10\cos 30° + 12.62\sin 33.7° - BC = 0$

$\underline{BC = 2.00 \text{ kN } T}$

Joint C:



$\Sigma F_x = 0: -CE\cos\alpha + CD\cos\alpha = 0$

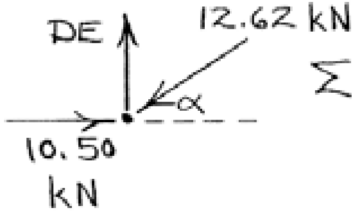
$CE = CD$

$\Sigma F_y = 0: 2 - 16 + (CE + CE)\sin 33.7° = 0$

$CE = 12.62 \text{ kN } T$

$CD = 12.62 \text{ kN } C$

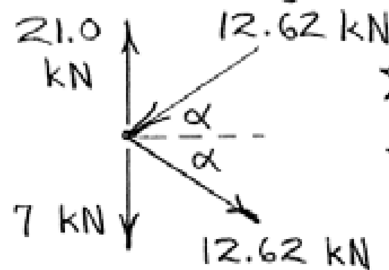
Joint D:



$\Sigma F_y = 0: DE - 12.62\sin 33.7° = 0$

$DE = 7 \text{ kN } T$

Check on joint E:



$\Sigma F_x = 0$ ✓

$\Sigma F_y = 21 - 7 - 2(12.62)\sin 33.7°$

$= 0$ ✓

4/9

As a whole: $\circlearrowleft + \Sigma M_A = 0: \; -8s - 4\frac{s}{2} - 2\frac{3s}{2} + C\frac{1}{2}(2s) = 0, \quad C = 13 \text{ kN}$

$\Sigma F_x = 0: \; A_x - 13\frac{\sqrt{3}}{2} = 0$

$A_x = 13\frac{\sqrt{3}}{2} \text{ kN}$

$\Sigma F_y = 0: \; A_y + 13\left(\frac{1}{2}\right) - 14 = 0$

$A_y = 7.5 \text{ kN}$

Joint A:

$\Sigma F_y = 0: \; 7.5 - AE\frac{\sqrt{3}}{2} = 0, \quad AE = 5\sqrt{3} \text{ kN T}$

$\Sigma F_x = 0: \; 13\frac{\sqrt{3}}{2} - AB + 5\sqrt{3}\left(\frac{1}{2}\right) = 0$

$AB = 9\sqrt{3} \text{ kN C}$

Joint E:

$\Sigma F_y = 0: \; 5\sqrt{3}\frac{\sqrt{3}}{2} - 4 - BE\frac{\sqrt{3}}{2} = 0$

$BE = \frac{7}{3}\sqrt{3} \text{ kN C}$

$\Sigma F_x = 0: \; -5\sqrt{3}\left(\frac{1}{2}\right) - \frac{7}{3}\sqrt{3}\left(\frac{1}{2}\right) + DE = 0$

$DE = \frac{11}{3}\sqrt{3} \text{ kN T}$

Joint D:

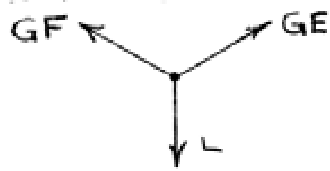
$\Sigma F_x = 0: \; BD\left(\frac{1}{2}\right) + CD\left(\frac{1}{2}\right) - \frac{11}{3}\sqrt{3} = 0$

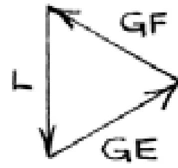
$\Sigma F_y = 0: \; -BD\frac{\sqrt{3}}{2} + CD\frac{\sqrt{3}}{2} - 2 = 0$

$\Rightarrow \begin{cases} CD = \frac{13}{3}\sqrt{3} \text{ kN T} \\ BD = 3\sqrt{3} \text{ kN C} \end{cases}$

4/10

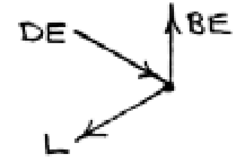
Joint G:

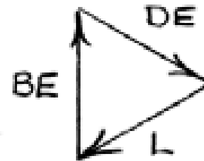




GE = GF = L C

Joint E:

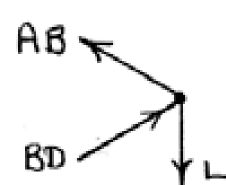


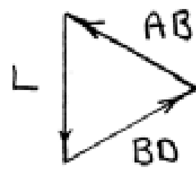


BE = L T

DE = L C

Joint B:



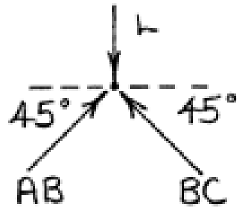


BD = L C

AB = L T

4/11

Joint B:

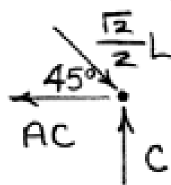


From $\Sigma F_x = 0$, $AB = BC$

$\Sigma F_y = 0$: $2AB\frac{\sqrt{2}}{2} - L = 0$

$AB = \frac{\sqrt{2}}{2}L = BC$

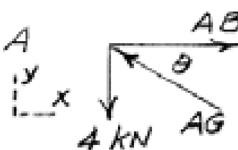
Joint C:



$\Sigma F_x = 0$: $\frac{\sqrt{2}}{2}L\left(\frac{\sqrt{2}}{2}\right) - AC = 0$

$AC = \frac{L}{2}\ T$

4/12 Joint A



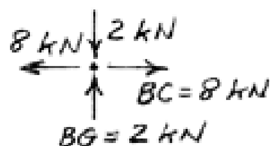
$\theta = \tan^{-1} 1/2 = 26.57°$

$\sin\theta = 1/\sqrt{5}$, $\cos\theta = 2/\sqrt{5}$

$\Sigma F_y = 0$; $AG/\sqrt{5} - 4 = 0$

$AG = 4\sqrt{5}$ kN C

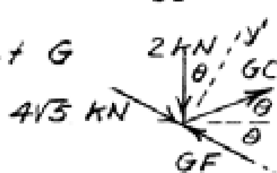
Joint B



$\Sigma F_x = 0$; $AB - 4\sqrt{5}(2/\sqrt{5}) = 0$

$AB = 8$ kN T

Joint G



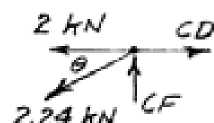
$\Sigma F_{y'} = 0$; $2\left(\frac{2}{\sqrt{5}}\right) - CG \sin 2\theta = 0$

$CG = \frac{4}{\sqrt{5}} \frac{1}{0.8} = \underline{2.24 \text{ kN T}}$

$\Sigma F_{x'} = 0$; $2.24 \cos 2\theta + 2/\sqrt{5} + 4\sqrt{5} - GF = 0$

$GF = 5\sqrt{5} = 11.18$ kN C

Joint C



$\Sigma F_y = 0$; $CF - 2.24 \sin\theta = 0$

$\underline{CF = 1.00 \text{ kN C}}$

4/13 Total weight of truss = 7(400) = 2800 lb

By symmetry, reactions at A & C are 1400 lb

Joint A (diagram labels: 400 lb, AE, 60°, AB, 1400 lb; axes y, x)

$\Sigma F_y = 0;\ AE\cos 30° + 400 - 1400 = 0$

$\underline{AE = 2000/\sqrt{3}}$ lb C

$\Sigma F_x = 0;\ AB - \frac{2000}{\sqrt{3}}\cos 60° = 0$

$\underline{AB = 1000/\sqrt{3}}$ lb T

By symmetry $\underline{BC = 1000/\sqrt{3}}$ lb T

$\underline{CD = 2000/\sqrt{3}}$ lb C

Joint E (diagram labels: 200(3) lb, ED, 60°, 60°, $\frac{2000}{\sqrt{3}}$ lb, BE)

$\Sigma F_y = 0;\ BE\sin 60° - \frac{2000}{\sqrt{3}}\sin 60° + 600 = 0$

$\underline{BE = 800/\sqrt{3}}$ lb T

& by symmetry $\underline{BD = 800/\sqrt{3}}$ lb T

$\Sigma F_x = 0;\ ED - \frac{2000}{\sqrt{3}}\sin 30° - \frac{800}{\sqrt{3}}\sin 30° = 0$

$\underline{ED = 1400/\sqrt{3}}$ lb C

4/14

$\circlearrowleft \Sigma M_A = 0: -4\cos 20°(10+20+\frac{30}{2}) + EI\cos 50°(8) + EI\sin 50°(30) = 0,\quad EI = 6.01$ kips

Joint E

$\Sigma F_x = 0: \; EF - 6.01\cos 50° = 0$

$EF = 3.87$ kips C

$\Sigma F_y = 0: \; -DE + 6.01\sin 50° = 0$

$DE = 4.61$ kips T

Joint D

$\Sigma F_y = 0: \; 4.61 - 2\sin 70° + DF\sin 38.7° = 0,\quad DF = -4.37$ kips (C)

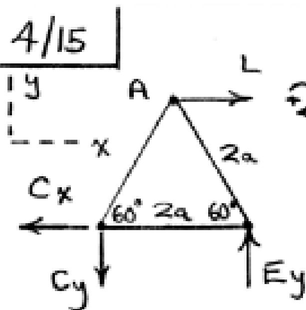
$\Sigma F_x = 0: \; 4.37\cos 38.7° - 2\cos 70° + CD = 0,\quad CD = -2.73$ kips (T)

Joint F

$\Sigma F_x = 0: \; -3.87 - 4.37\cos 38.7° + FG = 0$

$FG = 7.28$ kips C

4/15



Truss as a whole

$\circlearrowleft \Sigma M_C = 0: \; L(2a \sin 60°) - E_y(2a) = 0$

$E_y = \sqrt{3}L/2$

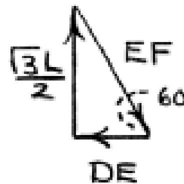
$\Sigma F_x, \Sigma F_y = 0: \; C_x = L, \; C_y = \frac{\sqrt{3}L}{2}$

Joint A:

$AB = L \; T$

$AF = L \; C$

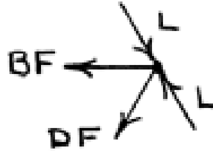
Joint E:



$EF = \frac{\sqrt{3}L/2}{\cos 30°} = L \; C$

$DE = EF \sin 30° = \frac{L}{2} \; T$

Joint F:



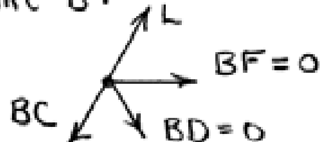
$\Sigma \underline{F} = \underline{0}: \quad BF = DF = 0$

Joint D:

$\Sigma F_y = 0: \; BD = 0$

$\Sigma F_x = 0: \; CD = \frac{L}{2} \; T$

Joint B:



$\Sigma \underline{F} = \underline{0}: \quad BC = L \; (T)$

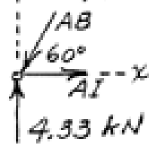
(Joint C checks)

4/16 Truss as a whole

$\Sigma M_F = 0;\ 2(a + \frac{a}{2}) + 4(2a + \frac{a}{2}) - A(3a) = 0$

$A = 13/3 = 4.33$ kN

Joint A



$\Sigma F_y = 0;\ AB \sin 60° - 4.33 = 0$

$AB = 5.00$ kN C

$\Sigma F_x = 0;\ AI - 5 \cos 60° = 0$

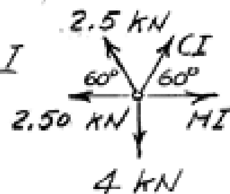
$AI = 2.50$ kN T

Joint B

$\Sigma F_{y'} = 0;\ 5.00 \cos 60° - BI = 0$

$BI = 2.50$ kN T

Joint I



$\Sigma F_y = 0;\ (CI + 2.5) \sin 60° - 4 = 0$

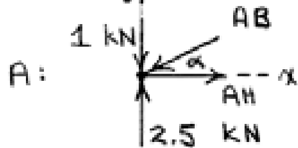
$CI = 2.12$ kN T

$\Sigma F_x = 0;\ HI + 2.12 \cos 60° - 2.50 - 2.5 \cos 60° = 0$

$HI = 2.69$ kN T

4/17 By symmetry, $A = E = 2.5$ kN; $\alpha = \tan^{-1}\left(\frac{2}{4}\right) = 26.6°$

A:



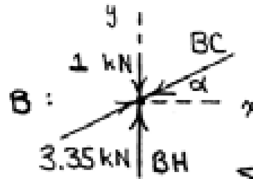
$\Sigma F_y = 0$: $2.5 - 1 - AB\sin\alpha = 0$

$AB = 3.35$ kN C

$\Sigma F_x = 0$: $-3.35\cos\alpha + AH = 0$

$AH = 3$ kN T

B:



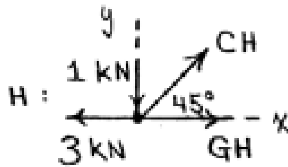
$\Sigma F_x = 0$: $3.35\cos\alpha - BC\cos\alpha = 0$

$BC = 3.35$ kN C

$\Sigma F_y = 0$: $-1 + (3.35 - 3.35)\sin\alpha + BH = 0$

$BH = 1$ kN C

H:



$\Sigma F_y = 0$: $-1 + CH\sin 45° = 0$

$CH = 1.414$ kN T

$\Sigma F_x = 0$: $-3 + 1.41\cos 45° + GH = 0$

$GH = 2$ kN T

By inspection of joint G and $\Sigma F_y = 0$, $CG = 0$.

By symmetry,

$DE = AB = 3.35$ kN C

$CD = BC = 3.35$ kN C

$EF = AH = 3$ kN T

$DF = BH = 1$ kN C

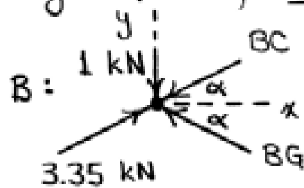
$CF = CH = 1.414$ kN T

$FG = GH = 2$ kN T

4/18 By symmetry, $A = E = 2.5$ kN; $\alpha = \tan^{-1}\left(\frac{2}{4}\right) = 26.6°$

Joint A analysis same as Prob. 4/16: $\begin{cases} AB = 3.35 \text{ kN } C \\ AH = 3.00 \text{ kN } T \end{cases}$

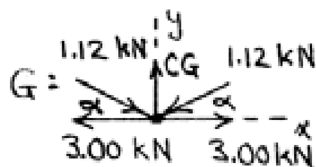
By inspection, $\underline{BH = 0}$ and $\underline{GH = AH}$.

B: 

$\Sigma F_y = 0$: $-1 + 3.35 \sin\alpha + BG \sin\alpha - BC \sin\alpha = 0$

$\Sigma F_x = 0$: $3.35 \cos\alpha - BC \cos\alpha - BG \cos\alpha = 0$

$\Rightarrow$ $\underline{BC = 2.24 \text{ kN } C, \quad BG = 1.118 \text{ kN } C}$

G: 

$\Sigma F_y = 0$: $CG - 2(1.12)\sin\alpha = 0$

$\underline{CG = 1.00 \text{ kN } T}$

By symmetry, $\begin{cases} DE = AB = 3.35 \text{ kN } C \\ CD = BC = 2.24 \text{ kN } C \\ EF = AH = 3.00 \text{ kN } T \\ DF = BH = 0 \\ FG = GH = 3.00 \text{ kN } T \\ DG = BG = 1.118 \text{ kN } C \end{cases}$

4/19 Joint E: DE = EF = 0

Joint D: ($\theta = \tan^{-1} \frac{13}{10} = 52.4°$)

$\Sigma F_x = 0$: $DF \sin\theta - 2000 = 0$

DF = 2520 lb T

$\Sigma F_y = 0$: $CD - 2520 \cos\theta = 0$, CD = 1538 lb, C

Joint F: ($\gamma = \tan^{-1} \frac{3}{10} = 16.70°$)

$\Sigma F_y = 0$: $2520 \cos\theta - FG \cos\gamma = 0$

FG = 1606 lb T

$\Sigma F_x = 0$: $2520 \sin\theta - CF - 1606 \sin\gamma = 0$

CF = 1538 lb C

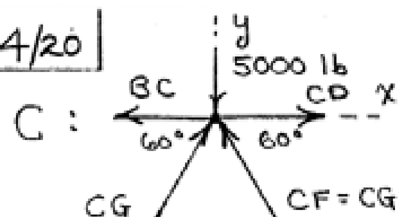
Joint C: ($\beta = \tan^{-1} \frac{16}{10} = 58.0°$)

$\Sigma F_x = 0$: $1538 + 2000 - CG \sin\beta = 0$

CG = 4170 lb T

4/20

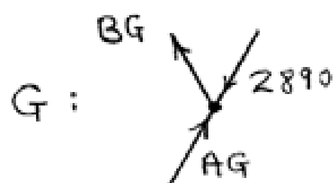
C :



Arguing symmetry,

$\Sigma F_y = 0: 2CG \sin 60° - 5000 = 0$, $\underline{CG = 2890 \text{ lb } C}$

G :



By inspection,

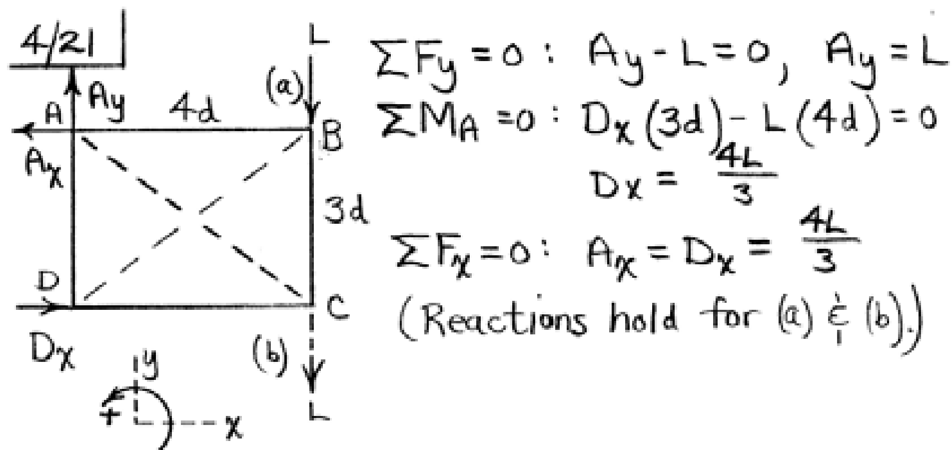
$\underline{AG = 2890 \text{ lb } C}$

$\underline{BG = 0}$

From joint B : $\underline{AB = 0, \ BC = 0}$.

(Right truss symmetric to left one.)

4/21



$\Sigma F_y = 0: \quad A_y - L = 0, \quad A_y = L$

$\Sigma M_A = 0: \quad D_x(3d) - L(4d) = 0$

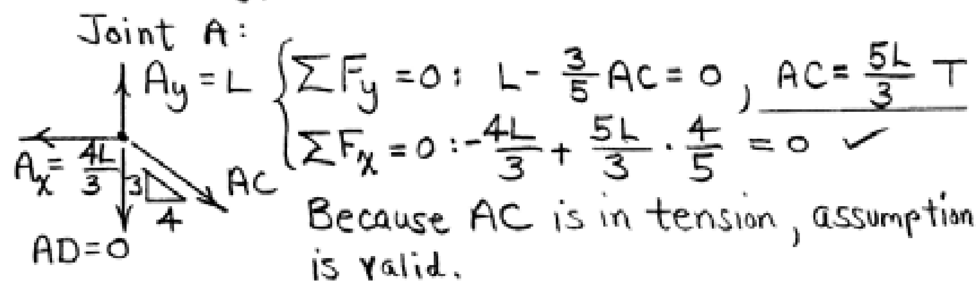
$D_x = \dfrac{4L}{3}$

$\Sigma F_x = 0: \quad A_x = D_x = \dfrac{4L}{3}$

(Reactions hold for (a) & (b).)

(a) Assume that BD goes slack. From an inspection of joint B, $\underline{AB = 0}$ and $\underline{BC = L\ C}$. Similarly, from joint D, $\underline{AD = 0}$ and $\underline{CD = \frac{4L}{3}\ C}$.

Joint A:



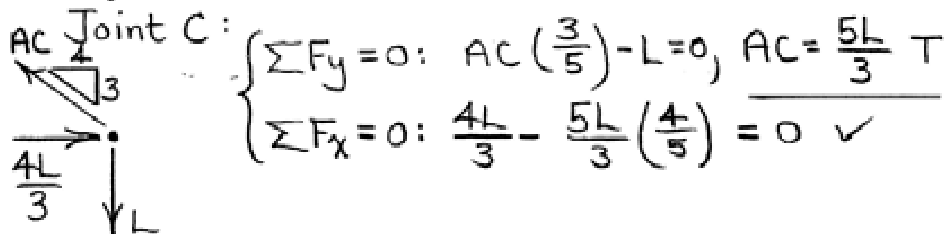
$\Sigma F_y = 0: \quad L - \frac{3}{5}AC = 0, \quad \underline{AC = \frac{5L}{3}\ T}$

$\Sigma F_x = 0: \quad -\frac{4L}{3} + \frac{5L}{3} \cdot \frac{4}{5} = 0 \quad \checkmark$

Because AC is in tension, assumption is valid.

(b) Assume that BD goes slack. From joint B, $\underline{AB = BC = 0}$. From joint D, $\underline{AD = 0}$ & $\underline{CD = \frac{4L}{3}\ C}$.

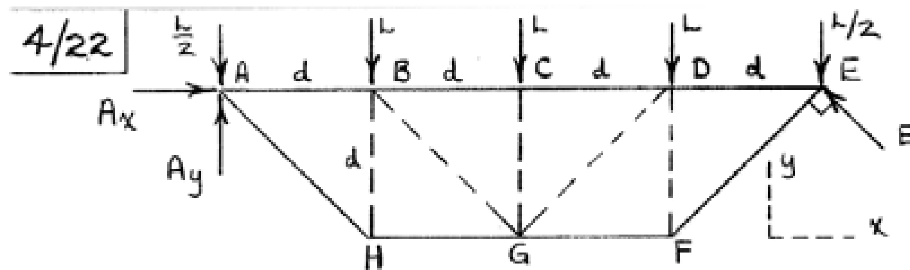
Joint C:



$\Sigma F_y = 0: \quad AC\left(\frac{3}{5}\right) - L = 0, \quad \underline{AC = \frac{5L}{3}\ T}$

$\Sigma F_x = 0: \quad \frac{4L}{3} - \frac{5L}{3}\left(\frac{4}{5}\right) = 0 \quad \checkmark$

4/22



Entire truss:

$$\circlearrowleft + \Sigma M_A = 0: \; -Ld - L(2d) - L(3d) - \frac{L}{2}(4d) + E\frac{\sqrt{2}}{2}(4d) = 0$$

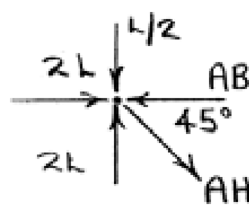
$$E = 2\sqrt{2}L$$

$$\Sigma F_x = 0: \; A_x - 2\sqrt{2}L\frac{\sqrt{2}}{2} = 0, \quad A_x = 2L$$

$$\Sigma F_y = 0: \; A_y - 4L + 2\sqrt{2}L\frac{\sqrt{2}}{2} = 0, \quad A_y = 2L$$

By inspection of joint C, $\underline{CG = L \ C}$

Joint A:



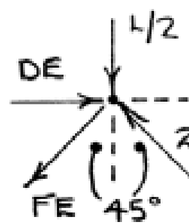
$$\Sigma F_y = 0: \; 2L - \frac{L}{2} - AH\frac{\sqrt{2}}{2} = 0$$

$$AH = \frac{3\sqrt{2}}{2}L \ \ T$$

$$\Sigma F_x = 0: \; 2L + \frac{3\sqrt{2}}{2}L\frac{\sqrt{2}}{2} - AB = 0$$

$$\underline{AB = \frac{7}{2}L \ \ C}$$

Joint E



$$\Sigma F_y = 0: \; -\frac{L}{2} + 2\sqrt{2}L\frac{\sqrt{2}}{2} - FE\frac{\sqrt{2}}{2} = 0$$

$$FE = \frac{3\sqrt{2}}{2}L \ \ T$$

$$\Sigma F_x = 0: \; DE - \frac{3\sqrt{2}}{2}L\frac{\sqrt{2}}{2} - 2\sqrt{2}L\frac{\sqrt{2}}{2} = 0$$

$$\underline{DE = \frac{7L}{2} \ \ C}$$

4/23 By symmetry, AJ = AB, CH = CD, BC = JC.

C: (free-body diagram: 120 kN left, 120 kN right, CJ, BC, slope 3/4, $(60)(9.81)$ kN down)

$\Sigma F_y = 0: \frac{4}{5}(2BC) - 60(9.81) = 0$

$BC = 368$ kN C

D: (free-body diagram: 120 kN, BD, DE at 45°)

$\Sigma F_x = 0: \frac{DE}{\sqrt{2}} - 120 = 0$, $DE = 169.7$ kN T

$\Sigma F_y = 0:$ $BD = 120$ kN C

E: (free-body diagram: 169.7 kN at 45°, BE, EF)

$\Sigma F_x = 0:$ $BE = 120$ kN C

$\Sigma F_y = 0:$ <u>$EF = 120$ kN T</u>

B: (free-body diagram: 368 kN, 120 kN, 120 kN, BG, BF)

$\Sigma F_x = 0: \frac{4}{5}BF + 120 - \frac{3}{5}(368) = 0$

<u>$BF = 188.4$ kN C</u>

4/24 m = no. of two-force members
j = no. of joints

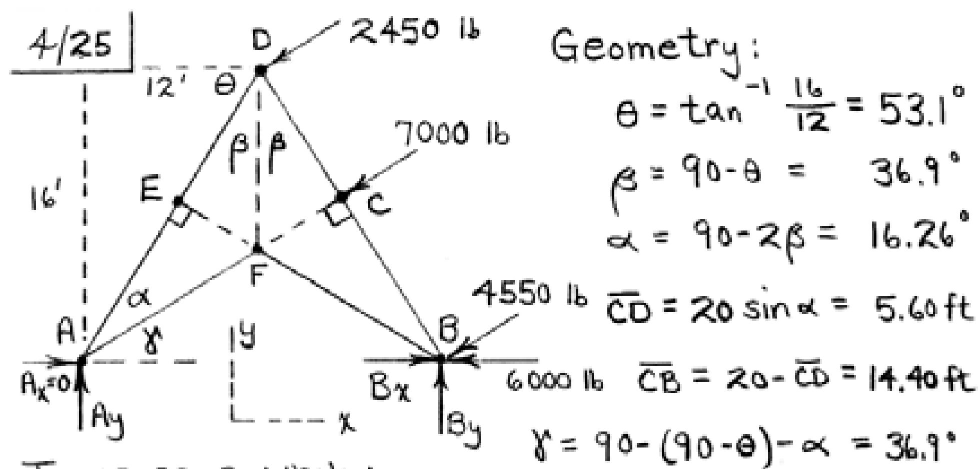
(a) $[m+3=13] > [2j=12]$ so redundant members.
Remove one member connecting B, C, D, and E.

(b) $[m+3=12] = [2j=12]$ so sufficient no. of members, but redundancy in external supports. Place A or F on roller.

(c) $[m+3=9] > [2j=8]$ so redundant members. Supports are also redundant. Remove AE or BE. Supports are then OK.

(d) $[m+3=12] = [2j=12]$ so sufficient no. of members, but redundancy in external supports. Place B on roller or remove member CD.

4/25



Geometry:

$\theta = \tan^{-1} \frac{16}{12} = 53.1°$

$\beta = 90 - \theta = 36.9°$

$\alpha = 90 - 2\beta = 16.26°$

$\overline{CD} = 20 \sin\alpha = 5.60 \text{ ft}$

$\overline{CB} = 20 - \overline{CD} = 14.40 \text{ ft}$

$\gamma = 90 - (90 - \theta) - \alpha = 36.9°$

Truss as a whole:

$\circlearrowleft \Sigma M_B = 0: \; 7000(14.4) + 2450(20) - 24A_y = 0$

$A_y = 6240 \text{ lb}$

$\Sigma F_y = 0: \; B_y + 6240 - (2450 + 7000 + 4550)\sin 36.9°$

$= 0, \quad B_y = 2160 \text{ lb}$

$\Sigma F_x = 0: \; B_x - (2450 + 7000 + 4550)\cos 36.9°$

$- 6000 = 0, \quad B_x = 17{,}200 \text{ lb}$

Joint A:

$\Sigma F_x = 0: -AE \cos 53.1° + AF \cos 36.9° = 0$

$\Sigma F_y = 0: 6240 - AE \sin 53.1° + AF \sin 36.9° = 0$

$\Rightarrow AF = 13{,}380$ lb T, $AE = 17{,}830$ lb C

From joint E, $ED = AE = 17{,}830$ lb C

Joint D:

$\Sigma F_x = 0: 17{,}830 \sin 36.9° - CD \sin 36.9°$

$- 2450 \cos 36.9° = 0$, $CD = 14{,}570$ lb C

$\Sigma F_y = 0: (14{,}570 + 17{,}830) \cos 36.9° - FD$

$- 2450 \sin 36.9° = 0$, $FD = 24{,}500$ lb T

Joint F:

$\Sigma F_x = 0: BF \cos 36.9° - (13{,}380 + 7000) \cos 36.9° = 0$, $BF = 20{,}400$ lb T

(Joint B checks.)

The maximum force occurs in member FD:

$FD = 24{,}500$ lb T

4/26 Structure is statically indeterminate externally; member AE in main vertical tower is indeterminate.

F: (free-body diagram: FJ at 45°, FG, 5 tons)

$\Sigma F_y = 0: \; FJ\frac{\sqrt{2}}{2} - 5 = 0$

$FJ = 5\sqrt{2}$ tons T

$\Sigma F_x = 0: \; FG - 5\sqrt{2}\left(\frac{\sqrt{2}}{2}\right) = 0$

$FG = 5$ tons C

J: (free-body diagram: IJ at α, $5\sqrt{2}$ tons at 45°, GJ)

$\alpha = \tan^{-1}\left(\frac{10}{90}\right) = 6.34°$

$\Sigma F_x = 0: \; -IJ\cos\alpha + 5\sqrt{2}\left(\frac{\sqrt{2}}{2}\right) = 0$

$IJ = 5.03$ tons T

$\Sigma F_y = 0: \; 5.03\sin\alpha - 5\sqrt{2}\left(\frac{\sqrt{2}}{2}\right) + GJ = 0,$ $\underline{GJ = 4.44 \text{ tons C}}$

G: (free-body diagram: GI at β, 4.44 tons, GH, 5 tons, 5 tons)

$\beta = \tan^{-1}\left(\frac{11.67}{15}\right) = 37.9°$

$\Sigma F_y = 0: \; -5 - 4.44 + GI\sin\beta = 0$

$\underline{GI = 15.38 \text{ tons T}}$

▶4/27

Joint A

$\theta = \tan^{-1}\frac{1}{2} = 26.6°$

$\Sigma F_y = 0;\ AB \sin 26.6° - 1.8 \cos 15° = 0$

$AB = 3.89$ kN C

$\Sigma F_x = 0;\ AE = 1.8 \sin 15° + 3.89 \cos\theta$

$AE = 3.94$ kN T

Joint E gives EB = 0

Joint B $\Sigma F_{y'} = 0$ so DB = 0

CB = 3.89 kN C

without diagonals, FD would lengthen & CJ shorten, so FD is tension & CJ = 0

Joint H

$\Sigma F_y = 0;\ HG \sin 26.6° - 1.8 \cos 15° = 0$

$HG = 3.89$ kN C

$\Sigma F_x = 0;\ HI + 1.8 \sin 15° - 3.89 \cos 26.6° = 0$

$HI = 3.01$ kN T

with IG = GJ = JC = 0, JD = HI

Joint D

$\Sigma F_x = 0;\ FD \cos 45° + 3.01 = 3.94$

$FD = 1.318$ kN T

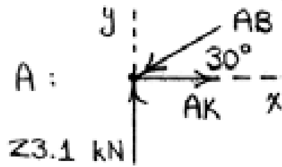
$\Sigma F_y = 0;\ CD = 1.318 \sin 45° =$ **0.932 kN C**

▸4/28 For entire truss,

$\Sigma M_I = 0$: $30A - 0.866[20(5+10+15) + 10(20)] = 0$

$A = 23.1$ kN

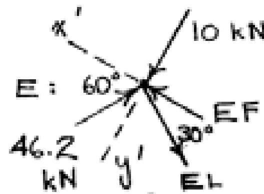
Forces in BP, PC, DN, CN, CO, ON, NE, EM (in this order) are seen to be zero.



$\Sigma F_y = 0$: $-0.5\,AB + 23.1 = 0$

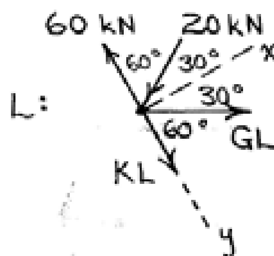
$AB = 46.2$ kN C

$DE = AB$



$\Sigma F_{y'} = 0$: $0.5\,EL + 10 - 0.866(46.2) = 0$, $EL = 60$ kN T

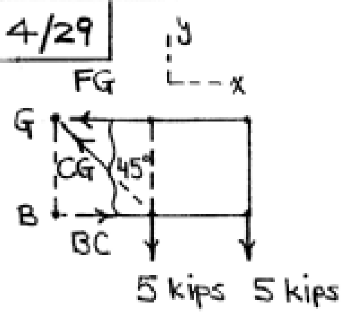
$\Sigma F_{x'} = 0$: $EF - 60(0.866) - 46.2(0.5) = 0$, $\underline{EF = 75.1 \text{ kN C}}$



$\Sigma F_x = 0$: $-20\cos 30° + GL\cos 20° = 0$, $\underline{GL = 20 \text{ kN T}}$

$\Sigma F_y = 0$: $KL - 60 + 20(0.5) + 20(0.5) = 0$, $\underline{KL = 40 \text{ kN T}}$

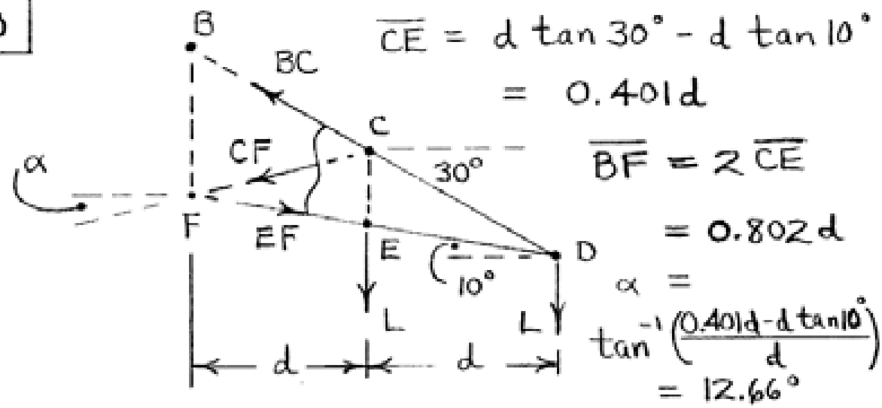
4/29



$\Sigma F_y = 0$: $CG \sin 45° - 5 - 5 = 0$

$\underline{CG = 14.14 \text{ kips } T}$

4/30



$\overline{CE} = d \tan 30° - d \tan 10°$

$= 0.401d$

$\overline{BF} = 2\,\overline{CE}$

$= 0.802d$

$\alpha = \tan^{-1}\left(\frac{0.401d - d\tan 10°}{d}\right) = 12.66°$

$\curvearrowleft + \sum M_C = 0: \; -Ld + EF\cos 10°(0.401d) = 0$

$\underline{EF = 2.53L \;\; C}$

$\curvearrowleft + \sum M_F = 0: \; -Ld - L(2d) + BC\cos 30°(0.802d) = 0$

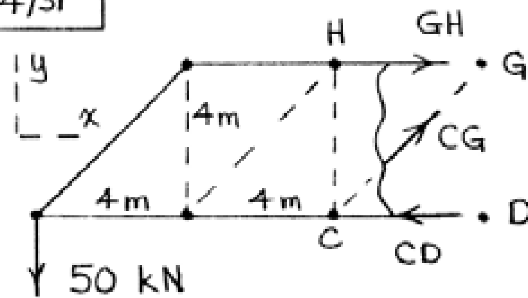
$\underline{BC = 4.32L \;\; T}$

$\curvearrowleft + \sum M_D = 0: \; Ld + CF\cos 12.66°(d \tan 30°)$

$+ CF \sin 12.66°(d) = 0 \qquad CF = -1.278L$

or $\underline{CF = 1.278L \;\; C}$

4/31

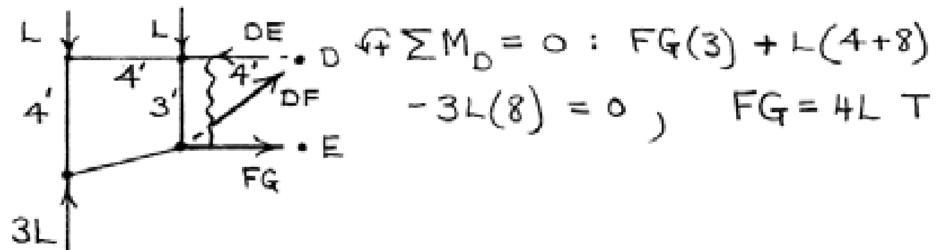


$\Sigma F_y = 0$: $CG \sin 45° - 50 = 0$, $\underline{CG = 70.7 \text{ kN } T}$

$\Sigma M_C = 0$: $GH(4) - 50(8) = 0$, $\underline{GH = 100 \text{ kN } T}$

All members except EF are statically determinate, so above solution is unaffected by the redundant support.

4/32 From entire truss, $A = B = 3L$.



$\circlearrowleft + \Sigma M_D = 0$: $FG(3) + L(4+8) - 3L(8) = 0$, $FG = 4L$ T

Joint G:

$\theta = \tan^{-1}\left(\frac{1}{4}\right) = 14.04°$

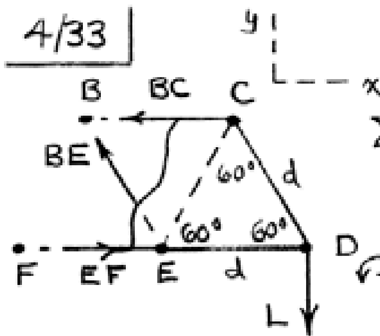
$\Sigma F_x = 0$: $-4L + GH \cos\theta = 0$

$GH = 4.12L$ T

$\Sigma F_y = 0$: $DG - 4.12 \sin\theta = 0$

$\underline{DG = 1.000L \text{ T}}$

4/33

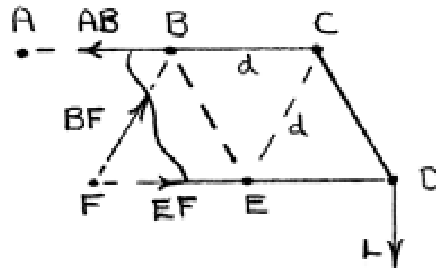


$\Sigma F_y = 0: \; BE \sin 60° - L = 0$

$BE = \frac{2L}{\sqrt{3}}$ T

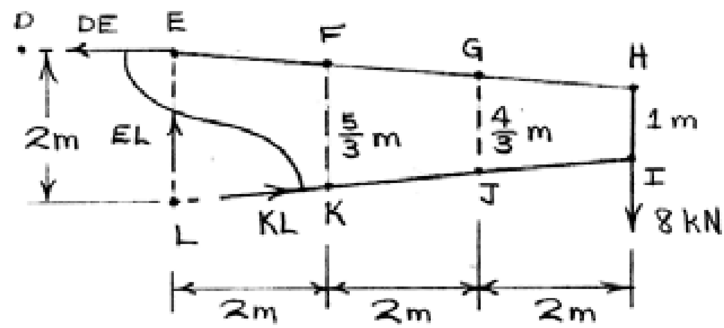
$\circlearrowleft + \Sigma M_E = 0: \; BC(d \cos 30°) - Ld = 0$

$BC = \frac{2L}{\sqrt{3}}$ T



$\Sigma F_y = 0: \; BF \sin 60° - L = 0, \quad BF = \frac{2L}{\sqrt{3}}$ C

4/34



$\Sigma M_L = 0$: $DE(2) - 8(6) = 0$, $\underline{DE = 24 \text{ kN } T}$

Joint D:

24 kN

45°

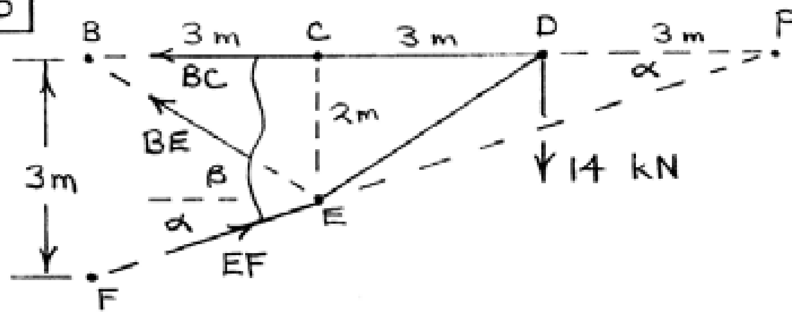
DL

DC

$\rightarrow \Sigma F_x = 0$: $24 - DL \cos 45° = 0$

$\underline{DL = 33.9 \text{ kN } C}$

4/35



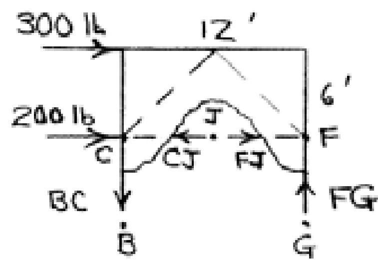
$\alpha = \tan^{-1}\left(\frac{2}{6}\right) = 18.43°, \quad \beta = \tan^{-1}\frac{2}{3} = 33.7°$

$\curvearrowleft + \Sigma M_E = 0: \quad BC(2) - 14(3) = 0, \quad \underline{BC = 21 \text{ kN T}}$

$\curvearrowleft + \Sigma M_P = 0: -BE \sin\beta(9) + 14(3) = 0, \quad \underline{BE = 8.41 \text{ kN T}}$

$\curvearrowleft + \Sigma M_B = 0: \quad EF\cos\alpha(3) - 14(6) = 0, \quad \underline{EF = 29.5 \text{ kN C}}$

4/36



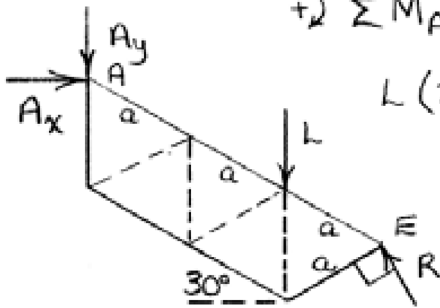
$\circlearrowleft + \Sigma M_C = 0: \; -300(6) + FG(12) = 0$

$\underline{FG = 150 \text{ lb } C}$

$\Sigma M_F = 0: \; -300(6) + BC(12) = 0$

$\underline{BC = 150 \text{ lb } T}$

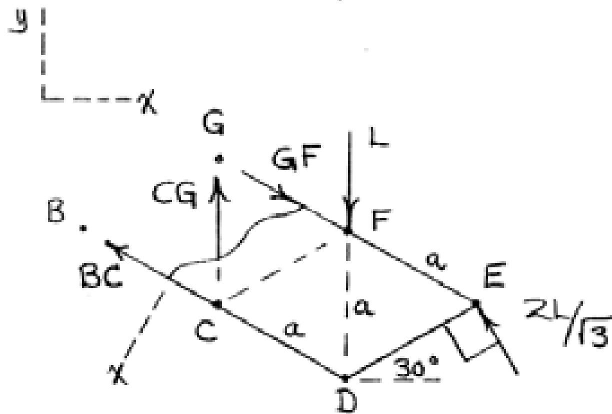
4/37



$\circlearrowright \Sigma M_A = 0:$

$$L(2a\cos 30°) - R(3a\sin 30°) = 0$$

$$R = 2L/\sqrt{3}$$



$\Sigma M_G = 0: \; La\cos 30° + BC\, a\cos 30° - \frac{2L}{\sqrt{3}} 2a\sin 30° = 0$

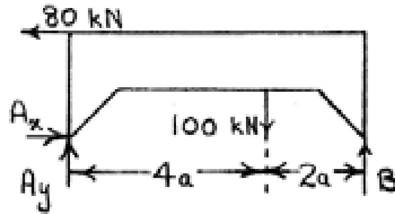
$\underline{BC = L/3 \quad T}$

$\Sigma F_x = 0: \; CG\cos 30° - L\cos 30° + \frac{2L}{\sqrt{3}} \sin 30° = 0$

$\underline{CG = L/3 \quad T}$

4/38 $\circlearrowleft \Sigma M_A = 0: \quad B(6a) - 100(4a) + 80(2a) = 0$

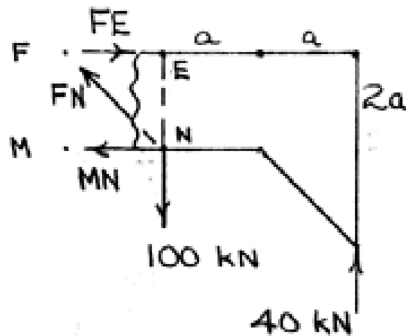
$B = 40 \text{ kN}$



For section,

$\Sigma F_y = 0: \quad \frac{FN}{\sqrt{2}} + 40 - 100 = 0$

$FN = 84.8 \text{ kN T}$



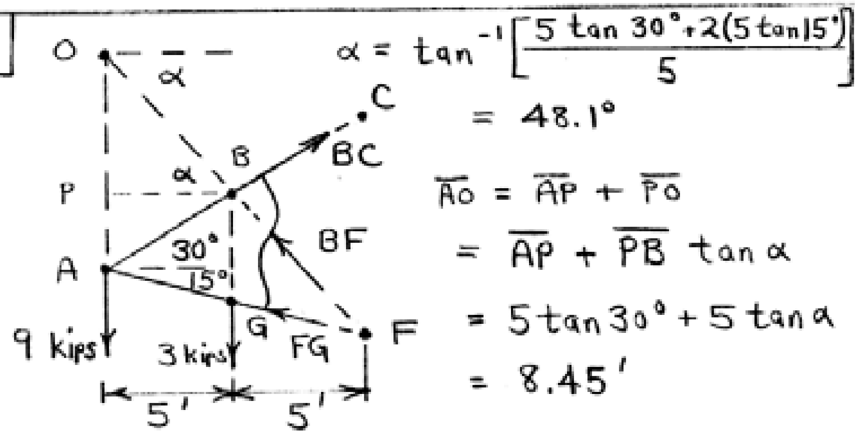
$\Sigma M_E = 0: \quad 40(2a) - 84.8\frac{a}{\sqrt{2}}$

$- MN(a) = 0, \quad MN = 20 \text{ kN T}$

For section through GF & LM, $\Sigma F_y = 0$ gives

$GM = 84.8 \text{ kN T}$.

4/39



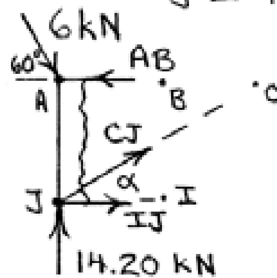
$$\alpha = \tan^{-1}\left[\frac{5\tan 30° + 2(5\tan 15°)}{5}\right] = 48.1°$$

$$\overline{AO} = \overline{AP} + \overline{PO} = \overline{AP} + \overline{PB}\tan\alpha = 5\tan 30° + 5\tan\alpha = 8.45'$$

$$\Sigma M_A = 0: \ (BF\cos\alpha)\,\overline{AO} - 3(5) = 0$$

<u>$BF = 2.66$ kips C</u>

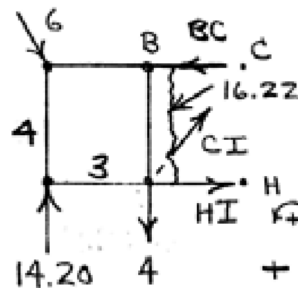
4/40 From truss as a whole and $\Sigma M_F = 0$, $J = 14.20$ kN.



$\Sigma F_y = 0: \; 14.20 - 6 \sin 60° + CJ \sin\alpha = 0$

where $\alpha = \tan^{-1}\left(\frac{4}{6}\right) = 33.7°$

$\therefore$ $\underline{CJ = -16.22 \text{ kN } C}$



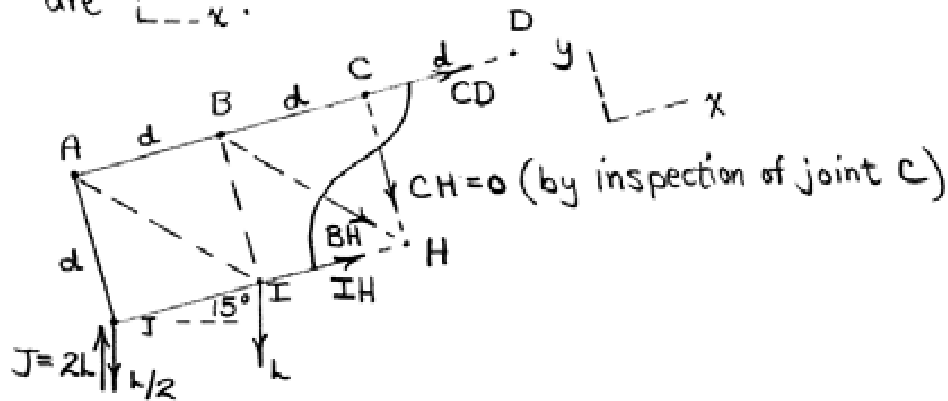
$\Sigma F_y = 0: -6 \sin 60° + 14.20 - 4 - 16.22 \sin\alpha + CI \frac{4}{5} = 0$

$\underline{CI = 5.00 \text{ kN } T}$

$\curvearrowleft\!+ \; \Sigma M_C = 0: (6 \sin 60°) 6 - (14.20) 6 + 4(3) + HI(4) = 0$, $\underline{HI = 10.50 \text{ kN } T}$

$\Sigma F_x = 0: \; 6 \cos 60° - 16.22 \cos\alpha + 5\left(\frac{3}{5}\right) + 10.5 - BC = 0$, $\underline{BC = 3.00 \text{ kN } C}$

4/41 From the truss as a whole, $F_x = 0$ and $J_y = F_y = 2L\ (\uparrow)$, where the coordinates are y | — x.



$\Sigma F_y = 0: \ -BH\frac{\sqrt{2}}{2} + (2L - \frac{L}{2} - L)\cos 15° = 0$

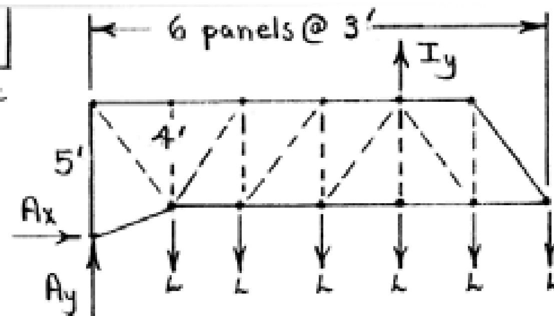
$\underline{BH = 0.683L \ T}$

$\Sigma M_H = 0: \ (\frac{L}{2} - 2L)(2d\cos 15°) + L(d\cos 15°) - CD(d) = 0, \quad CD = -1.932L$

So $\underline{CD = 1.932L \ C}$

4/42

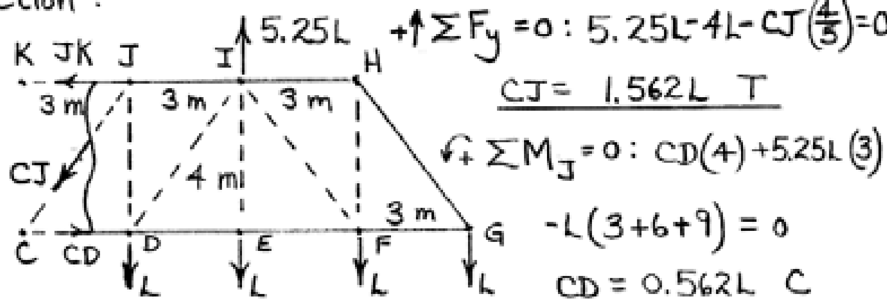
Entire truss:



$\curvearrowleft + \Sigma M_A = 0: \; I_y(12) - L(3+6+9+12+15+18) = 0$

$I_y = 5.25L$

Section :



$+\uparrow \Sigma F_y = 0: \; 5.25L - 4L - CJ\left(\frac{4}{5}\right) = 0$

$\underline{CJ = 1.562L \;\; T}$

$\curvearrowleft + \Sigma M_J = 0: \; CD(4) + 5.25L(3)$

$-L(3+6+9) = 0$

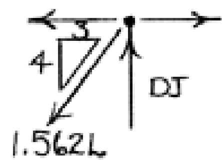
$\underline{CD = 0.562L \;\; C}$

From $\overset{+}{\rightarrow} \Sigma F_x = 0$, $JK = 0.562L \;\; T$.

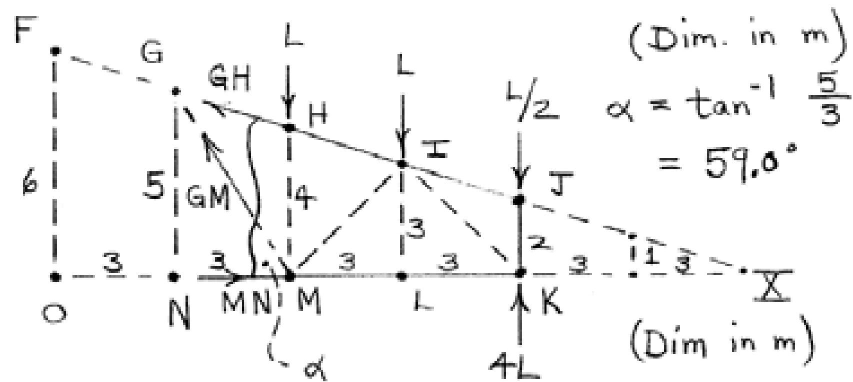
Joint J:

$+\uparrow \Sigma F_y = 0: \; DJ - 1.562L\left(\frac{4}{5}\right) = 0$

$\underline{DJ = 1.250L \;\; C}$



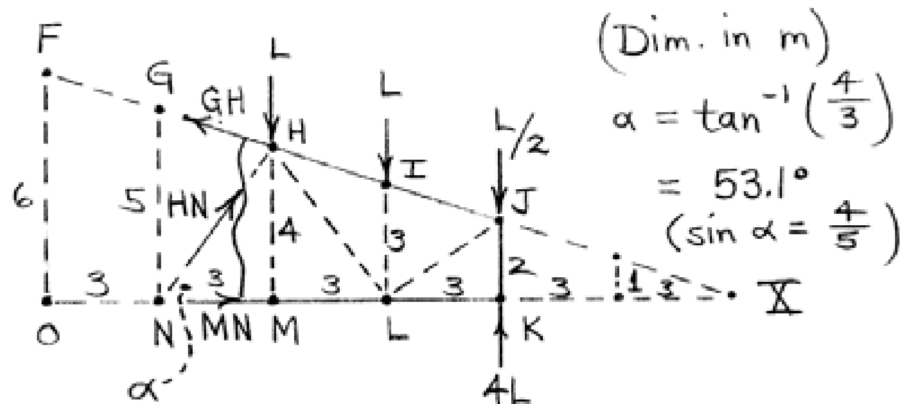
4/43 From truss as a whole, the reactions at A and K are 4L (up).



$$\curvearrowleft + \Sigma M_X = 0: \left(\frac{L}{2} - 4L\right)6 + L(9) + L(12) - GM \sin(59.0°)(12) = 0$$

$\underline{GM = 0}$

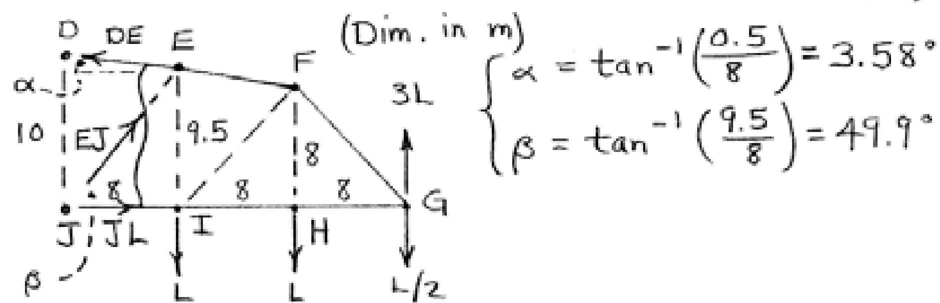
4/44 From the truss as a whole, the external reactions at A and K are 4L (up)



$$\circlearrowleft + \Sigma M_X = 0 : \left(\frac{L}{2} - 4L\right)6 + L(9) + L(12)$$

$$- HN\left(\frac{4}{5}\right)(15) = 0, \quad \underline{HN = 0}$$

4/45 From the truss as a whole, the external reactions at A and G are 3L (up)



$\alpha = \tan^{-1}\left(\frac{0.5}{8}\right) = 3.58°$

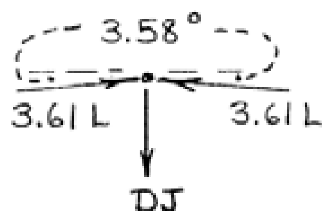
$\beta = \tan^{-1}\left(\frac{9.5}{8}\right) = 49.9°$

$\circlearrowleft + \sum M_J = 0: \ -L(8) - L(16) - \frac{L}{2}(24) + 3L(24) + DE\cos 3.58°\,(10) = 0, \quad DE = -3.61L$

$+\uparrow \sum F = 0: \ -3.61L\sin 3.58° - \frac{5}{2}L + 3L + EJ\sin 49.9° = 0$

$EJ = -0.360L$ or $\underline{EJ = 0.360L\ T}$

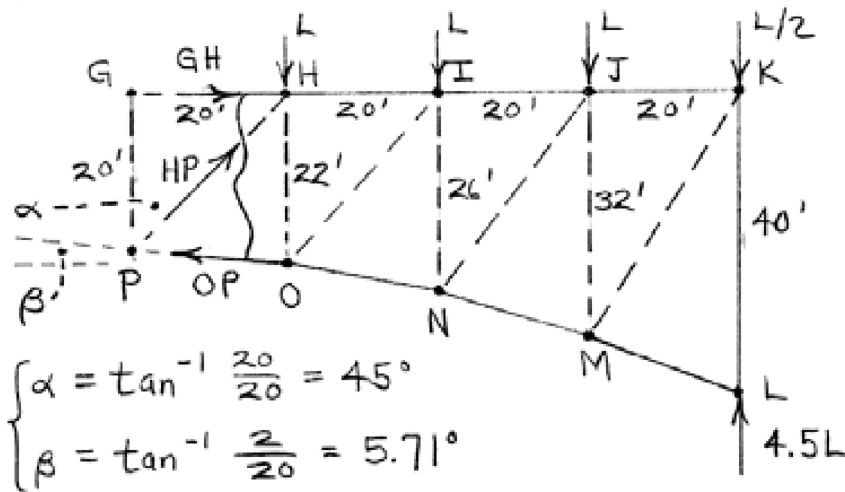
Joint D (using symmetry):



$+\uparrow \sum F = 0: \ 2(3.61L\sin 3.58°) - DJ = 0$

$\underline{DJ = 0.45L\ T}$

4/46 From the truss as a whole, the external reactions at A and L are 4.5L (up).



$$\begin{cases} \alpha = \tan^{-1} \frac{20}{20} = 45° \\ \beta = \tan^{-1} \frac{2}{20} = 5.71° \end{cases}$$

$\circlearrowleft + \Sigma M_H = 0: -L(20) - L(40) - \frac{L}{2}(60) + 4.5L(60)$

$-OP\cos 5.71°(22') = 0, \quad OP = 8.22L \ T$

$\uparrow \Sigma F = 0: -3.5L + 4.5L + 8.22L\sin 5.71°$

$+ HP\sin 45° = 0, \quad HP = -2.57L$

or $\underline{HP = 2.57L\ T}$

4/47 By symmetry, A = G = 150 kN

$\circlearrowleft \Sigma M_F = 0: \; 150(4) + HI\,(7.902) = 0$

$HI = -75.9$ kN T

HI FH F FG H γ α β 150 kN

(Note: $\alpha = \tan^{-1}\left(\frac{14}{4}\right) = 74.1°$

$\beta = \tan^{-1}\left(\frac{14}{16}\right) = 41.2°$

$\gamma = \alpha - \beta = 32.9°$

Then $d_\perp = FG \sin\gamma = \sqrt{14^2+4^2}\,\sin\gamma = 7.902$ m)

D DE 75 E δ EI 25 FI I F 75.9 H G 150

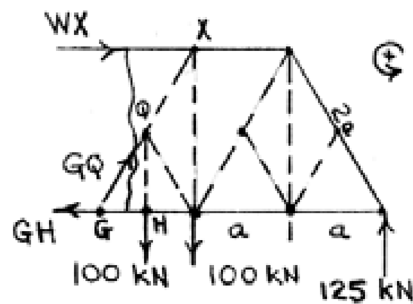
$\delta = \tan^{-1}\left(\frac{3}{10}\right) = 16.7°$

$\circlearrowleft \Sigma M_E = 0: -25(6) + 150(10)$
$- FI(4) - (75.9\sin\beta)(6) - (75.9\cos\beta)(4)$
$= 0$, $FI = 205$ kN T

$\Sigma M_I = 0: -75(6) - 25(12) + 150(16)$
$-(DE\cos\delta)(4) - (DE\sin\delta)(6) = 0$
$DE = 297$ kN C

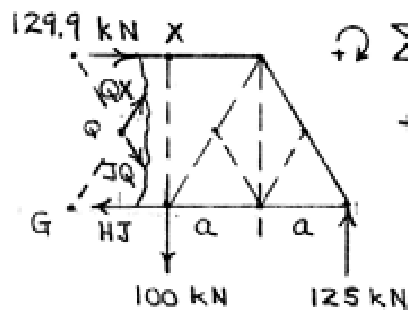
$\Sigma F_y = 0: -75 - 25 + 150 - 297\sin\delta + 75.9\sin\beta$
$+ EI\,\frac{4}{\sqrt{52}} = 0$, $EI = -26.4$ kN T

4/48 From truss as a whole, $\Sigma M_A = 0$ gives $N = 125$ kN.



$$\circlearrowleft^{+} \Sigma M_G = 0: \ 125(3a) - 100\left(\frac{3a}{2}\right) - WX\left(2a\frac{\sqrt{3}}{2}\right) = 0$$

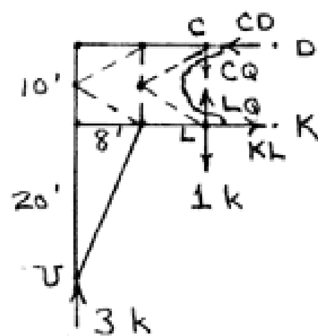
$WX = 129.9$ kN C



$$\circlearrowleft^{+} \Sigma M_G = 0: \ 100(a) + 129.9(a\sqrt{3}) + JQ\left(a\frac{\sqrt{3}}{2}\right) - 125(3a) = 0$$

$\underline{JQ = 57.7 \text{ kN C}}$

►4/49 From truss as a whole, $\begin{cases} U = 3 \text{ kips} \\ V = 4 \text{ kips} \end{cases}$



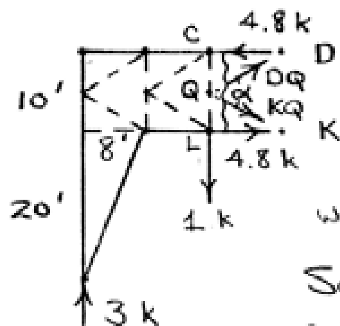
$$\circlearrowleft{+}\ \Sigma M_C = 0:\ KL(10) - 3(16) = 0$$

$$KL = 4.8 \text{ kips T}$$

$$\Sigma M_L = 0:\quad CD(10) - 3(16) = 0$$

$$CD = 4.8 \text{ kips C}$$

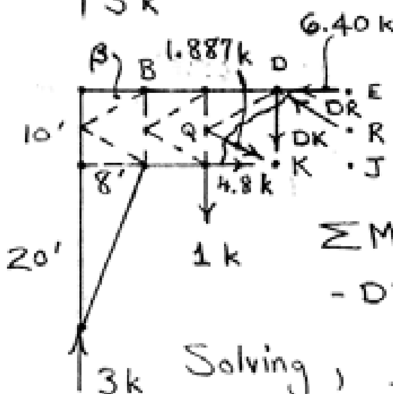
(From a similar right-hand section, $DE = 6.40$ kips C.)



$$\Sigma M_D = 0:\ -3(24) + 1(8) + KQ\left(\sqrt{8^2+5^2}\ \sin\alpha\right) + 4.8(10) = 0,$$

where $\alpha = 180 - 2\tan^{-1}\left(\frac{8}{5}\right) = 64.0°$

Solving, $KQ = 1.887$ kips T



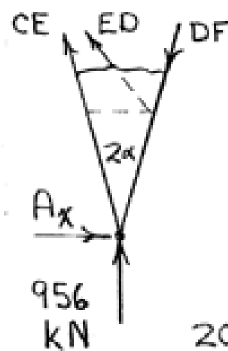
$$\beta = \tan^{-1}\left(\frac{5}{8}\right) = 32.0°$$

$$\Sigma F_x = 0:\ -6.40 + 1.887\cos\beta + 4.8 - DR\cos\beta = 0,\quad DR = 0$$

$$\Sigma M_B = 0:\ -3(8) - 1(8) + 4.8(10) - DK(16) = 0$$

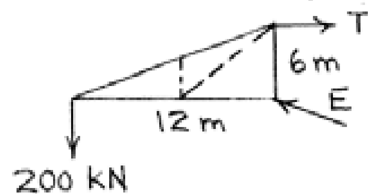
Solving, $\underline{DK = 1 \text{ kip T}}$

4/50 Crane as a whole: $\Sigma M_B = 0$: $1000(24) + 200(52) = 36\,A_y$, $A_y = 956$ kN



$\Sigma M_A = 0$ requires ED = 0 (also CF = 0)

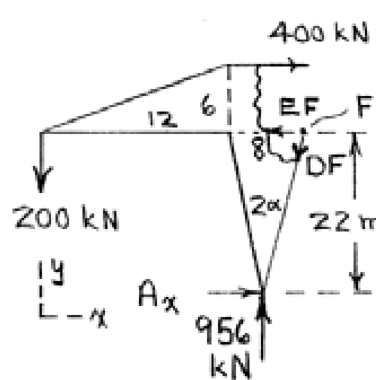
$\alpha = \tan^{-1}\left(\frac{4}{22}\right) = 10.30$, $\cos\alpha = 0.984$



$\Sigma M_E = 0$:

$6T - 12(200) = 0$

$T = 400$ kN



$\Sigma M_F = 0$: $200(20) - 400(6) + A_x(22) - 956(4) = 0$

$A_x = 101.1$ kN (to right)

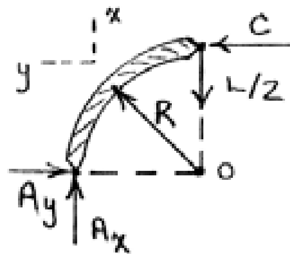
$\Sigma M_A = 0$: $200(16) + EF(22) - 400(28) = 0$

EF = 364 kN C

$\Sigma F_y = 0$: $0.984\,DF + 200 - 956 = 0$

DF = 768 kN C

▸4/51 By symmetry, the force which the right half exerts on the left half at C is horizontal:

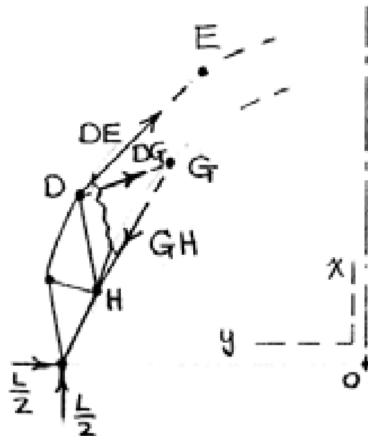


$$\Sigma M_A = 0: \; CR - \frac{L}{2}R = 0$$

$$C = L/2$$

$$\Sigma F_y = 0: \; -A_y + \frac{L}{2} = 0, \; A_y = \frac{L}{2}$$

$$\Sigma F_x = 0: \; A_x - \frac{L}{2} = 0, \; A_x = \frac{L}{2}$$



$$\underline{r}_{OD} + \underline{r}_{DE} = \underline{r}_{OE}$$

$$\therefore \underline{r}_{DE} = \underline{r}_{OE} - \underline{r}_{OD}$$

$$= 1.1R(\cos 30° \underline{i} + \sin 30° \underline{j})$$

$$- 1.1R(\cos 60° \underline{i} + \sin 60° \underline{j})$$

$$= R(0.403 \underline{i} - 0.403 \underline{j})$$

So force $\underline{DE} = DE \dfrac{\underline{r}_{DE}}{r_{DE}}$

$$= DE(0.707\underline{i} - 0.707\underline{j})$$

Similarly, force $\underline{GH} = GH(-0.866\underline{i} + 0.500\underline{j})$

force $\underline{DG} = DG(0.264\underline{i} - 0.965\underline{j})$

$$\Sigma F_x = 0: \; \frac{L}{2} + 0.707\,DE - 0.866\,GH + 0.264\,DG = 0 \quad (1)$$

$$\Sigma F_y = 0: \; -\frac{L}{2} - 0.707\,DE + 0.500\,GH - 0.965\,DG = 0 \quad (2)$$

$$\Sigma M_O = 0: \; -\frac{L}{2}R\underline{k} + \underline{r}_{OD} \times (\underline{DE} + \underline{DG})$$

$$+ \underline{r}_{OH} \times \underline{GH},$$

where $\underline{r}_{OH} = 0.9R(\cos 75° \underline{i} + \sin 75° \underline{j})$

Carrying out the cross products and collecting terms:

$$-1.063\,DE + 0.869\,GH - 0.782\,DG = \frac{L}{2} \quad (3)$$

Simultaneous solution of Eqs. (1)–(3):

$DE = 0.839L$ T, $GH = 1.090L$ C, $\underline{DG = -0.569L \;\; C}$

▶ 4/52 FI = 0, by inspection of joint I.

Joint G:

GH, FG, 60°, 30°, y, x, 25 kN

$$\begin{cases} \Sigma F_x = 0: -GH\left(\tfrac{1}{2}\right) + FG\left(\tfrac{\sqrt{3}}{2}\right) = 0 \\ \Sigma F_y = 0: GH\left(\tfrac{\sqrt{3}}{2}\right) + FG\left(\tfrac{1}{2}\right) - 25 = 0 \end{cases}$$

1st eq.: $GH = \sqrt{3}\, FG$

2nd eq.: $\sqrt{3}\, FG\left(\tfrac{\sqrt{3}}{2}\right) + \tfrac{1}{2} FG = 25$

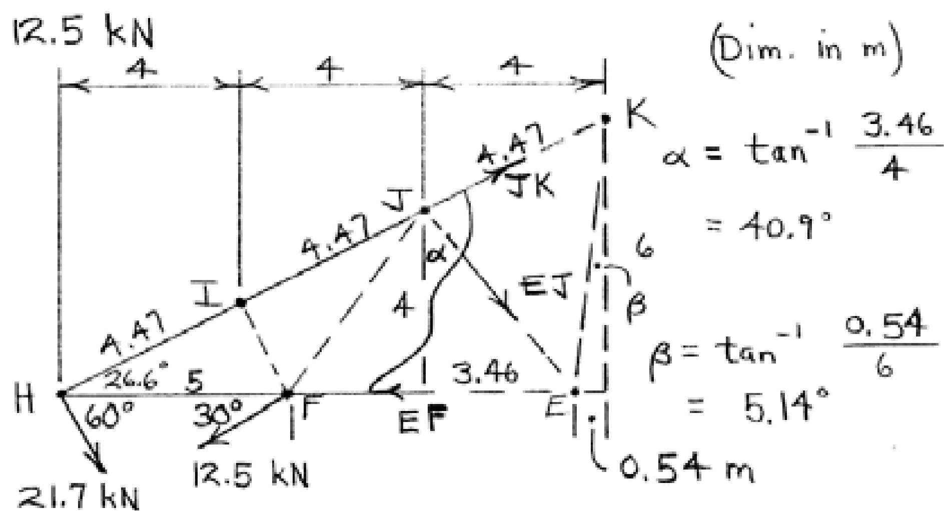
$\Rightarrow FG = 12.5$ kN T, $GH = 21.7$ kN T

Joint F:

FJ, FH, 53.1°, 30°, EF, 12.5 kN

$\Sigma F_y = 0$: $FJ(\sin 53.1°) - 12.5 \sin 30° = 0$

$FJ = 7.81$ kN T



$\alpha = \tan^{-1} \dfrac{3.46}{4} = 40.9°$

$\beta = \tan^{-1} \dfrac{0.54}{6} = 5.14°$

4/53

(Dim. in m)

$$\underline{BA} = BA\left(\frac{-\underline{i} - 3\underline{j} - 6\underline{k}}{\sqrt{1^2 + 3^2 + 6^2}}\right)$$

$$= BA\left(-\underline{i} - 3\underline{j} - 6\underline{k}\right)/\sqrt{46}$$

$$\underline{CA} = CA\left(\frac{+2\underline{i} - 6\underline{k}}{\sqrt{2^2 + 6^2}}\right)$$

$$= CA\left(+2\underline{i} - 6\underline{k}\right)/\sqrt{40}$$

$$\underline{DA} = DA\left(\frac{-\underline{i} + 3\underline{j} - 6\underline{k}}{\sqrt{1^2 + 3^2 + 6^2}}\right)$$

$$= DA\left(-\underline{i} + 3\underline{j} - 6\underline{k}\right)/\sqrt{46}$$

$$\Sigma \underline{F} = \underline{0}: \quad \underline{BA} + \underline{CA} + \underline{DA} - 5\left(\cos 30^\circ \underline{k} + \sin 30^\circ \underline{j}\right) = \underline{0}$$

yields

$$\underline{i}: -\frac{1}{\sqrt{46}} BA + \frac{2}{\sqrt{40}} CA - \frac{1}{\sqrt{46}} DA = 0$$

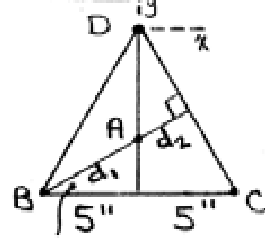
$$\underline{j}: -\frac{3}{\sqrt{46}} BA + \frac{3}{\sqrt{46}} DA - 5\left(\tfrac{1}{2}\right) = 0$$

$$\underline{k}: -\frac{6}{\sqrt{46}} BA - \frac{6}{\sqrt{40}} CA - \frac{6}{\sqrt{46}} DA - 5\frac{\sqrt{3}}{2} = 0$$

Solution:

$$\begin{cases} BA = -4.46 \text{ kN} \\ CA = -1.521 \text{ kN} \\ DA = 1.194 \text{ kN} \end{cases}$$

4/54 Top view of base:



$\cos 30° = \frac{d_1 + d_2}{10}$, $d_1 + d_2 = 8.66"$

$\cos 30° = \frac{5}{d_1}$, $d_1 = 5.77"$

$d_2 = 8.66 - 5.77 = 2.89"$

For joint A, assuming symmetry:

$\underline{F}_{BA} = P\left[\frac{5\underline{i} + 2.89\underline{j} + 16\underline{k}}{(5^2 + 2.89^2 + 16^2)^{1/2}}\right] = P(0.294\underline{i} + 0.1697\underline{j} + 0.941\underline{k})$

$\underline{F}_{CA} = P(-0.294\underline{i} + 0.170\underline{j} + 0.941\underline{k})$, $\underline{F}_{DA} = P(-0.339\underline{j} + 0.941\underline{k})$

$\Sigma F_z = 0$ at A: $3(0.941\,P) - 800 = 0$, $P = 283$ lb

For joint C, assuming symmetry:

$\underline{F}_{BC} = -Q\underline{i}$, $\underline{F}_{CD} = Q(-\sin 30°\,\underline{i} + \cos 30°\,\underline{j})$

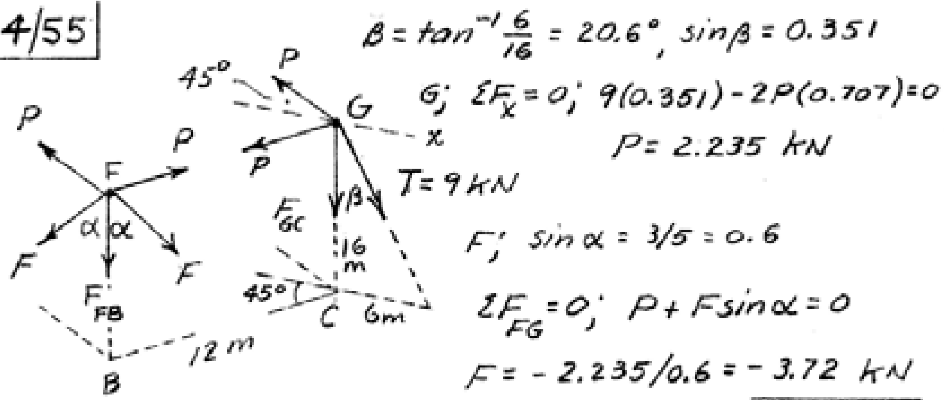
Normal $\underline{N} = 267\underline{k}$ lb

$\Sigma \underline{F} = \underline{0}$ at C: $\underline{N} + \underline{F}_{BC} + \underline{F}_{CD} + \underline{F}_{AC} = \underline{0}$

$267\underline{k} - Q\underline{i} + 283(0.294\underline{i} - 0.1697\underline{j} - 0.941\underline{k})$

$+ Q(-0.5\underline{i} + 0.866\underline{j}) = \underline{0}$

Solving, $Q = 55.6$ lb $\Rightarrow$ <u>BC = BD = CD = 55.6 lb T</u>



4/55
β = tan⁻¹ 6/16 = 20.6°, sin β = 0.351
G; ΣFx = 0; 9(0.351) − 2P(0.707) = 0
P = 2.235 kN
F; sin α = 3/5 = 0.6
ΣFFG = 0; P + F sin α = 0
F = − 2.235/0.6 = − 3.72 kN
P
F
α α
FFB
B
12 m
45°
G
x
T = 9 kN
β
FGC
16 m
C
6 m

4/56 From truss as a whole $\Sigma M_y = 0$ gives tension in vertical wire at C $T_C = \frac{1}{3} mg$

By symmetry & $\Sigma F_z = 0$; $T_A = T_B = \frac{1}{3} mg$

Joint A;

$\underline{F}_{AB} = F_{AB}\,\underline{j}$

$\underline{F}_{AC} = \frac{F_{AC}}{5}(3\underline{i} + 4\underline{j})$

$\underline{F}_{AD} = \frac{F_{AD}}{\sqrt{53}}(\underline{i} + 4\underline{j} - 6\underline{k})$

$\overline{AD} = \sqrt{4^2 + 1^2 + 6^2}$
$= \sqrt{53}$ m

$\Sigma \underline{F} = \underline{0}$; $\frac{mg}{3}\underline{k} + \underline{F}_{AB} + \underline{F}_{AC} + \underline{F}_{AD} = \underline{0}$

Substitute & collect terms to get

$$\left(\frac{3F_{AC}}{5} + \frac{F_{AD}}{\sqrt{53}}\right)\underline{i} + \left(F_{AB} + \frac{4F_{AC}}{5} + \frac{4F_{AD}}{\sqrt{53}}\right)\underline{j} + \left(\frac{mg}{3} - \frac{6F_{AD}}{\sqrt{53}}\right)\underline{k} = \underline{0}$$

Equate coefficients of $\underline{i}$-, $\underline{j}$-, & $\underline{k}$-terms to zero & get

$F_{AD} = \frac{\sqrt{53}}{18} mg$, $\underline{F_{AC} = -\frac{5}{54} mg}$, $\underline{F_{AB} = -\frac{4}{27} mg}$

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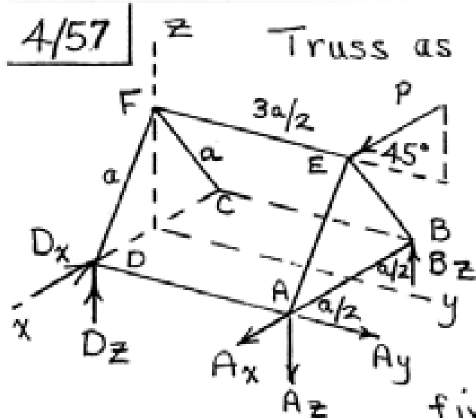

Truss as a whole is statically determinate with 6 supporting constraints. $j = 6$, $m = 12$, $3j = m + 6$ so sufficient members for stability. A, B & D are fixed so F is fixed. E & C are also fixed, so truss is a rigid unit.

$\Sigma M_{A_z} = 0: \quad \frac{P}{\sqrt{2}}\frac{a}{2} - D_x\frac{3a}{2} = 0, \quad D_x = \frac{P}{3\sqrt{2}}, \quad A_x = \frac{P}{3\sqrt{2}}$

$\Sigma M_{AB} = 0: \quad \frac{P}{\sqrt{2}}\frac{a\sqrt{3}}{2} - D_z\frac{3a}{2} = 0, \quad D_z = P/\sqrt{6}$

$\Sigma M_{AD} = 0: \quad \frac{P}{\sqrt{2}}\frac{a}{2} - B_z a = 0, \quad B_z = \frac{P}{2\sqrt{2}}$

$\Sigma F_z = 0$ gives $A_z = \frac{2-\sqrt{3}}{2\sqrt{6}}P$

Forces at C are all zero. From joint E, $\Sigma F_y = 0$ gives $EF = P/\sqrt{2}$ C

Joint F:

$FC = 0 \quad EF = \frac{P}{\sqrt{2}}$

DF AF BF

$$\begin{cases} \underline{AF} = AF(\underline{i} + 3\underline{j} - \sqrt{3}\,\underline{k})/\sqrt{13} \\ \underline{BF} = BF(-\underline{i} + 3\underline{j} - \sqrt{3}\,\underline{k})/\sqrt{13} \\ \underline{DF} = DF(\underline{i} - \sqrt{3}\,\underline{k})/2, \quad EF = -\frac{P}{\sqrt{2}}\underline{j} \end{cases}$$

$$\Sigma \underline{F} = \underline{0} = \left(\frac{AF}{\sqrt{13}} - \frac{BF}{\sqrt{13}} + \frac{DF}{2}\right)\underline{i} + \left(-\frac{P}{\sqrt{2}} + \frac{3AF}{\sqrt{13}} + \frac{3BF}{\sqrt{13}}\right)\underline{j} + \left(-\frac{\sqrt{3}}{\sqrt{13}}AF - \frac{\sqrt{3}}{\sqrt{13}}BF - \frac{\sqrt{3}}{2}DF\right)\underline{k}$$

Solve to obtain $BF = 0$, $DF = -\frac{\sqrt{2}}{3}P$

$\underline{AF = \frac{\sqrt{13}}{3\sqrt{2}}P}$

4/58 | Number of joints is $j = 7$, so $m + 6 = 3j = 21$ and $m = 15$. Since the figure shows only 13 members, 2 more are necessary to ensure stability. Inspection shows that panel ADGE needs a diagonal support AG in order to prevent motion of G toward E. Also, F needs the support of a member DF to fix it in space. Joints E and C then become fixed, and the truss is rigid. (Other possibilities exist for creating stability by adding two members.)

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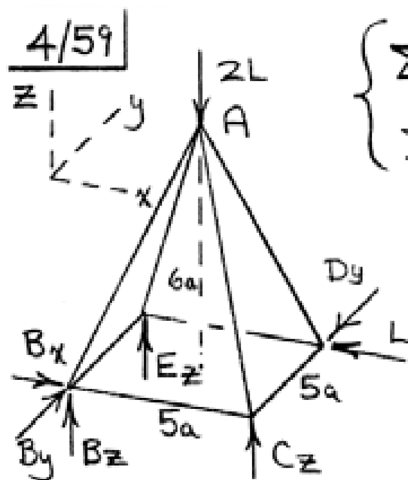
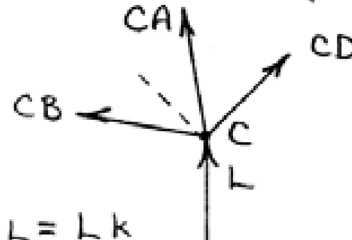


$$\begin{cases} \Sigma M_{BE} = 0 \Rightarrow C_z = L \\ \Sigma M_{Bz} = 0 \Rightarrow D_y = L \end{cases}$$

Joint C: (tensions assumed)



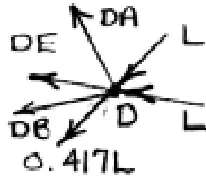
$\underline{CB} = -CB\,\underline{i}$, $\underline{CD} = CD\,\underline{j}$, $\underline{L} = L\,\underline{k}$

$$\underline{CA} = CA\left(\frac{-2.5a\underline{i} + 2.5a\underline{j} + 6a\underline{k}}{\sqrt{(2.5^2 + 2.5^2 + 6^2)a^2}}\right) = CA(-0.359\underline{i} + 0.359\underline{j} + 0.862\underline{k})$$

$\Sigma \underline{F} = \underline{0}$ yields:

$$\left.\begin{aligned} \underline{i}&: -CB - 0.359\,CA = 0 \\ \underline{j}&: \quad CD + 0.359\,CA = 0 \\ \underline{k}&: \quad L + 0.862\,CA = 0 \end{aligned}\right\} \begin{aligned} CA &= -0.1667L \\ CD &= +0.417L \end{aligned}$$

Joint D: $\Sigma \underline{F} = \underline{0}$ yields



$$\begin{cases} \underline{i}: -DE - L - 0.359\,DA - \frac{DB}{\sqrt{2}} = 0 \\ \underline{j}: -0.417L - L - 0.359\,DA - \frac{DB}{\sqrt{2}} = 0 \\ \underline{k}: \quad 0.862\,DA = 0 \end{cases}$$

$DA = 0$, $\underline{DB = -2.00L}$

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Joint F:

$$\underline{L} = \frac{L}{\sqrt{2}}(-\underline{i}+\underline{j}), \quad \underline{F}_{EF} = F_{EF}\,\underline{i}$$

$$\underline{F}_{CF} = \frac{F_{CF}}{\sqrt{5}}(-\underline{j}-2\underline{k})$$

$$\underline{F}_{DF} = \frac{F_{DF}}{\sqrt{5}}(\underline{j}-2\underline{k})$$

$$\Sigma \underline{F} = \underline{L} + \underline{F}_{EF} + \underline{F}_{CF} + \underline{F}_{DF} = \underline{0}$$

$\underline{i}$: $-\frac{L}{\sqrt{2}} + F_{EF} = 0$, $\quad F_{EF} = \frac{L}{\sqrt{2}}$

$\underline{j}$: $\frac{L}{\sqrt{2}} - \frac{F_{CF}}{\sqrt{5}} + \frac{F_{DF}}{\sqrt{5}} = 0$

$\underline{k}$: $-\frac{2}{\sqrt{5}}F_{CF} - \frac{2}{\sqrt{5}}F_{DF} = 0$

$$F_{CF} = \frac{\sqrt{5}\,L}{2\sqrt{2}}, \quad F_{DF} = -\frac{\sqrt{5}\,L}{2\sqrt{2}}$$

Joint C:

$$\underline{F}_{CD} = F_{CD}\,\underline{j}, \quad \underline{F}_{BC} = F_{BC}\,\underline{i}$$

$$\underline{F}_{CE} = F_{CE}(\underline{i}+\underline{j}+2\underline{k})/\sqrt{6}$$

$$\underline{F}_{CF} = F_{CF}(\underline{j}+2\underline{k})/\sqrt{5} = \frac{L}{2\sqrt{2}}(\underline{j}+2\underline{k})$$

$\Sigma F = 0$ yields:

$\underline{i}$: $0 + F_{BC} + \frac{F_{CE}}{\sqrt{6}} = 0$

$\underline{j}$: $F_{CD} + \frac{F_{CE}}{\sqrt{6}} + \frac{L}{2\sqrt{2}} = 0$

$\underline{k}$: $\frac{2F_{CE}}{\sqrt{6}} + \frac{L}{\sqrt{2}} = 0$

$$F_{BC} = \frac{L\sqrt{2}}{4}$$

$$F_{CD} = 0$$

$$F_{CE} = -\frac{L\sqrt{3}}{2}$$

4/61

$$\underline{T}_{AB} = T_{AB}\frac{0.36\underline{i} + 0.8\underline{k}}{\sqrt{0.36^2 + 0.8^2}} = T_{AB}\left[0.410\underline{i} + 0.912\underline{k}\right]$$

$$\underline{T}_{AE} = T_{AE}\frac{-0.36\underline{i} - 0.3\underline{j} + 0.44\underline{k}}{\sqrt{0.36^2 + 0.3^2 + 0.44^2}} = T_{AE}\left[-0.560\underline{i} - 0.467\underline{j} + 0.684\underline{k}\right]$$

$$\underline{T}_{AF} = T_{AF}\frac{-0.36\underline{i} + 0.3\underline{j} + 0.44\underline{k}}{\sqrt{0.36^2 + 0.3^2 + 0.44^2}} = T_{AF}\left[-0.560\underline{i} + 0.467\underline{j} + 0.684\underline{k}\right]$$

$$\left.\begin{aligned}
\Sigma F_x = 0:&\ 0.410T_{AB} - 0.560T_{AE} - 0.560T_{AF} = 0\\
\Sigma F_y = 0:&\ -0.467T_{AE} + 0.467T_{AF} = 0\\
\Sigma F_z = 0:&\ 0.912T_{AB} + 0.684T_{AE} + 0.684T_{AF} - 5 = 0
\end{aligned}\right\}$$

Solution: $T_{AB} = 3.54$ kN, $T_{AE} = T_{AF} = 1.296$ kN

$$\underline{T}_{BC} = T_{BC}\frac{-0.72\underline{i} + 0.3\underline{j} + 0.36\underline{k}}{\sqrt{0.72^2 + 0.3^2 + 0.36^2}} = T_{BC}\left[-0.838\underline{i} + 0.349\underline{j} + 0.419\underline{k}\right]$$

$$\underline{T}_{BD} = T_{BD}\left[-0.838\underline{i} - 0.349\underline{j} + 0.419\underline{k}\right]$$

$$\underline{T}_{BE} = T_{BE}\left[-0.838\underline{i} - 0.349\underline{j} - 0.419\underline{k}\right]$$

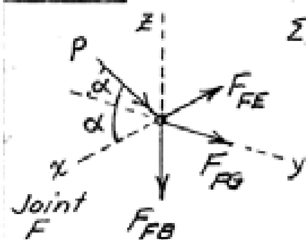
$$\underline{T}_{BF} = T_{BF}\left[-0.838\underline{i} + 0.349\underline{j} - 0.419\underline{k}\right]$$

Note: $T_{BD} = T_{BC}$; $T_{BE} = T_{BF}$ (Symmetry)

Set $\Sigma\underline{F} = \underline{0}$ to obtain $T_{BD} = T_{BC} = 1.491$ kN

$\underline{T_{BE} = T_{BF} = -2.36 \text{ kN (C)}}$

4/62 $\alpha = \cos^{-1}\frac{1}{\sqrt{3}}$; By symmetry $F_{FE} = F_{FG} = F_{FB} = F$

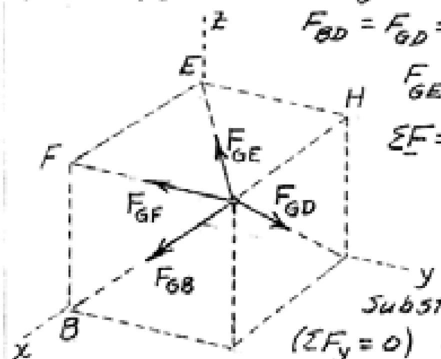


$\Sigma \underline{F} = \underline{0}$; $F(-\underline{i}+\underline{j}-\underline{k}) + \frac{P}{\sqrt{3}}(-\underline{i}+\underline{j}-\underline{k}) = \underline{0}$

$F = -P/\sqrt{3}$ so $\underline{F_{FE} = -P/\sqrt{3}}$ (compression)

By inspection, forces on joints A, C, & H are zero, and by symmetry forces at D are

$F_{BD} = F_{GD} = F_{ED} = R$. Also by symmetry



$F_{GE} = F_{GB}$; $\underline{F}_{GF} = F_{GF}(-\underline{j}) = -\frac{P}{\sqrt{3}}(-\underline{j})$

$\Sigma \underline{F} = \underline{0}$; $-\frac{P}{\sqrt{3}}(-\underline{j}) + \frac{F_{GE}}{\sqrt{2}}(-\underline{i}-\underline{j})$

$+ \frac{F_{GB}}{\sqrt{2}}(-\underline{j}-\underline{k}) + \frac{F_{GD}}{\sqrt{2}}(-\underline{i}-\underline{k}) = \underline{0}$

Substitute $F_{GB} = F_{GE}$, collect j-terms

$(\Sigma F_y = 0)$ & get $\left(\frac{P}{\sqrt{3}} - \frac{2F_{GE}}{\sqrt{2}}\right) = 0$

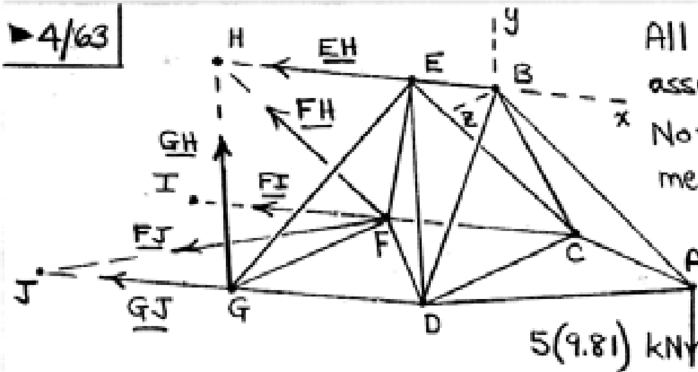
$\underline{F_{EG} = P/\sqrt{6}}$ (tension)

No. of members is $m = 18$

No. of joints is $j = 8$

$(m + 6 = 24) = (3j = 24)$ so internally stable

▶4/63



All members assumed in tension. Note that six members are cut!

$\underline{GJ} = -GJ\,\underline{i}$, $\underline{FI} = -FI\,\underline{i}$, $\underline{FJ} = \frac{FJ}{\sqrt{2}}(-\underline{i}+\underline{k})$

$\Sigma \underline{M}_H = \underline{0}$: $-49.05(5)\underline{k} + (-2\cos 30°\,\underline{j} + 2\sin 30°\,\underline{k}) \times (-GJ)\underline{i} + (-2\cos 30°\,\underline{j} - 2\sin 30°\,\underline{k}) \times (-FI)\underline{i} + (\underline{i} - 2\cos 30°\,\underline{j} - \underline{k}) \times \frac{FJ}{\sqrt{2}}(-\underline{i}+\underline{k}) = \underline{0}$.

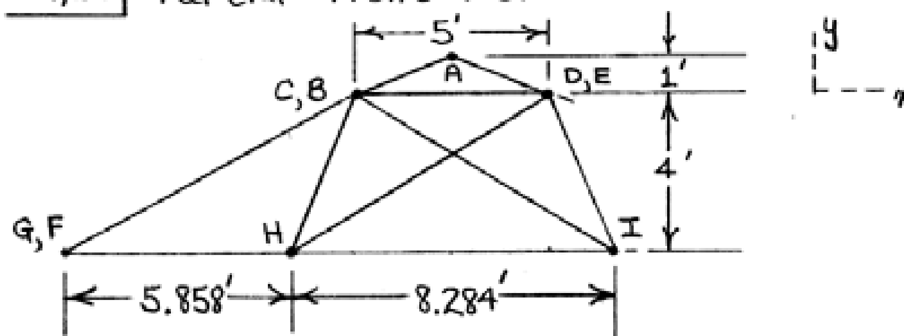
Equating unit vector coefficients to zero:

$-1.225\ FJ = 0 \Rightarrow \underline{FJ = 0}$

$\left.\begin{array}{l} -GJ + FI = 0 \\ -1.732\,GJ - 1.732\,FI = 245 \end{array}\right\}$ $FI = GJ = -70.8$ kN

∴ <u>Force in GJ = 70.8 kN C</u>

▶4/64 Partial front view:



For equilibrium of joint A, force vectors are

$\underline{L} = -L\underline{j}$

$$\underline{BA} = P\left[\frac{2.5\underline{i} + \underline{j} + 2.5\underline{k}}{\sqrt{2.5^2 + 1^2 + 2.5^2}}\right] = P(0.680\underline{i} + 0.272\underline{j} + 0.680\underline{k})$$

Similarly,

$$\underline{CA} = P(0.680\underline{i} + 0.272\underline{j} - 0.680\underline{k})$$

$$\underline{DA} = P(-0.680\underline{i} + 0.272\underline{j} - 0.680\underline{k})$$

$$\underline{EA} = P(-0.680\underline{i} + 0.272\underline{j} + 0.680\underline{k})$$

where P is the force in the 4 members joined at A, all of which are assumed to be in compression.

$\Sigma F_y = 0$ at A : $4P(0.272) - L = 0$, $P = 0.919\,L$

For equilibrium of joint B, force vectors are

$\underline{BC} = -Q\underline{k}$, $\underline{CD} = Q\underline{i}$

$\underline{AC} = 0.919L\,(-0.680\,\underline{i} - 0.272\,\underline{j} + 0.680\,\underline{k})$

$$\underline{CF} = R\left[\frac{-(10-2.5)\underline{i} - 4\underline{j} - (8.28/2 + 5/2)\underline{k}}{\sqrt{7.5^2 + 4^2 + 6.64^2}}\right]$$

$$= R\,(-0.695\,\underline{i} - 0.371\,\underline{j} - 0.616\,\underline{k})$$

Similarly,

$\underline{CG} = S\,(-0.866\,\underline{i} - 0.462\,\underline{j} + 0.190\,\underline{k})$

$\underline{CH} = S\,(-0.190\,\underline{i} - 0.462\,\underline{j} + 0.866\,\underline{k})$

$\underline{CI} = R\,(0.616\,\underline{i} - 0.371\,\underline{j} + 0.695\,\underline{k})$

where Q, R, and S are force magnitudes and where all unknowns are assumed in tension.

$\Sigma \underline{F} = \underline{0}$ at joint B: $\underline{AC} + \underline{BC} + \underline{CD} + \underline{CF} + \underline{CG} + \underline{CH} + \underline{CI} = \underline{0}$, or

$$[(0.919L)(-0.680) + Q - 0.695R - 0.866S - 0.190S + 0.616R]\,\underline{i}$$

$$+[(0.919L)(-0.272) - 0.371R - 0.462S - 0.462S - 0.371R]\,\underline{j}$$

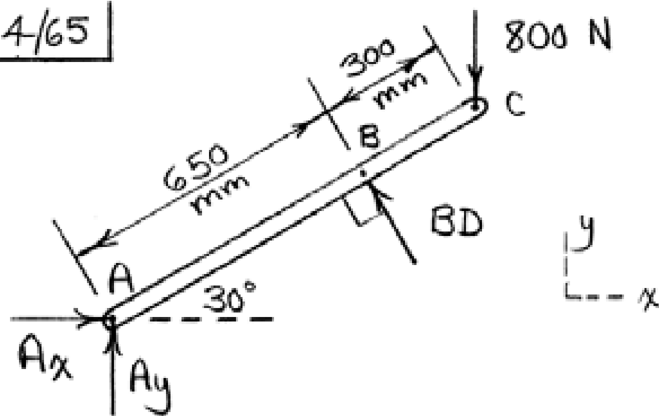
$$+[-Q + (0.919L)(0.680) - 0.616R + 0.190S + 0.866S + 0.695R]\,\underline{k}$$

$= \underline{0}$ (note dependency between $\underline{i}$ & $\underline{k}$ components!)

With $Q = 0.3L$, solve x- and y- equations to obtain $R = 0.051L$, $S = -0.312L$

$\therefore$ $\underline{CF = 0.051L\ T}$ and $\underline{CG = 0.312L\ C}$

4/65



$\curvearrowleft + \Sigma M_A = 0: \quad BD(650) - 800(950 \cos 30°) = 0$

$BD = 1013 \text{ N}$

So pin-reaction magnitudes at B and D are $\underline{B = D = 1013 \text{ N}}$.

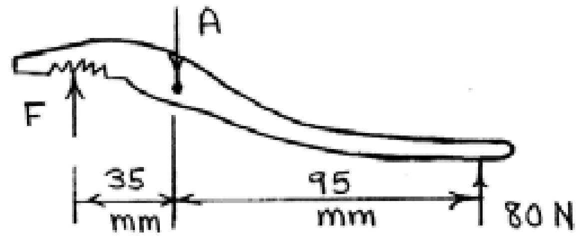
$\Sigma F_x = 0: \quad A_x - 1013 \sin 30° = 0, \; A_x = 506 \text{ N}$

$\Sigma F_y = 0: \quad A_y + 1013 \cos 30° - 800 = 0$

$A_y = -76.9 \text{ N}$

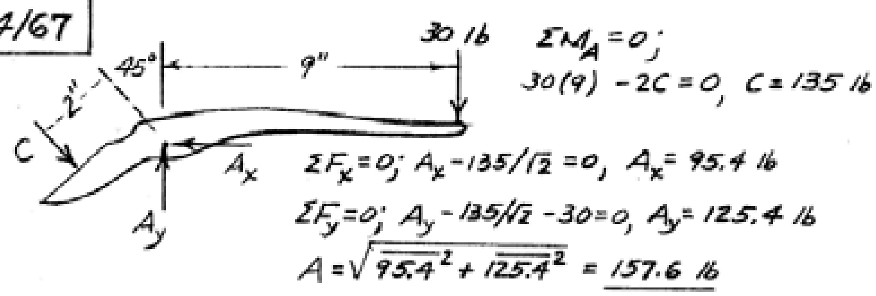
$A = \sqrt{A_x^2 + A_y^2} = \underline{512 \text{ N}}$

4/66

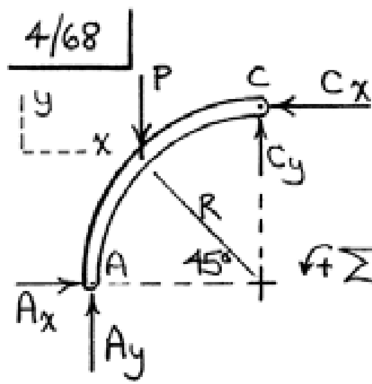


$\circlearrowleft + \Sigma M_A = 0: \; 80(95) - F(35) = 0, \quad \underline{F = 217 \text{ N}}$

$+\uparrow \Sigma F = 0: \; 217 - A + 80 = 0, \quad \underline{A = 297 \text{ N}}$


4/67
30 lb
45°
9"
2"
C
A_x
A_y
ΣM_A = 0;
30(9) − 2C = 0, C = 135 lb
ΣF_x = 0; A_x − 135/√2 = 0, A_x = 95.4 lb
ΣF_y = 0; A_y − 135/√2 − 30 = 0, A_y = 125.4 lb
A = √(95.4² + 125.4²) = 157.6 lb

4/68



$\underline{C_y = 0}$ due to symmetry of overall structure.

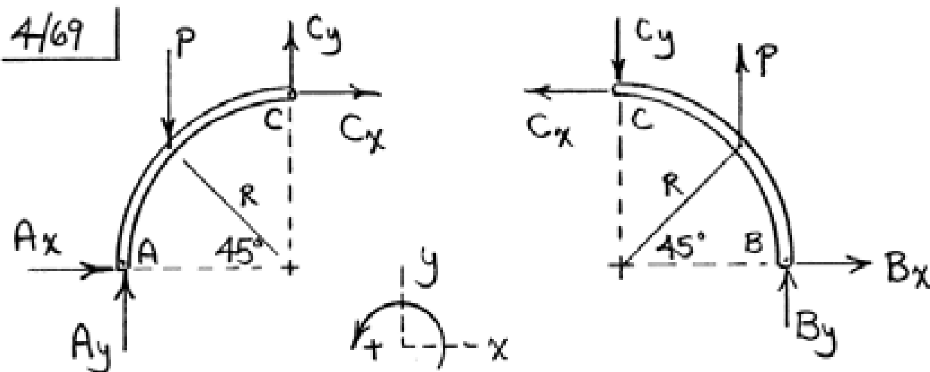
Then $\Sigma F_y = 0$ yields $\underline{A_y = P}$

$\circlearrowleft + \Sigma M_A = 0$: $-PR(1 - \cos 45°) + C_x(R) = 0$

$\underline{C_x = 0.293R}$

Finally, $\Sigma F_x = 0$ yields $\underline{A_x = 0.293R}$

(Forces on member BC are symmetric)



AC:

$$\begin{cases} \Sigma F_x = 0: & A_x + C_x = 0 & (1) \\ \Sigma F_y = 0: & A_y + C_y - P = 0 & (2) \\ \Sigma M_A = 0: & C_y(R) - C_x(R) - PR(1-\cos 45°) = 0 & (3) \end{cases}$$

BC:

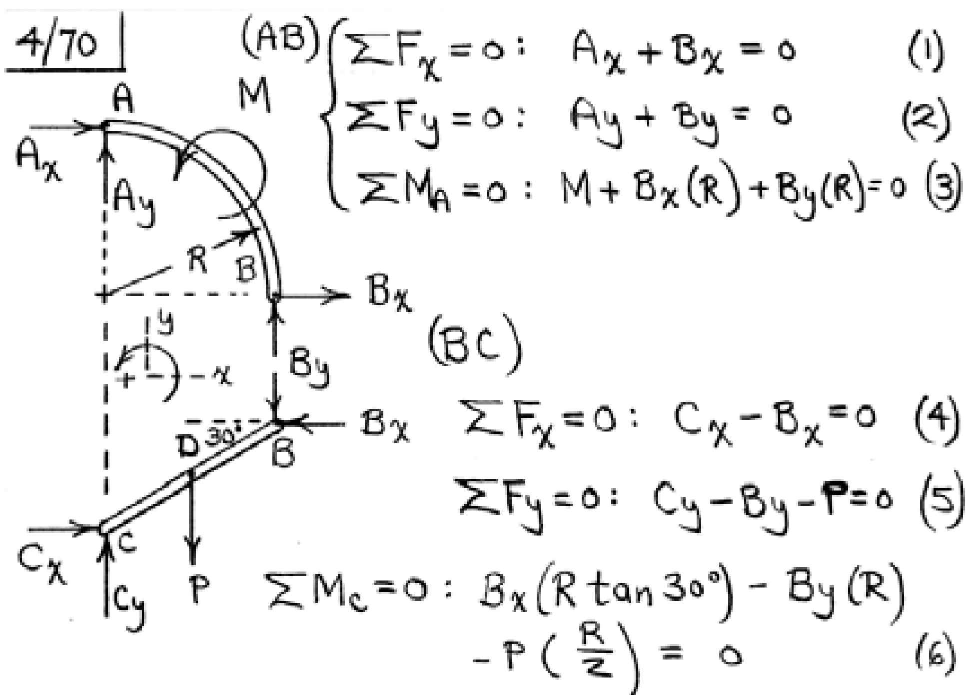
$$\begin{cases} \Sigma F_x = 0: & -C_x + B_x = 0 & (4) \\ \Sigma F_y = 0: & -C_y + B_y + P = 0 & (5) \\ \Sigma M_B = 0: & C_y(R) + C_x(R) - PR(1-\cos 45°) = 0 & (6) \end{cases}$$

Solve Eqs. (1)–(6):

Unlike Prob. 4/68, this problem is not symmetric.

$$\begin{cases} A_x = C_x = B_x = 0 \\ A_y = 0.707P, \quad B_y = -0.707P \\ C_y = 0.293P \end{cases}$$

4/70



(AB)

$\Sigma F_x = 0: \quad A_x + B_x = 0 \qquad (1)$

$\Sigma F_y = 0: \quad A_y + B_y = 0 \qquad (2)$

$\Sigma M_A = 0: \quad M + B_x(R) + B_y(R) = 0 \qquad (3)$

(BC)

$\Sigma F_x = 0: \quad C_x - B_x = 0 \qquad (4)$

$\Sigma F_y = 0: \quad C_y - B_y - P = 0 \qquad (5)$

$\Sigma M_C = 0: \quad B_x(R \tan 30°) - B_y(R) - P\left(\frac{R}{2}\right) = 0 \qquad (6)$

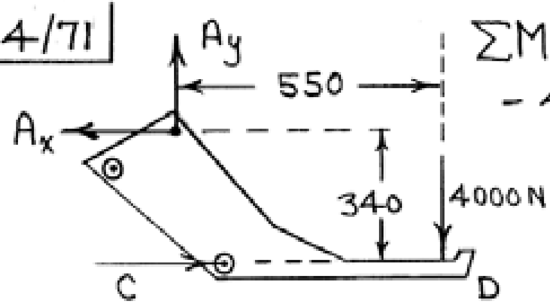
Solve Eqs. (1)-(6) for B_x and B_y:

$B_x = 0.634\left(\frac{P}{2} - \frac{M}{R}\right), \quad B_y = -0.366\frac{M}{R} - 0.317P$

(a) For $B_x = 0$, $M = \frac{PR}{2}$ (CCW)

(b) For $B_y = 0$, $M = -0.866PR$ (CW)

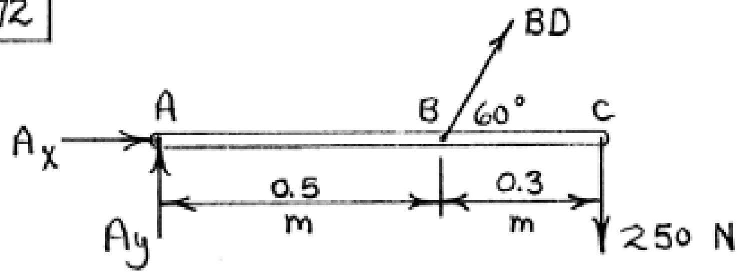
4/71



$\Sigma M_A = 0$: $C(340) - 4000(550) = 0$

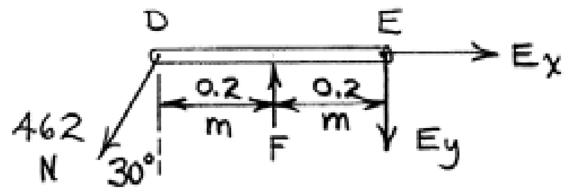
$\underline{C = 6470 \text{ N}}$

4/72



$\circlearrowleft+ \; \Sigma M_A = 0: \; (BD \sin 60°)(0.5) - 250(0.8) = 0$

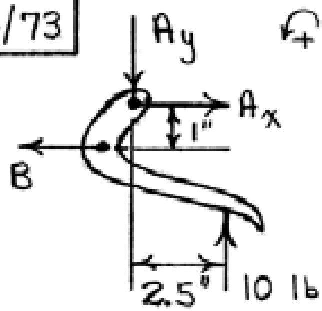
$BD = 462 \text{ N}$



$\circlearrowleft+ \; \Sigma M_E = 0: \; 462 \cos 30° (0.4) + F(0.2) = 0$

$\underline{F = 800 \text{ N}}$

4/73

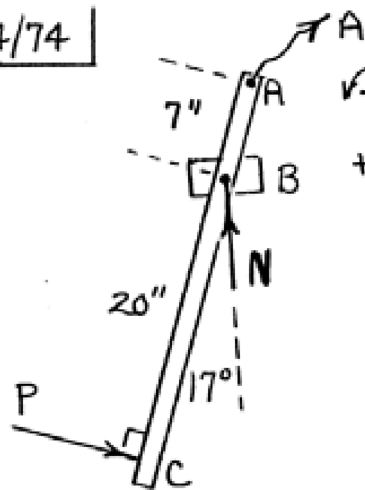


$\circlearrowleft^{+} \Sigma M_A = 0$: $10(2.5) - B(1) = 0$

$B = 25$ lb

$\therefore$ Force F on brad = $\underline{25 \text{ lb}}$

4/74



$\circlearrowleft + \Sigma M_A = 0:$

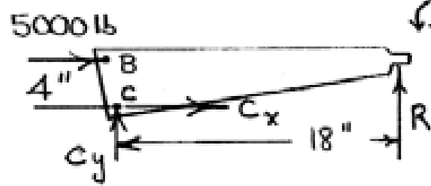
$$+P(27) - N \sin 17°(7) = 0$$

$\underline{N = 13.19P}$

4/75 Piston force = $(500)(20) = 10{,}000$ lb

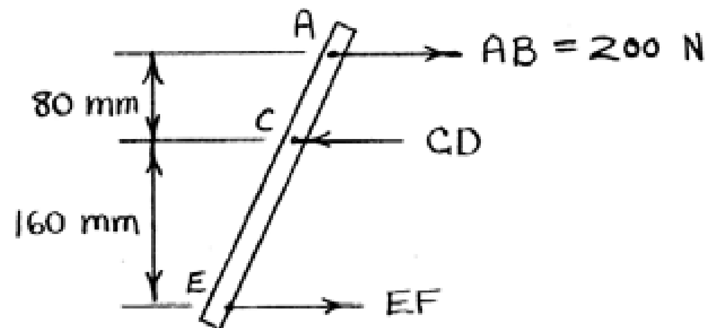
Force in link AB $= 10{,}000/2 = 5000$ lb

Lower jaw:



$\circlearrowleft + \sum M_C = 0$: $R(18) - 5000(4) = 0$, $\underline{R = 1111 \text{ lb}}$

4/76



$\circlearrowleft + \Sigma M_C = 0: \ -200(80) + EF(160) = 0$

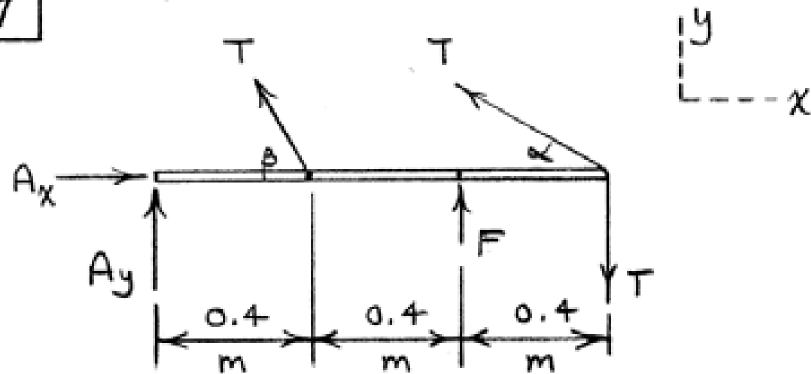
$EF = 100 \text{ N} \ T$

$\overset{+}{\rightarrow} \Sigma F = 0: \ 200 - CD + 100 = 0$

$CD = 300 \text{ N}$

So force supported by pin C is $F = 300 \text{ N}$

4/77



$$\begin{cases} T = 60(9.81) = 589 \text{ N} \\ \alpha = \tan^{-1}\left(\frac{0.5}{1.2}\right) = 22.6° \\ \beta = \tan^{-1}\left(\frac{0.5}{0.4}\right) = 51.3° \end{cases}$$

$\circlearrowleft + \; \Sigma M_A = 0: \; T\sin\beta(0.4) + T\sin\alpha\,(1.2)$

$-T\,(1.2) + F\,(0.8) = 0, \quad \underline{F = 314 \text{ N}}$

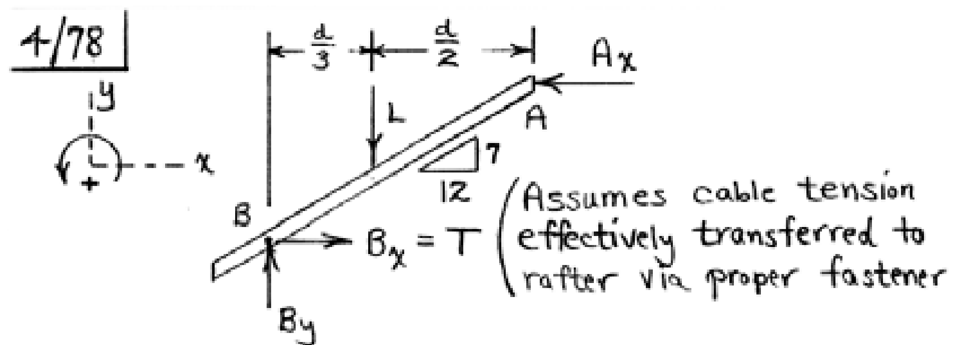
(Contact at bottom roller)

$\Sigma F_x = 0: \; A_x - T\cos\beta - T\cos\alpha = 0, \; A_x = 911 \text{ N}$

$\Sigma F_y = 0: \; A_y + T\sin\beta + T\sin\alpha - T + F = 0$

$A_y = -411 \text{ N}$

$A = \sqrt{A_x^2 + A_y^2} = \underline{999 \text{ N}}$

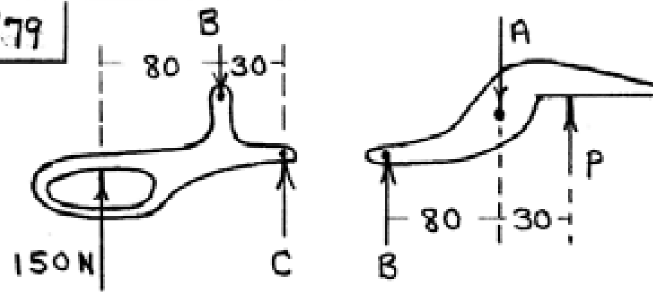


$$\Sigma F_x = 0 \Rightarrow \quad T = A_x$$

$$\Sigma M_B = 0: \quad -L\left(\frac{d}{3}\right) + T\left(\frac{5d}{6} \cdot \frac{7}{12}\right) = 0$$

$$\underline{T = \frac{24}{35} L}$$

4/79

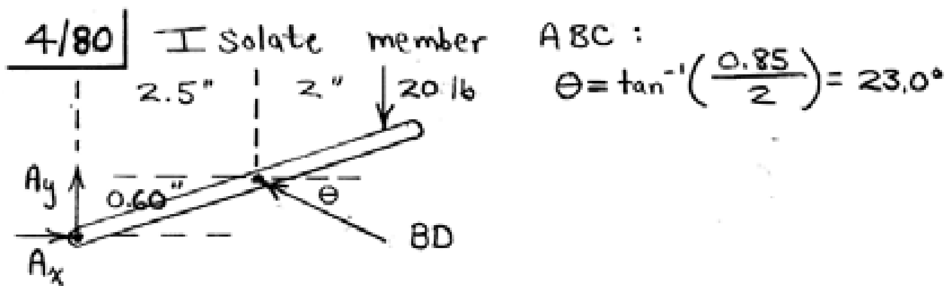


(Handle) $\Sigma M_C = 0: \; 150(110) - B(30) = 0$

$B = 550 \text{ N}$

(Jaw) $\Sigma M_A = 0: \; 550(80) - P(30) = 0$

$\underline{P = 1467 \text{ N}}$



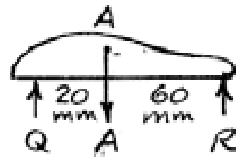
$\circlearrowleft \Sigma M_A = 0: \; -20(4.5) + (BD\cos\theta)(0.6) + (BD\sin\theta)(2.5)$

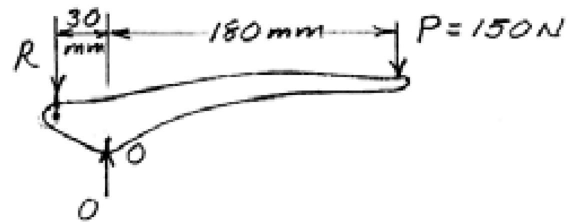
$= 0, \quad BD = 58.8 \text{ lb}$

$\Sigma F_x = 0: \; A_x - 58.8\cos\theta = 0, \quad A_x = 54.1 \text{ lb}$

Thus squeezing force $\underline{P = 54.1 \text{ lb.}}$

4/81



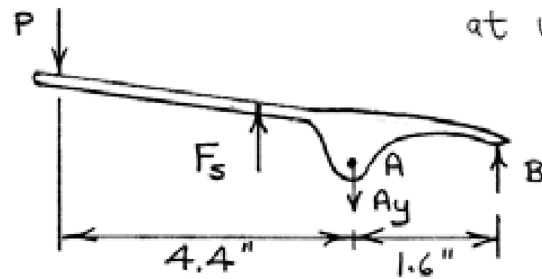


Handle: $\Sigma M_O = 0$; $30R - 180(150) = 0$, $R = 900$ N

Jaw: $\Sigma M_A = 0$; $20Q - 60(900) = 0$, $Q = 2700$ N

or $\underline{Q = 2.7 \text{ kN}}$

4/82 Upper handle: (Spring force F_s acts at unknown location)



When clamp is released, $B = 0$. $\circlearrowleft \Sigma M_A = 0$:

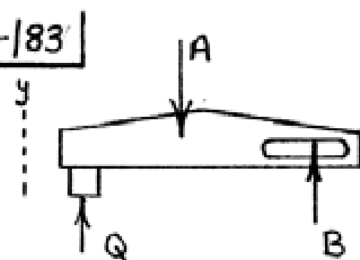
$P(4.4) - M_{F_s} = 0$, $M_{F_s} = P(4.4) = 6(4.4) = 26.4$ lb-in.

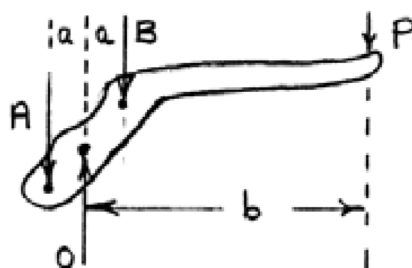
(M_{F_s} is moment exerted by spring on handle)

With $P = 0$, $\Sigma M_A = 0$: $B(1.6) - 26.4 = 0$

$\underline{B = 16.50 \text{ lb}}$

4/83





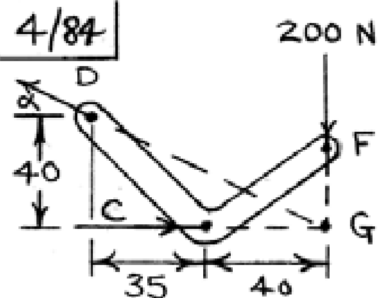
(Handle) $\Sigma M_O = 0$: $Pb = (A - B)a$

(Jaw) $\Sigma F_y = 0$: $Q = A - B$

$Pb = Qa$

$\underline{Q = P\frac{b}{a}}$

4/84



DCF is a three-force body; forces intersect at G.

$\alpha = \tan^{-1}\left(\frac{40}{75}\right) = 28.1°$

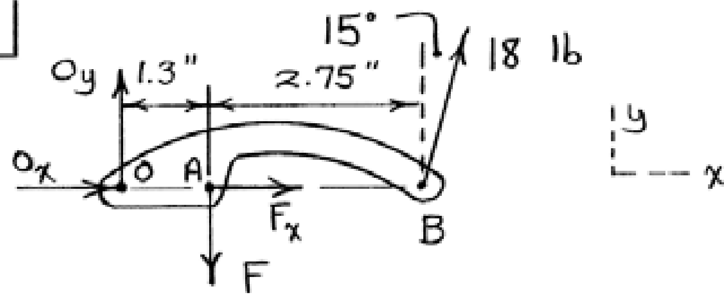
$\Sigma F_y = 0: -200 + D\sin\alpha = 0$

$\underline{D = 425 \text{ N}}$

$\Sigma F_x = 0: -D\cos\alpha + C = 0,$ $\underline{C = 375 \text{ N}}$

(BC in compression)

4/85



$$\curvearrowleft + \sum M_O = 0 : -F(1.3) + 18 \cos 15°(2.75 + 1.3) = 0$$

$$\underline{F = 54.2 \text{ lb}}$$

(Note: Treatment of member OC as a three-force body would yield a constraint relationship between O_x and O_y.)

4/86

$$F_{EF} = k\delta = 3600\,[0.6 - 0.3] = 1080 \text{ N}$$

From $\xrightarrow{+} \Sigma F = 0$, $T = 540$ N

$\circlearrowleft + \Sigma M_O = 0: -540(1300) - 540(1100) + T_{AB}(500) = 0$, $T_{AB} = 2590$ N

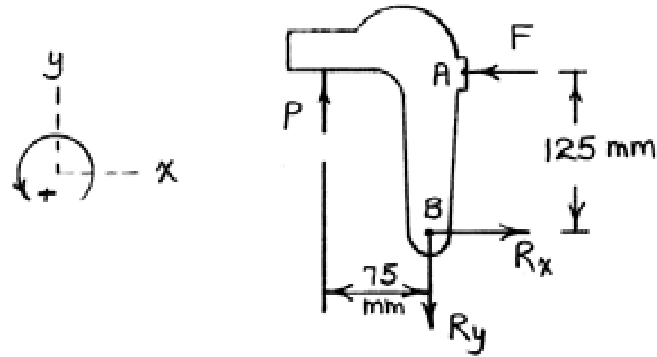
$\xrightarrow{+} \Sigma F = 0: O_x - 2590 + 2(540) = 0$

$O_x = 1512$ N

$O_y = 0$

So $\underline{O = 1512 \text{ N}}$

4/87



For $P = 3$ kN :

$\Sigma M_B = 0$: $125F - 3(75) = 0$, $F = 1.8$ kN

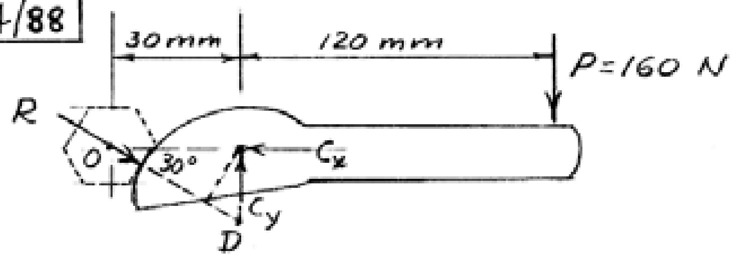
For $F = 2(1.8) = 3.6$ kN , $P = 3(2) = 6$ kN

$\Sigma F_x = 0$: $R_x - 3.6 = 0$, $R_x = 3.6$ kN

$\Sigma F_y = 0$: $-R_y + 6 = 0$, $R_y = 6$ kN

$R = \sqrt{3.6^2 + 6^2} = \underline{7.00 \text{ kN}}$

4/88

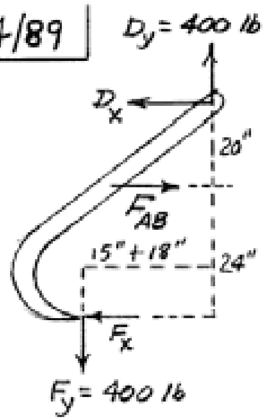


$\Sigma M_O = 0$; $30 C_y - 150(160) = 0$, $C_y = 800$ N

$\Sigma M_D = 0$; $(30 \tan 30°) C_x - 120(160) = 0$, $C_x = 1109$ N

$C = \sqrt{\overline{800}^2 + \overline{1109}^2} = \underline{1367 \text{ N}}$

4/89



From tongs as a whole

$D_y = \frac{1}{2}(800) = 400$ lb $= F_y$

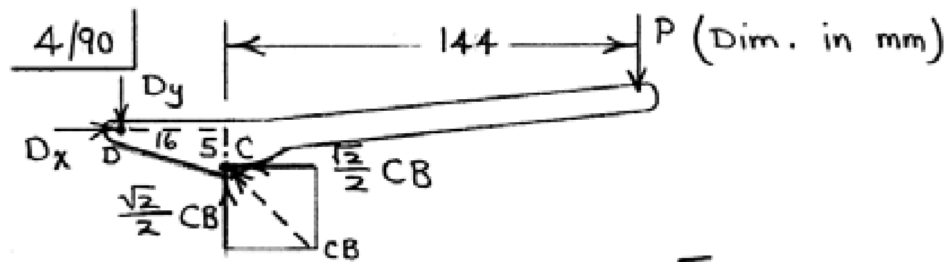
From ED, $D_x = \frac{18}{12} D_y = \frac{18}{12}(400) = 600$ lb

From DF, $\Sigma M_F = 0$;

$F_{AB}(24) - 600(44) - 400(18 + 15) = 0$

$F_{AB} = \underline{1650 \text{ lb}}$ tension

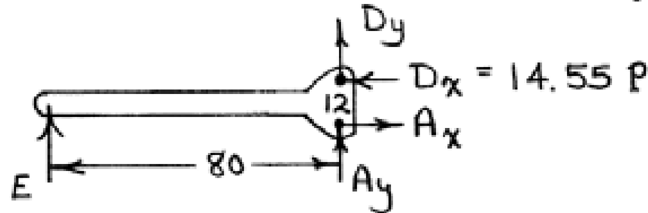
4/90



$$\circlearrowleft + \sum M_D = 0: \quad P(160) + \frac{\sqrt{2}}{2} CB(5) - \frac{\sqrt{2}}{2} CB(16) = 0$$

$$CB = 20.6\,P$$

$$\xrightarrow{+} \sum F = 0: \quad D_x - 20.6P\,\frac{\sqrt{2}}{2} = 0, \quad D_x = 14.55P$$

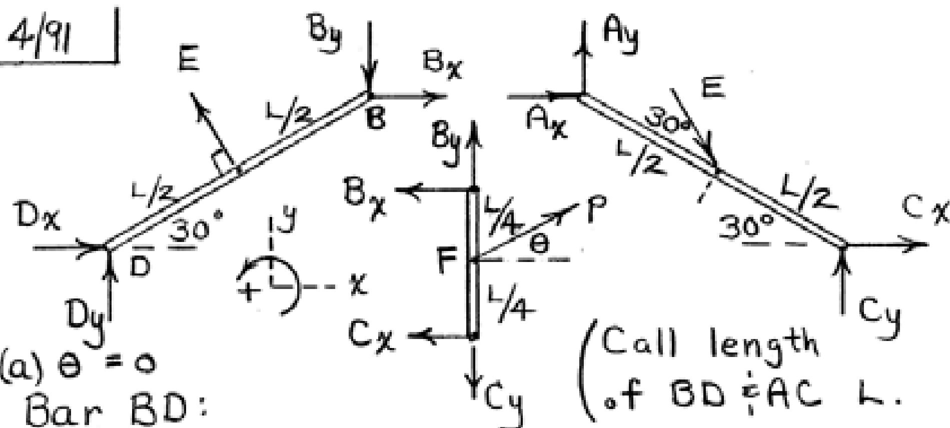


$$\circlearrowleft + \sum M_A = 0: \quad E(80) - 14.55P(12) = 0$$

$$\underline{E = 2.18\,P}$$

(Note: Mechanical advantage will increase as CB becomes more aligned with CD.)

4/91



(Call length of BD & AC L.)

(a) $\theta = 0$

Bar BD:

$$\Sigma F_x = 0: \quad B_x + D_x - E\left(\tfrac{1}{2}\right) = 0 \qquad (1)$$

$$\Sigma F_y = 0: \quad -B_y + D_y + E\left(\tfrac{\sqrt{3}}{2}\right) = 0 \qquad (2)$$

$$\Sigma M_D = 0: \quad E\left(\tfrac{L}{2}\right) - B_y\left(L\tfrac{\sqrt{3}}{2}\right) - B_x\left(L\tfrac{1}{2}\right) = 0 \qquad (3)$$

Bar AC:

$$\Sigma F_x = 0: \quad A_x + C_x + E\left(\tfrac{1}{2}\right) = 0 \qquad (4)$$

$$\Sigma F_y = 0: \quad A_y + C_y - E\left(\tfrac{\sqrt{3}}{2}\right) = 0 \qquad (5)$$

$$\Sigma M_A = 0: \quad -E\left(\tfrac{1}{2}\right)\left(\tfrac{L}{2}\right) + C_y\left(L\tfrac{\sqrt{3}}{2}\right) + C_x\left(L\tfrac{1}{2}\right) = 0 \qquad (6)$$

Bar BC:

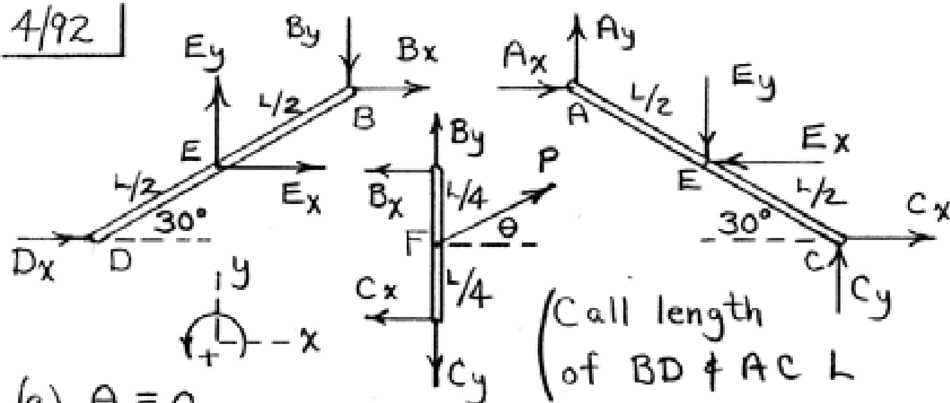
$$\Sigma F_x = 0: \quad -B_x - C_x + P = 0 \qquad (7)$$

$$\Sigma F_y = 0: \quad B_y - C_y = 0 \qquad (8)$$

$$\Sigma M_F = 0: \quad B_x\left(\tfrac{L}{4}\right) - C_x\left(\tfrac{L}{4}\right) = 0 \qquad (9)$$

Solve the above nine equations to obtain

4/92



(a) $\theta = 0$

Bar BD:

$$\begin{cases} \Sigma F_x = 0: & B_x + D_x + E_x = 0 & (1) \\ \Sigma F_y = 0: & -B_y + E_y = 0 & (2) \\ \Sigma M_D = 0: & E_y\left(\frac{L}{2}\frac{\sqrt{3}}{2}\right) - E_x\left(\frac{L}{2}\cdot\frac{1}{2}\right) - B_y\left(L\frac{\sqrt{3}}{2}\right) - B_x\left(L\frac{1}{2}\right) = 0 & (3) \end{cases}$$

Bar AC:

$$\begin{cases} \Sigma F_x = 0: & A_x + C_x - E_x = 0 & (4) \\ \Sigma F_y = 0: & A_y + C_y - E_y = 0 & (5) \\ \Sigma M_A = 0: & -E_y\left(\frac{L}{2}\frac{\sqrt{3}}{2}\right) - E_x\left(\frac{L}{2}\frac{1}{2}\right) + C_y\left(L\frac{\sqrt{3}}{2}\right) + C_x\left(L\frac{1}{2}\right) = 0 & (6) \end{cases}$$

Bar BC:

$$\begin{cases} \Sigma F_x = 0: & -B_x - C_x + P = 0 & (7) \\ \Sigma F_y = 0: & B_y - C_y = 0 & (8) \\ \Sigma M_F = 0: & B_x\left(\frac{L}{4}\right) - C_x\left(\frac{L}{4}\right) = 0 & (9) \end{cases}$$

Solve the above nine equations to obtain

$$\begin{cases} A_x = -P/2 & C_x = P/2 \\ A_y = 0 & C_y = -0.577P \\ B_x = P/2 & D_x = -P/2 \\ B_y = -0.577P & E_x = 0, \quad E_y = -0.577P \end{cases}$$

(b) $\theta = 30^\circ$. All equations remain the same except (7) & (8):

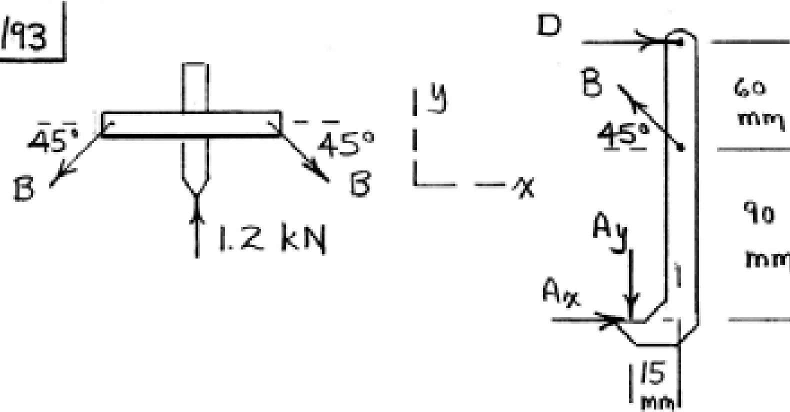
$$-B_x - C_x + P\frac{\sqrt{3}}{2} = 0 \quad (7)$$

$$B_y - C_y + P/2 = 0 \quad (8)$$

Resolve:

$$\begin{cases} A_x = 0.433P & C_x = 0.433P \\ A_y = -P/2 & C_y = -P/2 \\ B_x = 0.433P & D_x = -1.299P \\ B_y = -P & E_x = 0.866P \quad E_y = -P \end{cases}$$

4/93



(Upper bar & screw)

$\Sigma F_y = 0: -2B \sin 45° + 1.2 = 0, \quad B = 0.849 \text{ kN}$

(ABD)

$\Sigma M_A = 0: \; 150D - 0.849 \cos 45° (90) - 0.849 \sin 45° (15) = 0$

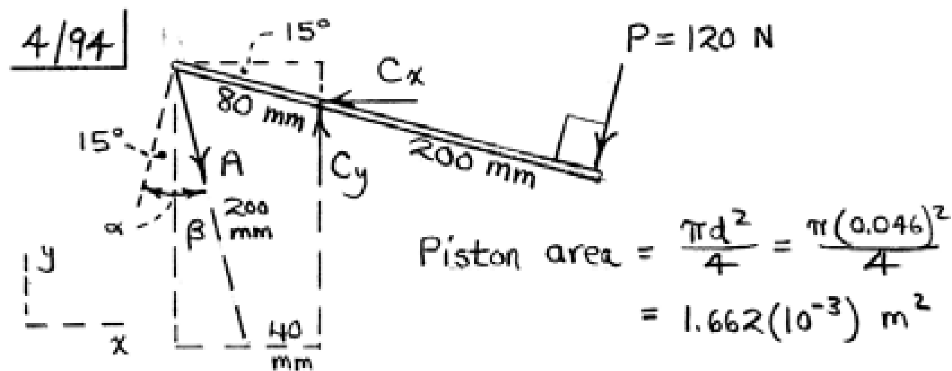
$D = 0.420 \text{ kN}$

$\Sigma F_x = 0: \; A_x - 0.849 \cos 45° + 0.420 = 0, \quad A_x = 0.1800 \text{kN}$

$\Sigma F_y = 0: -A_y + 0.849 \sin 45° = 0, \quad A_y = 0.6 \text{ kN}$

$A = \sqrt{A_x^2 + A_y^2} = \underline{0.626 \text{ kN}}$

4/94



Piston area $= \dfrac{\pi d^2}{4} = \dfrac{\pi (0.046)^2}{4}$

$= 1.662(10^{-3})\ \text{m}^2$

$\sin\beta = (80\cos 15° - 40)/200\,,\quad \beta = 10.74°$

$\alpha = \beta + 15° = 25.7°$

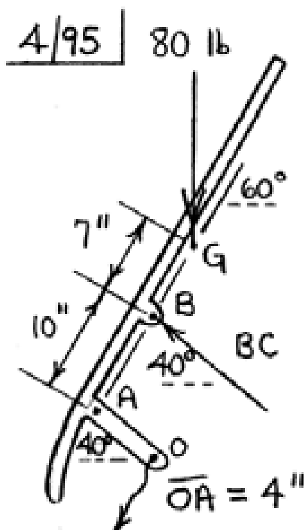
$\Sigma M_C = 0:\ 120(200) - A\cos 25.7°(80) = 0,\quad A = 333\ \text{N}$

$\Sigma F_y = 0:\ C_y - 120\cos 15° - 333\cos 10.74° = 0,\ C_y = 443\ \text{N}$

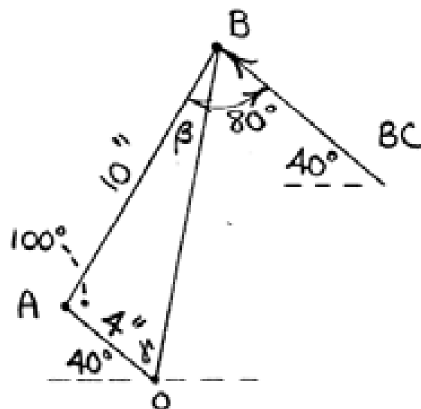
Piston : $p(\text{Area}) = C_y\,,\ p = \dfrac{443}{1.662(10^{-3})} = 267(10^3)\ \text{N/m}^2$

or $\underline{p = 267\ \text{kPa}}$

4/95



Blowup :

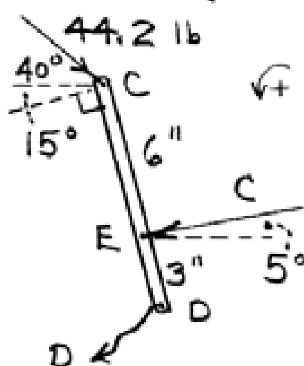


$\overline{OB}^2 = 4^2 + 10^2 - 2(4)(10)\cos 100°$, $\overline{OB} = 11.40$ in.

$\dfrac{\sin\beta}{4''} = \dfrac{\sin 100°}{11.40''}$, $\beta = 20.2°$

$\circlearrowleft + \Sigma M_O = 0:\ -80(17\cos 60° - 4\cos 40°)$

$+ BC\,(11.40\sin(80° - 20.2°)) = 0$, $BC = 44.2$ lb

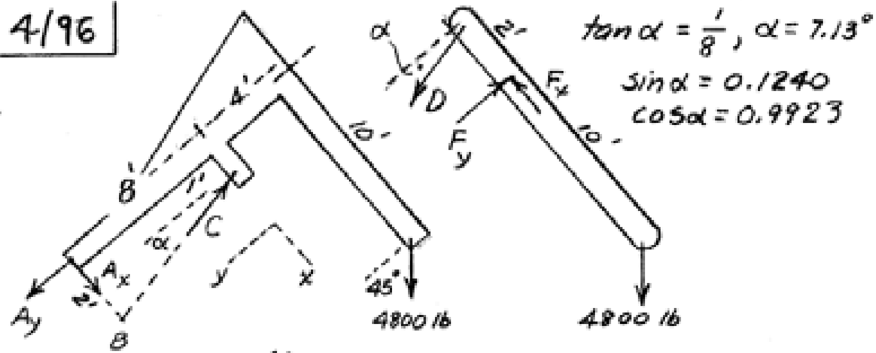


$\circlearrowleft + \Sigma M_D = 0:\ -44.2\cos 55°\,(9)$

$+ C\cos 10°\,(3) = 0$

$C = 77.2$ lb

4/96



$\tan\alpha = \frac{1}{8}$, $\alpha = 7.13°$

$\sin\alpha = 0.1240$

$\cos\alpha = 0.9923$

$\Sigma M_B = 0;\ 2A_y - \frac{4800}{\sqrt{2}}(10-2+12) = 0,\ A_y = 33{,}940$ lb

$\Sigma M_C = 0;\ 8A_x + 33900(1) - \frac{4800}{\sqrt{2}}(10-1+4) = 0,\ A_x = 1273$ lb

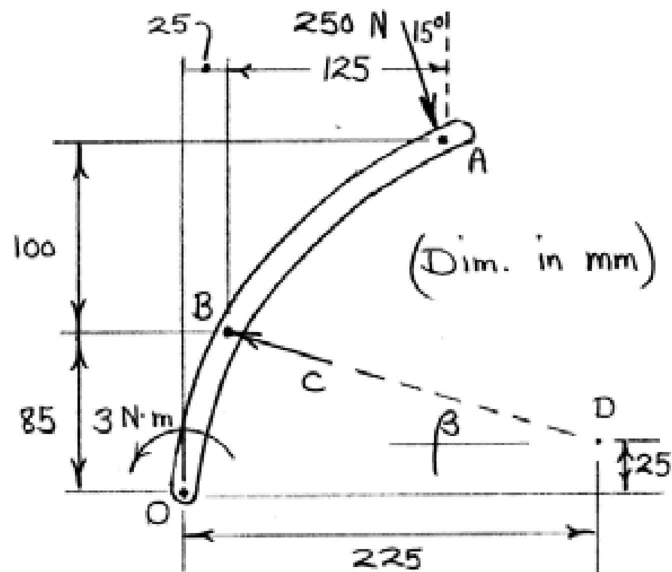
$A = \sqrt{1273^2 + 33{,}940^2} = \underline{34{,}000 \text{ lb}}$

$\Sigma M_F = 0;\ \frac{4800}{\sqrt{2}}(10) - 0.9923D(2) = 0,\ \underline{D = 17{,}100 \text{ lb}}$

4/97

$\beta = \tan^{-1} \frac{60}{200}$

$= 16.70°$



$\circlearrowleft + \sum M_O = 0: \; 3000 - 250 \cos 15° (150) -$

$250 \sin 15° (185) + C \cos\beta (85) + C \sin\beta (25) = 0$

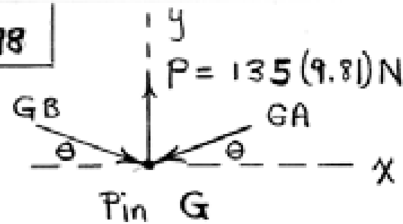
$\underline{C = 510 \text{ N}}$

$C = pA: \quad 510 = p\left(\frac{\pi \, 45^2}{4}\right)$

$p = 0.321 \frac{\text{N}}{\text{mm}^2}$ or $\underline{321\,000 \text{ Pa}}$

(gauge pressure)

4/98

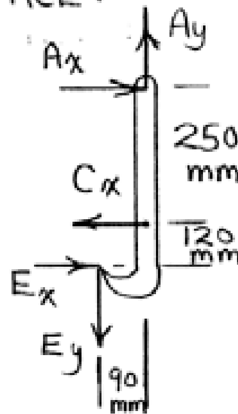


$\theta = \cos^{-1} \frac{340}{350} = 13.73°$

$\Sigma F_y = 0 : 135(9.81) - 2\,GA \sin 13.73° = 0$

$GA = GB = 2790$ N

ACE :



$A_x = 2790 \cos 13.73° = 2710$ N

$A_y = 2790 \sin 13.73° = 662$ N

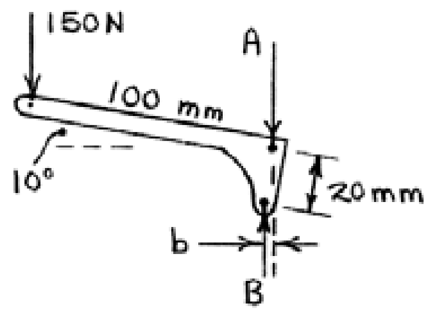
$\Sigma F_y = 0 \Rightarrow E_y = 662$ N

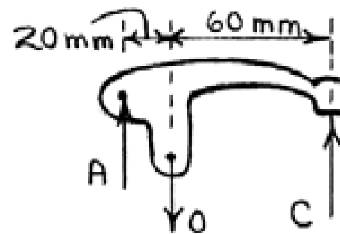
$\Sigma M_C = 0 : 2710(250) - 662(90) - E_x(120) = 0$, $E_x = 5150$ N

$E = \sqrt{E_x^2 + E_y^2} = 5190$ N

or $\underline{5.19 \text{ kN}}$

4/99

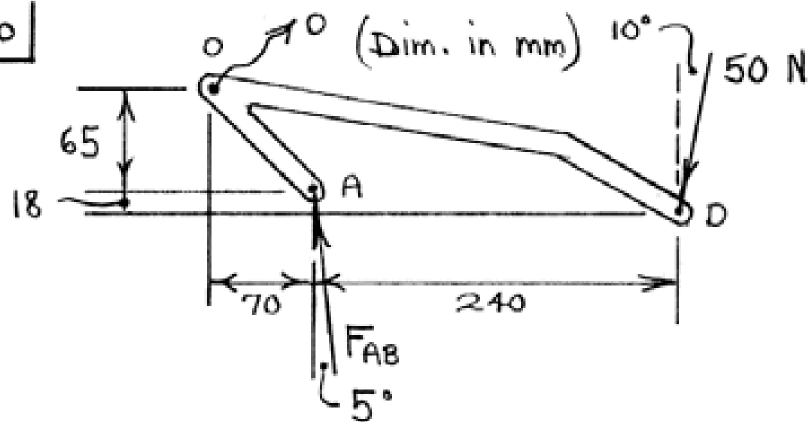




(Handle) $\Sigma M_B = 0$: $150\left[100\cos 10° - 20\sin 10°\right] - A\left[20\sin 10°\right] = 0$, $A = 4103$ N

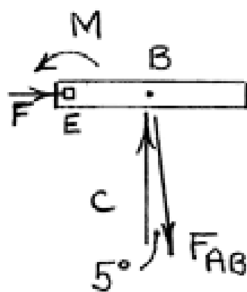
(Jaw) $\Sigma M_O = 0$: $60C - 20(4103) = 0$, $\underline{C = 1368 \text{ N}}$

4/100



$\circlearrowleft + \Sigma M_O = 0$: $F_{AB}\left[\cos 5°(70) - \sin 5°(65)\right]$
$- 50\left[\cos 10°(310) + \sin 10°(83)\right] = 0$

$F_{AB} = 250$ N



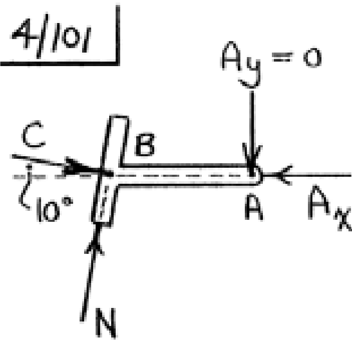
$M = 0$ (from $\Sigma M_B = 0$)

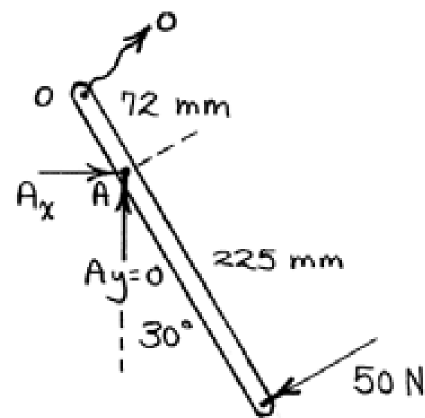
$+\uparrow \Sigma F = 0$: $C - 250\cos 5° = 0$

$C = 249$ N

(a factor $\frac{C}{P} = 4.97$)

4/101





$A_y = 0$ because AB is a three-force body.

Body OA:

$$\circlearrowleft + \Sigma M_O = 0: \quad A_x(72 \cos 30°) - 50(72 + 225) = 0$$

$$A_x = 238 \text{ N}$$

Body AB:

$$\xrightarrow{+} \Sigma F = 0: \quad C - 238 \cos 10° = 0$$

$$\underline{C = 235 \text{ N}}$$

$$\left(\text{a factor } \frac{C}{P} = 4.69 \right)$$

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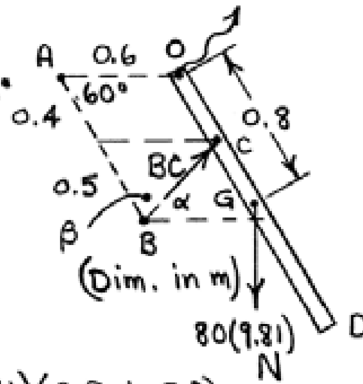


$$\overline{BC}^2 = 0.5^2 + 0.6^2 - 2(0.5)(0.6)\cos 60°$$

$BC = 0.557$ m

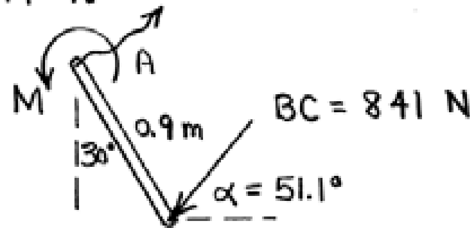
$$\frac{\sin\beta}{0.6} = \frac{\sin 60°}{0.557}, \quad \beta = 68.9°$$
$$\alpha = 51.1°$$

$\curvearrowleft\!+ \Sigma M_O = 0:$

$$BC(0.4)\cos(51.1° - 30°) - 80(9.81)(0.8 \sin 30°) = 0$$

$BC = 841$ N

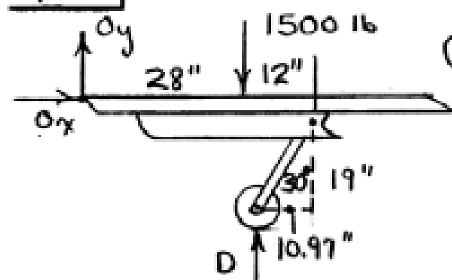
AB:



$\curvearrowleft\!+ \Sigma M_A = 0: \quad M - 841(0.9)\cos(51.1° - 30°) = 0$

$M = 706$ N·m

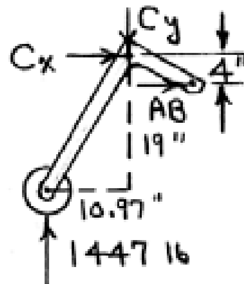
4/103 Entire ramp + mechanism :



$\circlearrowleft + \sum M_O = 0$: $D(28+12-10.97) - 1500(28) = 0$

$D = 1447$ lb

Crank BCD :



$\circlearrowleft + \sum M_C = 0$: $-1447(10.97) + AB(4) = 0$, $\underline{AB = 3970 \text{ lb}}$

(Cylinder is in compression)

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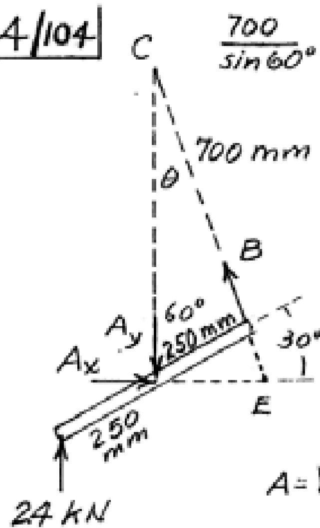

$$\frac{700}{\sin 60^\circ} = \frac{250}{\sin\theta}, \quad \theta = \sin^{-1} 0.3093 = 18.02^\circ$$

$$\cos\theta = 0.9510, \quad \tan\theta = 0.3252$$

$$\overline{AC} = 700(0.9510) + 250(0.5) = 791 \text{ mm}$$

$$\Sigma M_C = 0; \quad 24(250)(0.866) - 791 A_x = 0$$

$$A_x = 6.57 \text{ kN}$$

$$\overline{AE} = \overline{AC} \tan\theta = 791(0.3252) = 257 \text{ mm}$$

$$\Sigma M_E = 0; \quad A_y(257) - 24(250 \times .866 + 257) = 0$$

$$A_y = 44.2 \text{ kN}$$

$$A = \sqrt{\overline{6.57}^2 + \overline{44.2}^2} = \underline{44.7 \text{ kN}}$$

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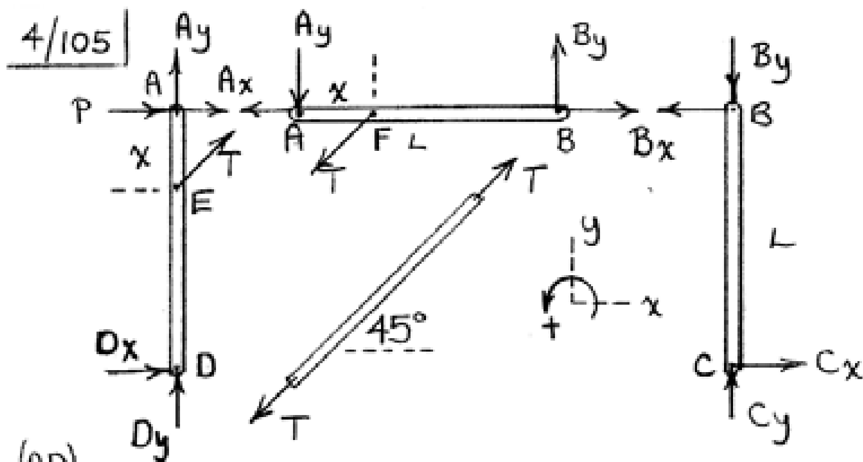


(AD)

$\Sigma F_x = 0: \quad D_x + A_x + P + T\frac{\sqrt{2}}{2} = 0$ (1)

$\Sigma F_y = 0: \quad D_y + A_y + T\frac{\sqrt{2}}{2} = 0$ (2)

$\Sigma M_A = 0: \quad D_x(L) + T\frac{\sqrt{2}}{2}(x) = 0$ (3)

(AB)

$\Sigma F_x = 0: \quad -A_x + B_x - T\frac{\sqrt{2}}{2} = 0$ (4)

$\Sigma F_y = 0: \quad -A_y + B_y - T\frac{\sqrt{2}}{2} = 0$ (5)

$\Sigma M_A = 0: \quad B_y(L) - T\frac{\sqrt{2}}{2}x = 0$ (6)

(BC)

$\Sigma F_x = 0: \quad -B_x + C_x = 0$ (7)

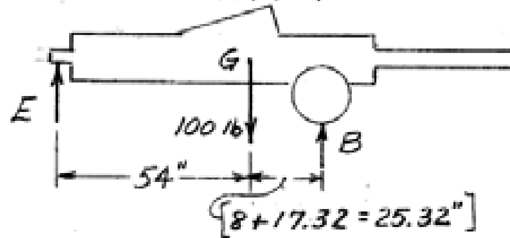
$\Sigma F_y = 0: \quad -B_y + C_y = 0$ (8)

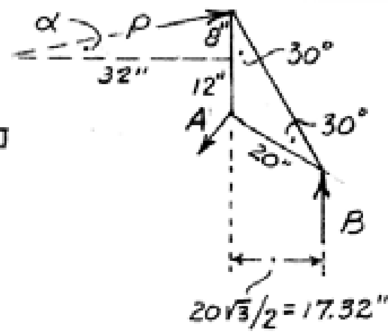
$\Sigma M_C = 0: \quad B_x(L) = 0$ (9)

Solve Eqs. (1)-(9) for $\underline{T = \frac{\sqrt{2}PL}{x}}$ $(x \neq 0)$

4/106 $\tan\alpha = \frac{8}{32}$, $\alpha = 14.04°$

$\cos\alpha = 0.9701$





$\Sigma M_E = 0$; $(54 + 25.32)B - 54(100) = 0$

$B = 68.1$ lb

$\Sigma M_A = 0$; $0.9701 P(20) - 68.1(17.32) = 0$

$P = 60.8$ lb

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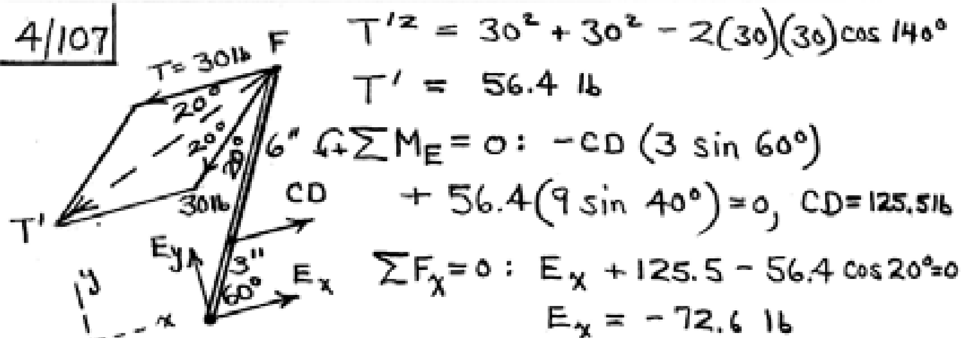


$$T'^2 = 30^2 + 30^2 - 2(30)(30)\cos 140°$$

$$T' = 56.4 \text{ lb}$$

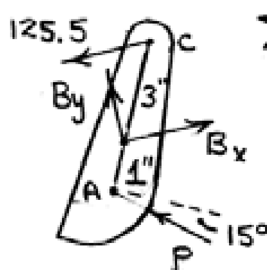
$$\circlearrowleft + \Sigma M_E = 0: \; -CD\,(3\sin 60°) + 56.4\,(9\sin 40°) = 0, \quad CD = 125.5\text{ lb}$$

$$\Sigma F_x = 0: \; E_x + 125.5 - 56.4\cos 20° = 0$$

$$E_x = -72.6 \text{ lb}$$

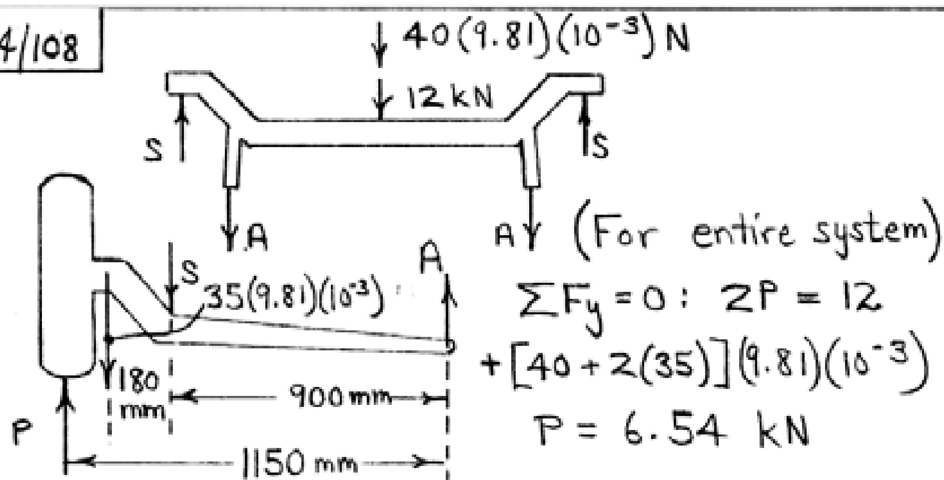
$$\Sigma F_y = 0: \; E_y - 56.4\sin 20° = 0, \quad E_y = 19.28 \text{ lb}$$

$$E = \sqrt{E_x^2 + E_y^2} = \underline{75.1 \text{ lb}}$$



$$\Sigma M_B = 0: \; 125.5\,(3\sin 60°) - P\cos 15°\,(1) = 0, \quad \underline{P = 338 \text{ lb}}$$

4/108



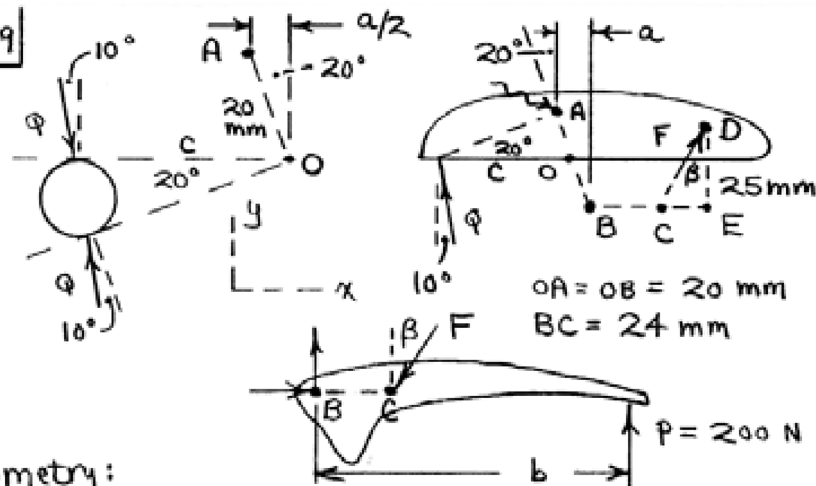
(For entire system) $\Sigma F_y = 0$: $2P = 12 + [40 + 2(35)](9.81)(10^{-3})$

$P = 6.54$ kN

(Wheel Assembly) $\Sigma M_S = 0$: $900A - 6.54(250) + 35(9.81)(10^{-3})(180) = 0$

$\underline{A = 1.748 \text{ kN}}$

4/109



Geometry:

$c = \dfrac{7.5}{\tan 10°} = 42.5 \text{ mm}$, $\dfrac{a}{2} = 20 \sin 20° = 6.84 \text{ mm}$

$a = 13.68 \text{ mm}$, $b = 190 - 13.68 = 176.3 \text{ mm}$

$\overline{CE} = 50 - 13.68 - 24 = 12.32 \text{mm}$, $\beta = \tan^{-1} \dfrac{12.32}{25} = 26.2°$

Lower handle:

$\Sigma M_B = 0: \; 200(176.3) - F\cos\beta\,(24) = 0$

$F\cos\beta = 1469 \text{ N}$, $F\sin\beta = 724 \text{ N}$

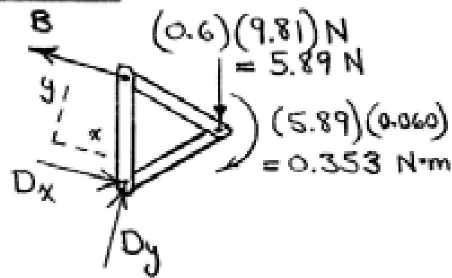
Upper jaw: (consider F to act at C)

$\Sigma M_A = 0: \; 1469(24 + 13.68) + 724(40 \cos 20°)$

$- Q\cos 10°\,(42.5 - 6.84) - Q\sin 10°\,(20 \cos 20°) = 0$

$Q = \underline{2150 \text{ N}}$ or $\underline{2.15 \text{ kN}}$

4/110 Note that AB is a two-force member.



$\Sigma F_y = 0: D_y - 5.89 \cos 15° = 0$

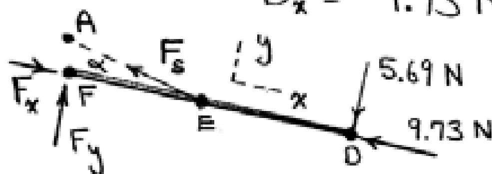
$D_y = 5.69$ N

$\Sigma M_D = 0: B(0.065)\cos 15° + 5.89(0.060) - 0.353 = 0$

$B = 11.25$ N

$\Sigma F_x = 0: -11.25 + D_x + 5.89 \sin 15° = 0$

$D_x = 9.73$ N

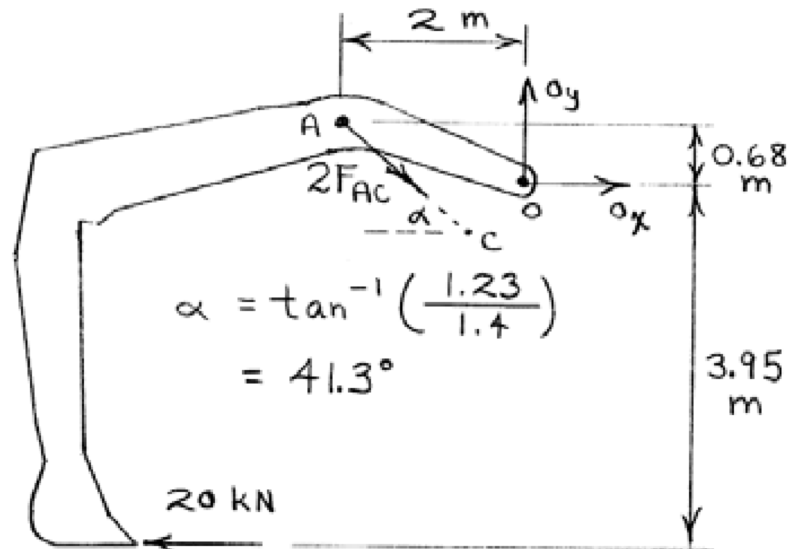


$\alpha = \tan^{-1}\left(\frac{65 \cos 15°}{225 + 65 \sin 15°}\right) = 14.55°$

$\Sigma M_F = 0: (F_S \sin\alpha)(0.225) - 5.69(0.450) = 0$

$F_S = 45.2$ N

4/111



$$\circlearrowleft + \sum M_O = 0 : \quad -20\,000(3.95) - 2F_{AC}\cos\alpha\,(0.68) + 2F_{AC}\sin\alpha\,(2) = 0$$

$F_{AC} = 48\,800$ N or $\underline{48.8 \text{ kN}}$

$F_{AC} = pA$: $48\,800 = p\left(\pi \frac{0.095^2}{4}\right)$

$p = 6.89(10^6)$ Pa or $\underline{6.89 \text{ MPa}}$

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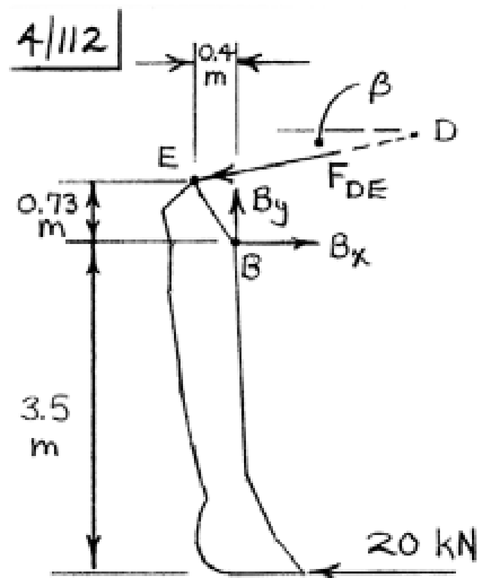


$\beta = \tan^{-1}\left(\frac{0.5}{2.5}\right)$

$= 11.31°$

$\circlearrowleft + \Sigma M_B = 0: \quad -20\,000(3.5) + F_{DE}\cos\beta\,(0.73)$

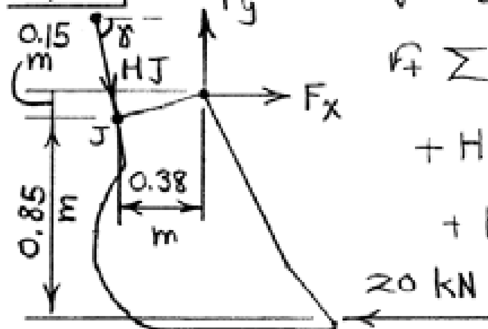
$+ F_{DE}\sin\beta\,(0.4) = 0$

$F_{DE} = 88\,100$ N or $\underline{88.1 \text{ kN}}$

$F_{DE} = pA: \quad 88\,100 = p\left(\pi\,\frac{0.105^2}{4}\right)$

$p = 10.18(10^6)$ Pa or $\underline{10.18 \text{ MPa}}$

4/113

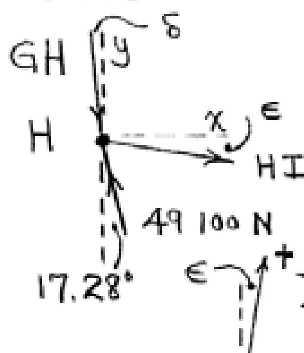


$\gamma = \tan^{-1}\left(\frac{0.45}{0.14}\right) = 72.7°$

$\sum M_F = 0: -20\,000(1) + HJ \sin\gamma\,(0.38) + HJ \cos\gamma\,(0.15) = 0$

$HJ = 49\ 100$ N C

Joint H:



$\delta = \tan^{-1}\left(\frac{0.18}{1.75}\right) = 5.87°$

$\epsilon = \tan^{-1}\left(\frac{0.09}{0.47}\right) = 10.84°$

$\sum F = 0:$

$49\,100 \cos(17.28° + 10.84°) - GH \cos(5.87° + 10.84°) = 0$

$GH = 45\,200$ N or 45.2 kN

$GH = pA: \; 45\,200 = p\left(\pi \frac{0.095^2}{4}\right)$

$p = 6.38(10^6)$ Pa or 6.38 MPa

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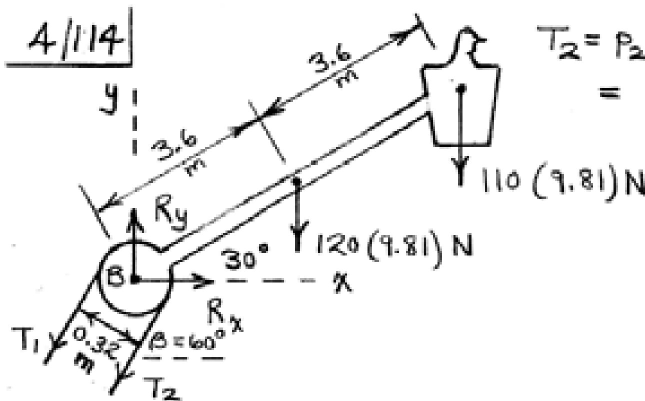


$T_2 = p_2 A = 80 \cdot 10^3 (7.85 \cdot 10^{-3})$
$= 628$ N

$\Sigma M_B = 0$: $110(9.81)(7.2 \cos 30°) + 120(9.81)(3.6 \cos 30°)$
$+ 628(0.160) - T_1 (0.160) = 0$, $T_1 = 65.6(10^3)$ N

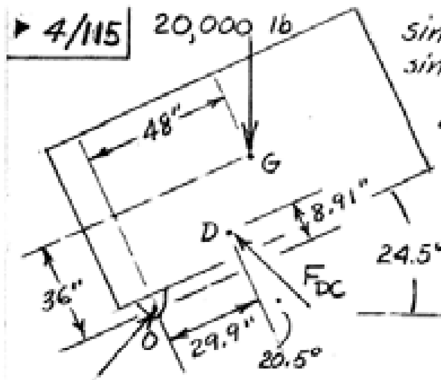
$p = T/A = 65.6(10^3)/(7.85 \cdot 10^{-3}) = 8360(10^3)$ Pa

or $\underline{p = 8360 \text{ kPa}}$

$\Sigma F_x = 0$: $R_x - [65.6(10^3) + 628] \cos 60° = 0$, $R_x = 33.1$ kN

$\Sigma F_y = 0$: $R_y - [65.6(10^3) + 628] \sin 60° - 230(9.81) = 0$
$R_y = 59.6$ kN

$R = \sqrt{R_x^2 + R_y^2} = \underline{68.2 \text{ kN}}$

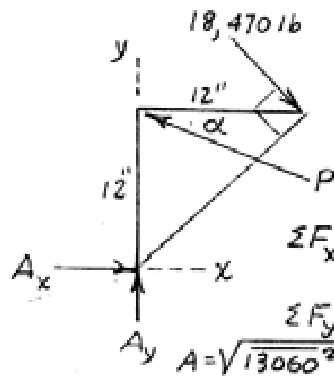


$\sin 24.5° = 0.4147, \quad \cos 24.5° = 0.9100$

$\sin 20.5° = 0.3502, \quad \cos 20.5° = 0.9367$

$\Sigma M_O = 0; \; 20{,}000(0.9100)48 - 20{,}000(0.4147)(36) - F_{DC}(0.9367)(29.9) - F_{DC}(0.3502)(8.91) = 0$

$F_{DC} = 18{,}470 \text{ lb}$



$\alpha = \tan^{-1}\frac{12}{48} = 14.04°,$

$\sin\alpha = 0.2425, \; \cos\alpha = 0.9701$

$\Sigma M_A = 0; \; 0.9701P(12) - 18{,}470(12\sqrt{2}) = 0$

$\underline{P = 26{,}900 \text{ lb}}$

$\Sigma F_x = 0; \; A_x + 18{,}470/\sqrt{2} - 26{,}900(0.9701) = 0$

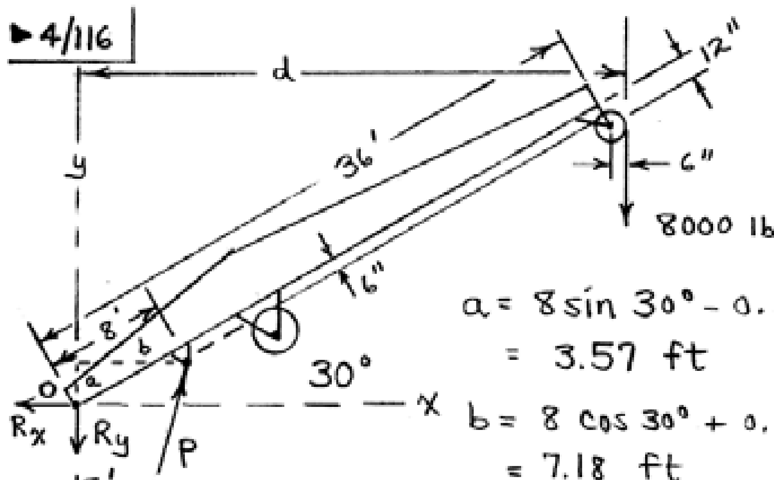
$A_x = 13{,}060 \text{ lb}$

$\Sigma F_y = 0; \; A_y + 26{,}900(0.2425) - 18{,}470/\sqrt{2} = 0$

$A_y = 6530 \text{ lb}$

$A = \sqrt{13060^2 + 6530^2} = \underline{14{,}600 \text{ lb}}$

▶ 4/116



$$a = 8 \sin 30° - 0.5 \cos 30° = 3.57 \text{ ft}$$

$$b = 8 \cos 30° + 0.5 \sin 30° = 7.18 \text{ ft}$$

18'

F 4' D

α

E

$$d = 36 \cos 30° + 1 \sin 30° + 0.5 = 32.2 \text{ ft}$$

$$\tan \alpha = \frac{7.18 - 4}{8 + 3.57}, \quad \alpha = 15.36°$$

$$\overline{FE} = 4/\tan\alpha = 14.56 \text{ ft}$$

$$\Sigma M_E = 0: \; 8000(32.2) - (14.56 + 8) R_x = 0$$

$$R_x = 11{,}410 \text{ lb}$$

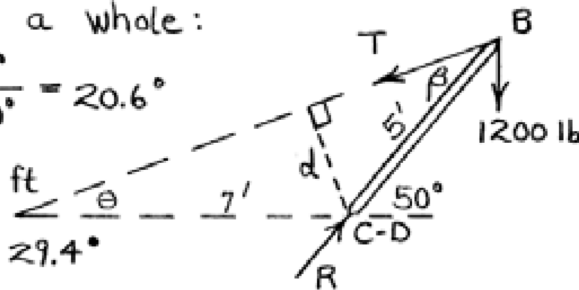
$$\Sigma M_O = 0: \; 8000(32.2) - P\cos(15.36° + 30°)\,8 - P\sin(15.36° + 30°)(0.5) = 0, \quad \underline{P = 43{,}100 \text{ lb}}$$

$$\Sigma F_y = 0: \; 43{,}100 \cos 15.36° - 8000 - R_y = 0$$

$$R_y = 33{,}500 \text{ lb}$$

$$R = \sqrt{R_x^2 + R_y^2} = \underline{35{,}400 \text{ lb}}$$

4/117 Frame as a whole:



$\theta = \tan^{-1} \frac{5 \sin 50°}{7 + 5 \cos 50°} = 20.6°$

$d = 7 \sin 20.6° = 2.46$ ft

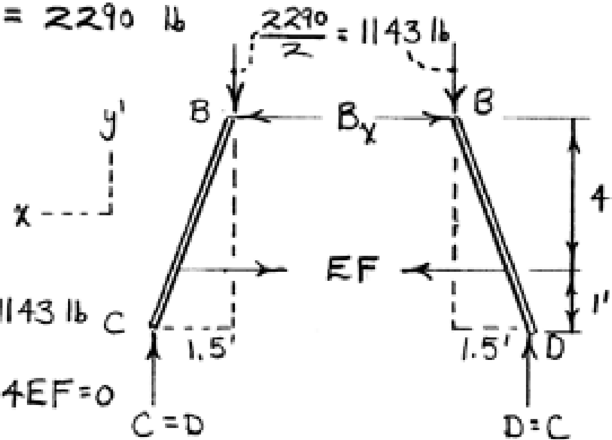
$\beta + 20.6° = 50°$, $\beta = 29.4°$

$\Sigma M_x = 0: (5 \cos 50°)(1200) - 2.46T = 0$, $T = 1569$ lb

$\Sigma F_{(C-D)-B} = 0: R - 1200 \cos 40° - 1569 \cos 29.4° = 0$

$R = 2290$ lb

Plane of frame:



$\Sigma F_{y'} = 0: C = D = 1143$ lb

$\Sigma M_B = 0: 1143(1.5) - 4EF = 0$

$EF = 429$ lb

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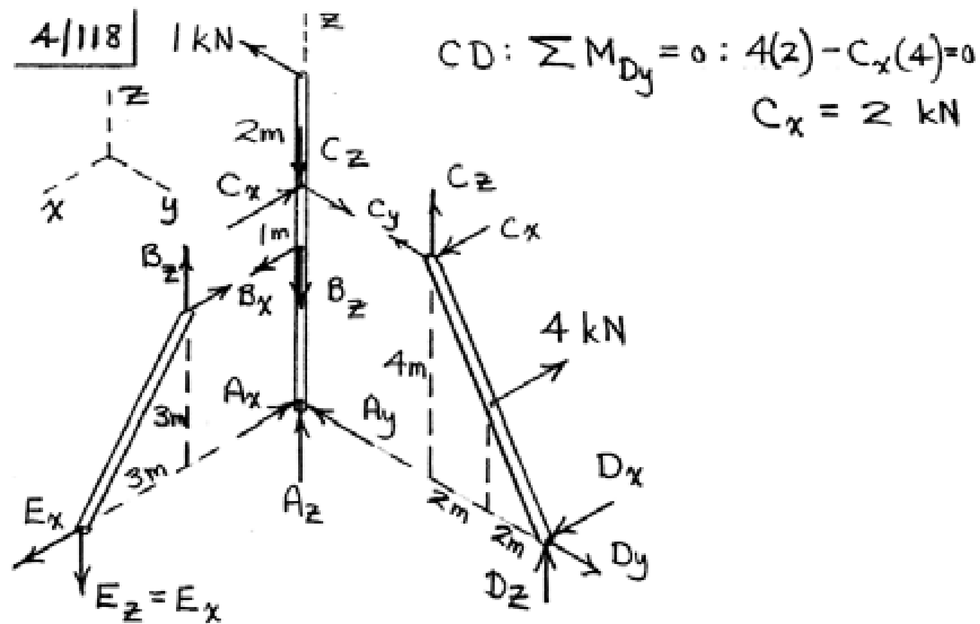

CD: $\Sigma M_{D_y} = 0: \; 4(2) - C_x(4) = 0$

$C_x = 2$ kN

ABC: $\Sigma M_{B_y} = 0: \; 2(1) - A_x(3) = 0, \qquad A_x = 0.667$ kN

$\Sigma F_x = 0: \; B_x - 0.667 - 2 = 0, \qquad B_x = 2.67$ kN

EB: $B_z = E_z = B_x = E_x = 2.67$ kN

ABC: $\Sigma M_{A_x} = 0: \; 1(6) - C_y(4) = 0, \qquad C_y = 1.50$ kN

CD: $\Sigma M_{D_x} = 0: \; C_z = C_y = 1.50$ kN

ABC: $\Sigma F_z = 0: \; A_z - 2.67 - 1.50 = 0, \qquad A_z = 4.17$ kN

$\Sigma F_y = 0: \; A_y + 1 - 1.50 = 0, \qquad A_y = 0.50$ kN

$A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \underline{4.25 \text{ kN}}$

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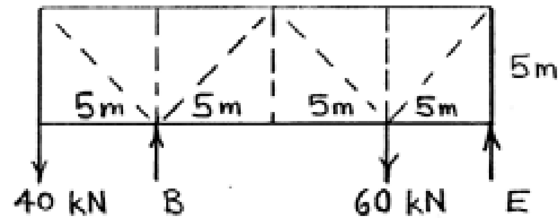
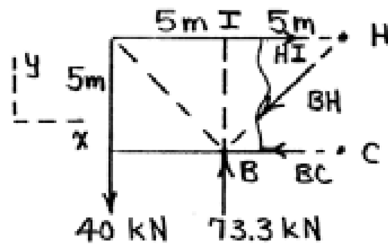


$\Sigma M_E = 0: \; 40(20) + 60(5) - 15B = 0, \quad B = 73.3 \text{ kN}$

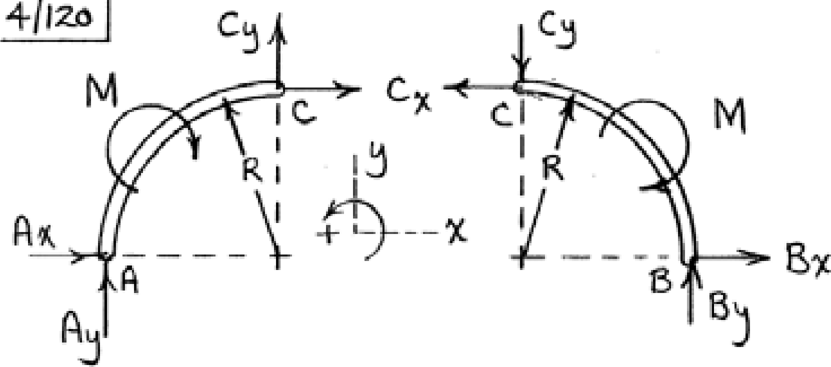


$\Sigma F_y = 0: -\frac{\sqrt{2}}{2} BH + 73.3 - 40 = 0, \quad \underline{BH = 47.1 \text{ kN } C}$

$\Sigma M_B = 0: \; HI(5) - 40(5) = 0, \quad \underline{HI = 40 \text{ kN } T}$

$\Sigma F_x = 0: \; 40 - 47.1 \frac{\sqrt{2}}{2} - BC = 0, \quad \underline{BC = 6.67 \text{ kN } C}$

4/120



Left member:

$$\begin{cases} \Sigma F_x = 0: \quad A_x + C_x = 0 & (1) \\ \Sigma F_y = 0: \quad A_y + C_y = 0 & (2) \\ \Sigma M_A = 0: \quad -M - C_x(R) + C_y(R) = 0 & (3) \end{cases}$$

Right member:

$$\Sigma F_x = 0: \quad -C_x + B_x = 0 \qquad (4)$$

$$\Sigma F_y = 0: \quad -C_y + B_y = 0 \qquad (5)$$

$$\Sigma M_B = 0: \quad -M + C_x(R) + C_y(R) = 0 \qquad (6)$$

Solution of Eqs. (1)–(6):

$$\begin{cases} C_y = B_y = \frac{M}{R}, \quad A_y = -\frac{M}{R} \\ A_x = B_x = C_x = 0 \end{cases}$$

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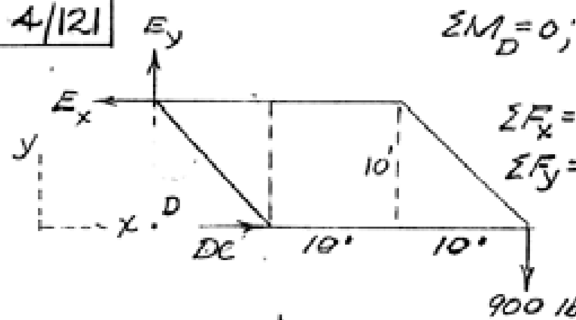


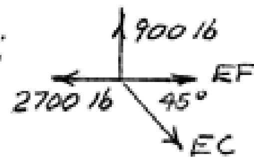
$\Sigma M_D = 0;\ 900(30) - 10E_x = 0$

$E_x = 2700 \text{ lb}$

$\Sigma F_x = 0;\ DC = 2700 \text{ lb}\ C$

$\Sigma F_y = 0;\ E_y = 900 \text{ lb}$

Joint E;



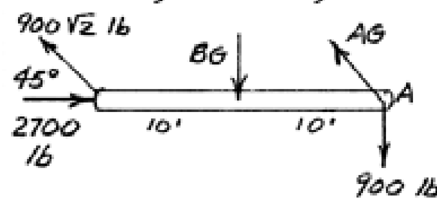
$\Sigma F_y = 0;\ EC/\sqrt{2} - 900 = 0$

$EC = 900\sqrt{2} \text{ lb}$

$\Sigma F_x = 0;\ EF + \dfrac{900\sqrt{2}}{\sqrt{2}} - 2700 = 0$

$EF = 1800 \text{ lb}\ T$

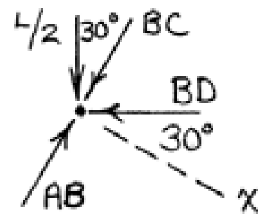
Joint F; $FG = EF,\ FC = 0$



$\Sigma M_A = 0;\ 10BG - \dfrac{900\sqrt{2}}{\sqrt{2}}20 = 0$

$\underline{BG = 1800 \text{ lb}\ C}$

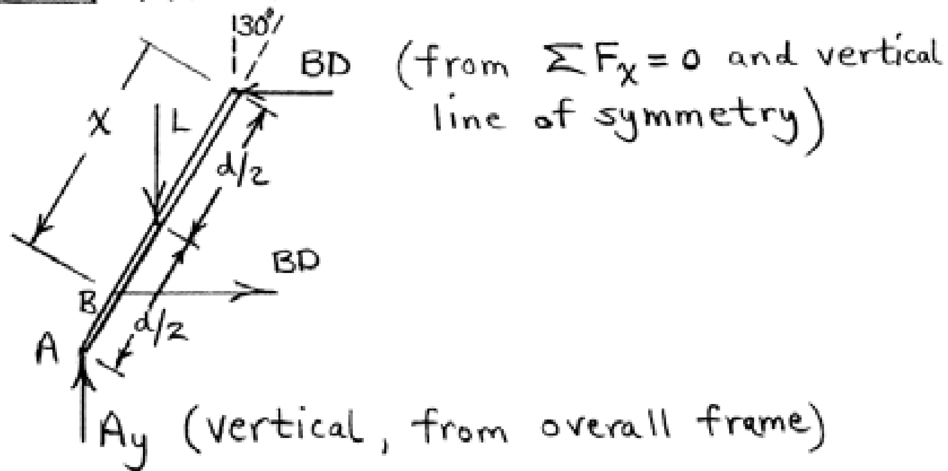
4/122 Joint B :



$\Sigma F_x = 0: \quad \frac{L}{2}\left(\frac{1}{2}\right) - BD\frac{\sqrt{3}}{2} = 0$

$BD = \frac{\sqrt{3}}{6} L$ C , independent of x.

4/123 Member ABC:



BD (from $\Sigma F_x = 0$ and vertical line of symmetry)

A_y (vertical, from overall frame)

$$\circlearrowleft + \Sigma M_A = 0: \quad BD(d \cos 30°) - BD(d-x)\cos 30° - L\left(\frac{d}{2}\sin 30°\right) = 0$$

$$BD = \frac{Ld}{2x}\tan 30° = \frac{0.289Ld}{x}$$

(x cannot be zero)

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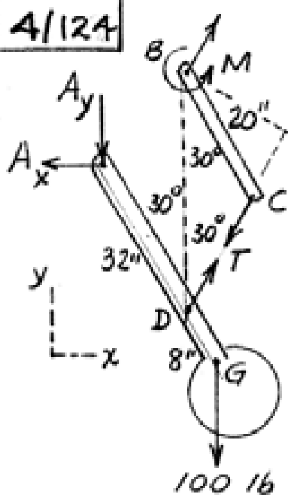


$ADG;\ \Sigma M_A = 0;\ 100\,(40\ \sin 30°)$

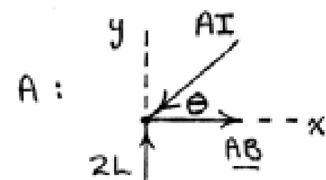
$-T(32 \cos 30°) = 0$

$T = 72.2\ lb$

$BC;\ \Sigma M_B = 0;\ 72.2\,(20 \cos 30°) - M = 0$

$\underline{M = 1250\ lb\text{-}in.}$

4/125 From whole truss, $A = F = 2L$

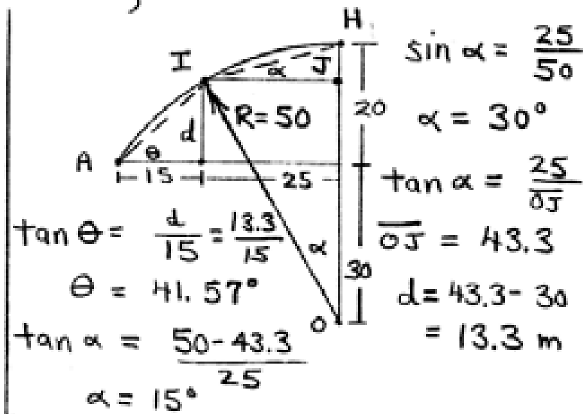


y: $2L - AI \sin\theta$

$AI = 3.01L$ C

x: $-3.01\cos\theta + AB = 0$

$AB = 2.26L$ T



$\sin\alpha = \frac{25}{50}$

$\alpha = 30°$

$\tan\alpha = \frac{25}{\overline{OJ}}$

$\overline{OJ} = 43.3$

$d = 43.3 - 30 = 13.3$ m

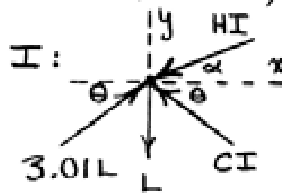
$\tan\theta = \frac{d}{15} = \frac{13.3}{15}$

$\theta = 41.57°$

$\tan\alpha = \frac{50 - 43.3}{25}$

$\alpha = 15°$

From joint B, $BI = L$ T



$$\begin{cases} \Sigma F_x = 0: (3.01L - CI)\cos\theta - HI\cos\alpha = 0 \\ \Sigma F_y = 0: (3.01L + CI)\sin\theta - HI\sin\alpha - L = 0 \end{cases}$$

Solve to obtain $CI = -0.458L$ T

$HI = 2.69L$ C

4/126 From whole structure, $A = F = 2L$

Half of structure:

$\circlearrowleft + \Sigma M_C = 0: \ 20H + 15L - 30(2L) = 0; \quad H = 2.25L$

$\Sigma F_x = 0 \Rightarrow \ CD = 2.25L \ T$

From joint B, $\underline{BI = L \ T}$

Member AIH:

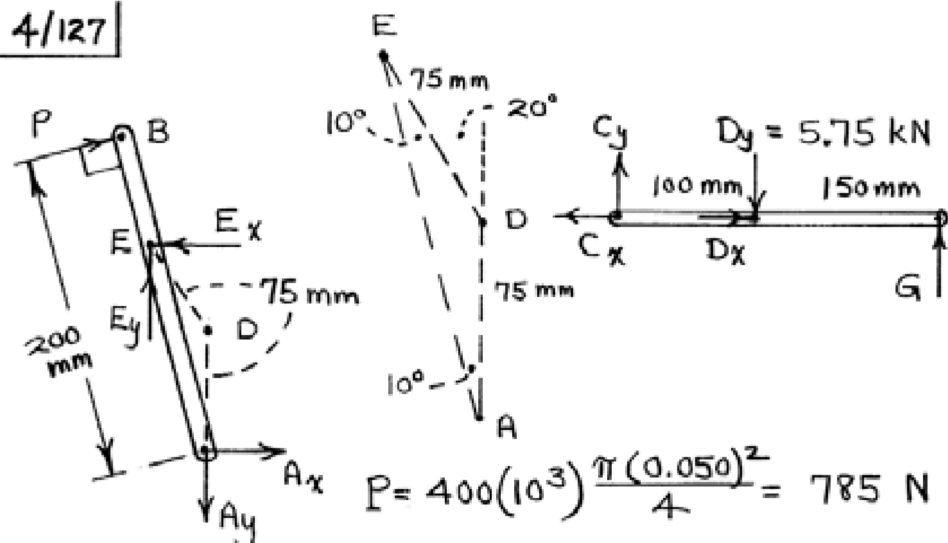
From previous solution, $\theta = 41.57, \ \alpha = \tan^{-1}\left(\frac{10}{20}\right) = 26.57°$

$\Sigma F_y = 0: \ 2L - L - CI \sin\theta - 0.5L \cos\alpha = 0, \quad \underline{CI = 0.833L \ T}$

$\Sigma F_x = 0: \ AB + 0.833L \cos\theta - 0.5L \sin\alpha - 2.25L = 0$

$\underline{AB = 1.850L \ T}$

Problem not solvable without CH data.



$P = 400(10^3)\dfrac{\pi(0.050)^2}{4} = 785\ \text{N}$

AB: $\Sigma M_A = 0$: $785(200) + E_y(75 \sin 20°)$
$- E_x(75 + 75 \cos 20°) = 0$

But ED is a two-force member so that

$E_x = E_y \tan 20° = 0.364 E_y$

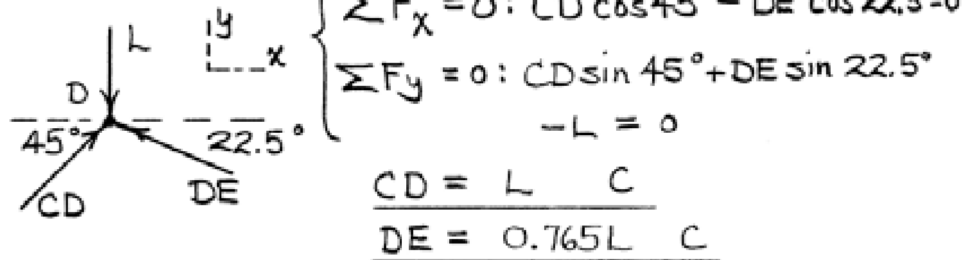
Solve moment equation : $E_y = 5.75$ kN

CG: $\Sigma M_C = 0$: $250 G - 100(5.75) = 0$, $\underline{G = 2.30\ \text{kN}}$

4/128 By inspection of joints B, K, and C,

$BK = CK = CE = 0.$

Joint D:



$$\Sigma F_x = 0: CD \cos 45° - DE \cos 22.5° = 0$$

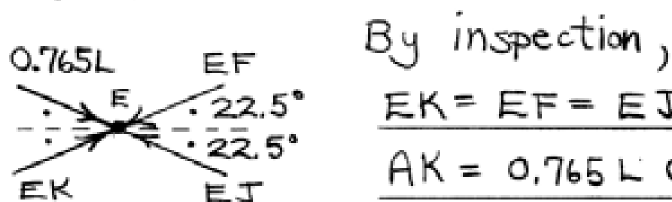
$$\Sigma F_y = 0: CD \sin 45° + DE \sin 22.5° - L = 0$$

$CD = L \quad C$

$DE = 0.765L \quad C$

Then $AB = BC = L \quad C$

Joint E:



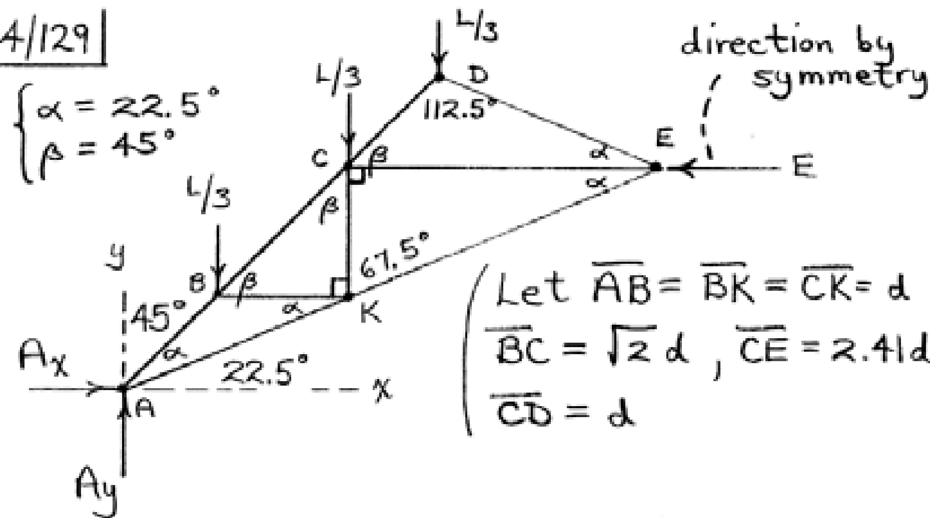
By inspection,

$EK = EF = EJ = 0.765L \; C$

$AK = 0.765L \; C$

By symmetry, $EG = GJ = HJ = 0$,

$FG = GH = HI = L \; C$, $IJ = 0.765L \; C$.

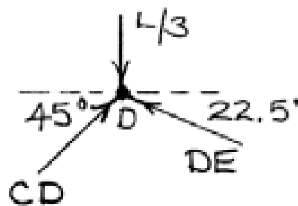


$\curvearrowleft + \Sigma M_A = 0$: $E(d+\sqrt{2}d)\sin 45° - \frac{L}{3}(d\cos 45°)$

$- \frac{L}{3}(d+\sqrt{2}d)\cos 45° - \frac{L}{3}(d+\sqrt{2}d+d)\cos 45° = 0$

$E = 0.943L$

Joint D:



$\Sigma F_x = 0$: $CD\cos 45° - DE\cos 22.5° = 0$

$\Sigma F_y = 0$: $CD\sin 45° + DE\sin 22.5° - \frac{L}{3} = 0$

$\underline{DE = 0.255L \quad C}$

$\underline{CD = 0.333L \quad C}$

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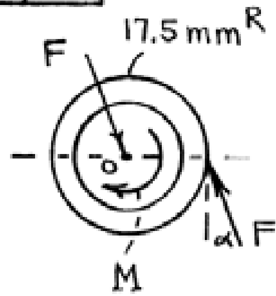
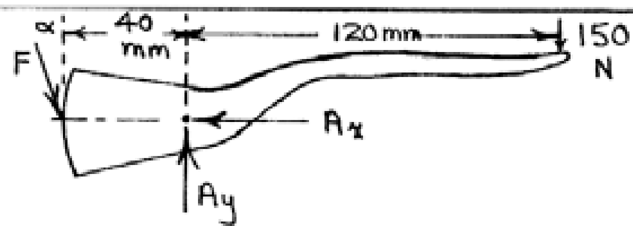
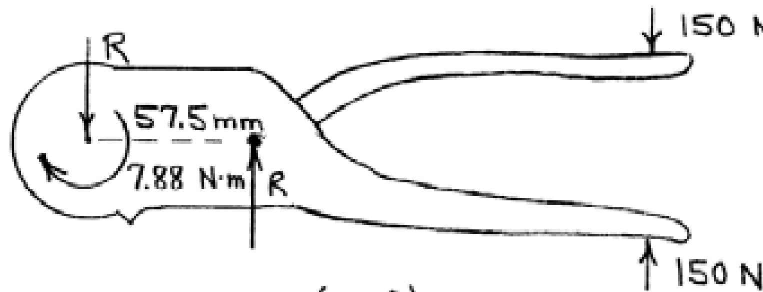




$\Sigma M_A = 0$: $40 F \cos\alpha = 120(150)$

$F \cos\alpha = 450$ N

$\Sigma M_O = 0$: $450(0.0175) = \underline{M = 7.88\ \text{N}\cdot\text{m}}$



$\Sigma M = 0$: $57.5(10^{-3})R = 7.88$

$\underline{R = 137.0\ \text{N}}$

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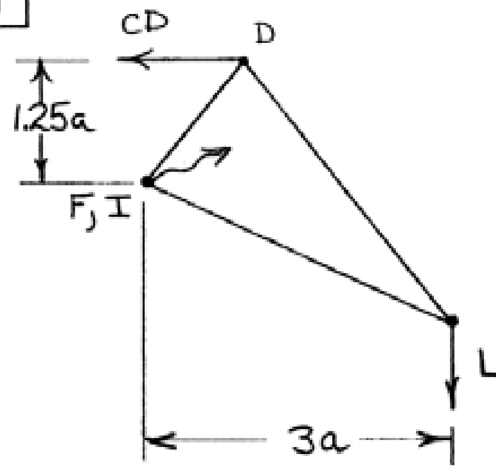
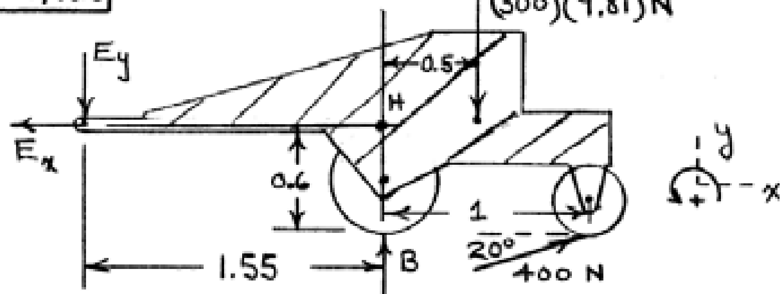


$\curvearrowleft + \Sigma M_F = 0: \quad CD(1.25a) - L(3a) = 0$

$CD = 2.4L \ \ T$

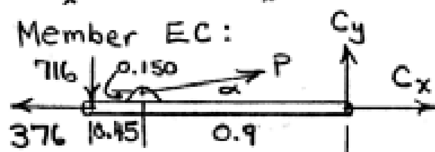
4/132 Entire machine :



$\Sigma M_H = 0$: $E_y(1.55) - (300)(9.81)(0.5) + (400 \sin 20°)(1)$
$+ (400 \cos 20°)(0.6) = 0$, $E_y = 716$ N

$\Sigma F_x = 0$: $-E_x + 400 \cos 20° = 0$, $E_x = 376$ N

Member EC:



$\alpha = \tan^{-1}\left(\frac{230}{1300}\right) = 10.03°$

$\Sigma M_C = 0$ yields $P = 3170$ N

$\Sigma F_x = 0$ yields $C_x = 2750$ N

$\Sigma F_y = 0$ yields $C_y = 162.9$ N

$C = \sqrt{C_x^2 + C_y^2} = 2750$ N

4/133

$$\begin{cases} \alpha = 22.5° \\ \measuredangle ABO = 67.5° \end{cases}$$

$\overline{BE}^2 = R^2 + (1.5R)^2 - 2R \times (1.5R) \cos 22.5°, \; \overline{BE} = 0.692R$

$\dfrac{\sin 22.5°}{0.692R} = \dfrac{\sin\beta}{R}, \; \beta = 33.6°$

Joint B:

$\Sigma F_y = 0: \; 2(0.8L)\cos 33.6° - 2AB\cos 67.5° - L = 0$

$\underline{AB = 0.434L \; T}$

For joint E, note that EF = DE & AE = BE.

$\Sigma F_{y'} = 0: \; -2(0.8L)\cos 56.1° + 2DE\cos 67.5° = 0$

$\underline{DE = 1.166L \; C}$

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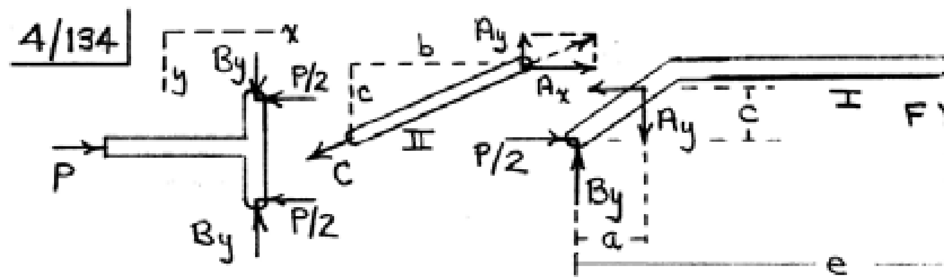


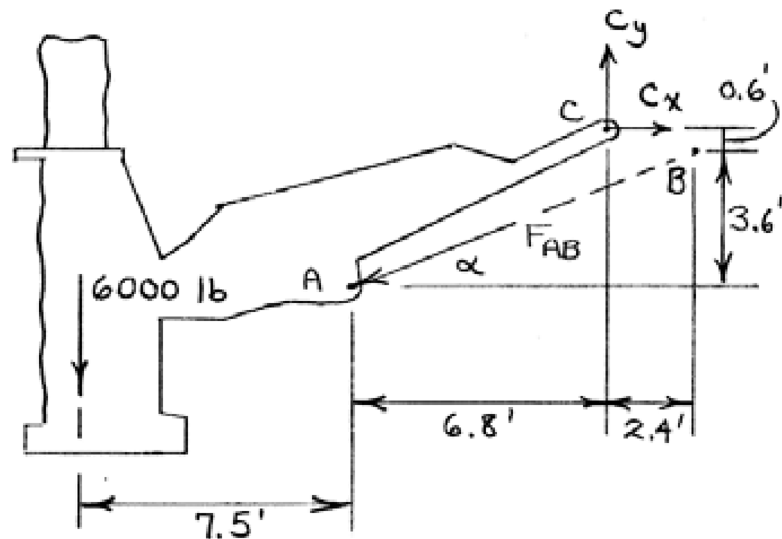
II. $A_x = \frac{b}{c} A_y$, $A_y = \frac{c}{b} A_x$

I. $\Sigma M_B = 0$: $Fe + \frac{c}{b} A_x a - A_x c = 0$

$$A_x = \frac{Fe}{c\left(1 - \frac{a}{b}\right)}$$

$\Sigma F_x = 0$: $\frac{Fe}{c(1 - a/b)} = \frac{P}{2}$, $\underline{P = \frac{2Fe}{c(1 - a/b)}}$

4/135



$$\alpha = \tan^{-1}\left(\frac{3.6}{9.2}\right) = 21.4°$$

$$\circlearrowleft + \Sigma M_C = 0: \; -F_{AB}\cos\alpha\,(0.6) - F_{AB}\sin\alpha\,(2.4) + 6000\,(14.3) = 0$$

$$\underline{F_{AB} = 59{,}900 \text{ lb}}$$

$$F_{AB} = pA: \quad 59{,}900 = p\left(\pi\,\frac{4.72^2}{4}\right)$$

$$\underline{p = 3420 \text{ lb/in.}^2}$$

▸4/136 Vector expressions for forces at A (treated as tensions) with $F_{AE} = F_{AF} = F_1$, $F_{BE} = F_{BF} = P$, $F_{BD} = F_{BC} = C$, are

$\underline{F}_{AE} = \frac{F_1}{1.552}(-1.2\underline{i} - 0.4\underline{j} + 0.9\underline{k})$, $\underline{F}_{AF} = \frac{F_1}{1.552}(-1.2\underline{i} + 0.4\underline{j} + 0.9\underline{k})$

$\underline{F}_{AB} = \frac{F_{AB}}{1.432}(-0.3\underline{i} + 1.4\underline{k})$, $\underline{F} = 2.2\underline{k}$. For joint A, $\Sigma\underline{F} = 0$ gives $\left[\frac{F_{AB}}{1.432}(-0.3) + \frac{2F_1}{1.552}(-1.2)\right]\underline{i}$

$+\left[2.2 + \frac{F_{AB}}{1.432}(1.4) + \frac{2F_1}{1.552}(0.9)\underline{k}\right] = \underline{0}$

Solve to get $\underline{F_{AB} = -2.681 \text{ kN}}$, $\underline{F_1 = 0.363 \text{ kN}}$

On B: $\underline{F}_{BE} = \frac{P}{1.105}(-0.9\underline{i} - 0.4\underline{j} - 0.5\underline{k})$

$\underline{F}_{BF} = \frac{P}{1.105}(-0.9\underline{i} + 0.4\underline{j} - 0.5\underline{k})$, $\underline{F}_{BD} = \frac{C}{1.105}(-0.9\underline{i} - 0.4\underline{j} + 0.5\underline{k})$

$\underline{F}_{BC} = \frac{C}{1.105}(-0.9\underline{i} + 0.4\underline{j} + 0.5\underline{k})$

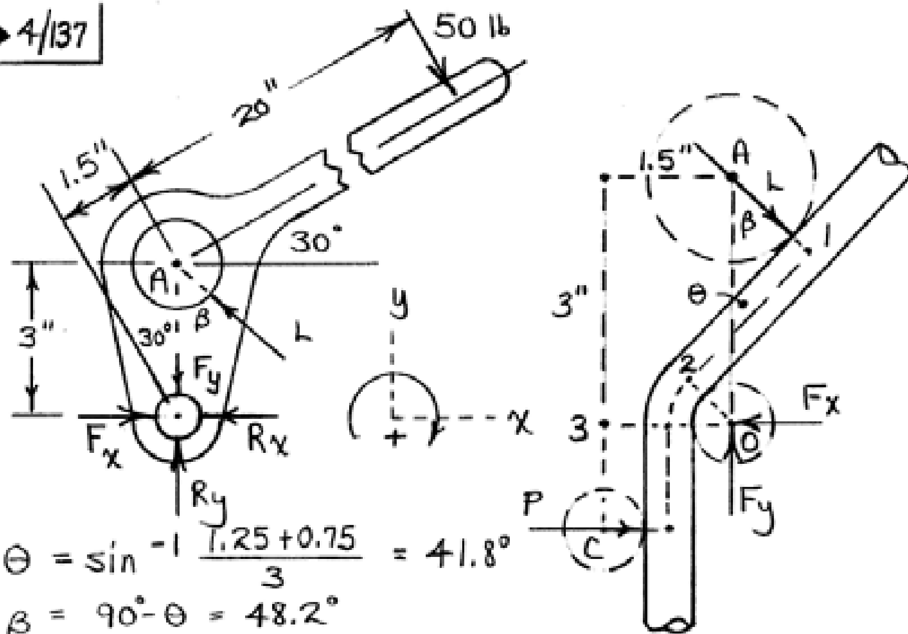
For joint B, $\Sigma\underline{F} = \underline{0}$ gives

$\left(\frac{-1.8P}{1.105} - \frac{1.8C}{1.105} + 0.3\frac{-2.681}{1.432}\right)\underline{i} + \left(\frac{-P}{1.105} + \frac{C}{1.105} - 1.4\frac{-2.681}{1.432}\right)\underline{k}$

$+ 0\underline{j} = \underline{0}$. Solve to get $P = 1.620$ kN,

$C = -1.275$ kN, $\underline{F_{BE} = P = 1.620 \text{ kN}}$

▶4/137



$\theta = \sin^{-1} \frac{1.25 + 0.75}{3} = 41.8°$

$\beta = 90° - \theta = 48.2°$

$1\text{-}2 = 3 \sin\beta = 2.24"$; $C\text{-}3 = \sqrt{2^2 - 1.5^2} = 1.323"$

Handle $\Sigma M_O = 0: 50(21.5) - L \sin 48.2°(3) = 0$, $L = 481$ lb

Bar
$\Sigma M_O = 0: 481(2.24) - P(1.323) = 0$, $P = 813$ lb

$\Sigma F_x = 0: 813 + 481 \sin 48.2° - F_x = 0$, $F_x = 1171$ lb

$\Sigma F_y = 0: F_y - 481 \cos 48.2° = 0$, $F_y = 321$ lb

Handle
$\Sigma F_x = 0: 1171 - R_x - 481 \sin 48.2° + 50 \sin 30° = 0$

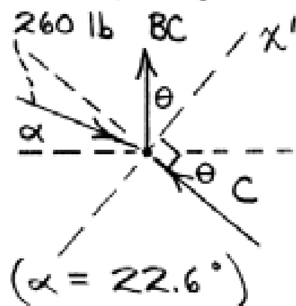
$\underline{R_x = 838 \text{ lb}}$

$\Sigma F_y = 0: R_y - 321 + 481 \cos 48.2° - 50 \cos 30° = 0$

$\underline{R_y = 43.3 \text{ lb}}$

*4/138 From the solution to Prob. 4/2, AC = 260 lb C and AB = 240 lb T, both constant.

Joint C (BC assumed to be in tension)



$\Sigma F_{x'} = 0$:

$BC \cos\theta + 260 \sin(\theta - \alpha) = 0$

$BC = -260 \dfrac{\sin(\theta-\alpha)}{\cos\theta}$

$BC \to \infty$ as $\theta \to 90°$, but truss is partially constrained at $\theta = 90°$. (b) BC is zero when $\sin(\theta - \alpha) = 0$, $\underline{\theta = \alpha = 22.6°}$. (Note that AC & C are collinear if $\theta = \alpha$, leaving BC = 0 by inspection.)

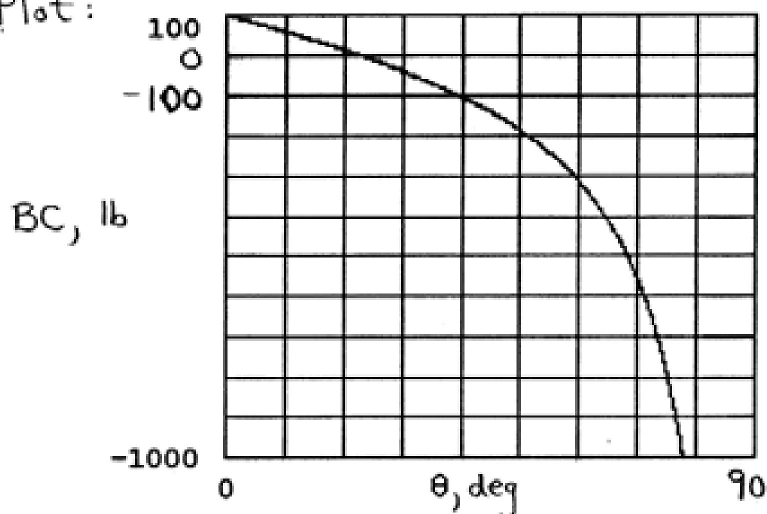
(c) Set $BC = -1000 = -260 \dfrac{\sin(\theta-\alpha)}{\cos\theta}$

$3.85 \cos\theta = \sin\theta\cos\alpha - \sin\alpha\cos\theta$

$\Rightarrow \tan\theta = \dfrac{3.85 + \sin\alpha}{\cos\alpha}$, $\underline{\theta = 77.7°}$

(Note: BC is never +1000 lb over $0 \le \theta \le 90°$.)

Plot:



*4/139

Joint B: $\begin{cases} \Sigma F_y = 0: & BC\left(\frac{1}{2}\right) - 3 = 0, \ \underline{BC = 6 \text{ kN C}} \\ \Sigma F_x = 0: & -AB + 6\frac{\sqrt{3}}{2} = 0, \ \underline{AB = 3\sqrt{3} \text{ kN T}} \end{cases}$

AB, 30°, BC, 3 kN, y, x

Joint C: $\begin{cases} \Sigma F_x = 0: -AC\sin\alpha - 6\left(\frac{\sqrt{3}}{2}\right) + CD\left(\frac{\sqrt{3}}{2}\right) = 0 \\ \Sigma F_y = 0: AC\cos\alpha - 6\left(\frac{1}{2}\right) + CD\left(\frac{1}{2}\right) - 3 = 0 \end{cases}$

AC, α, 6 kN, 30°, 30°, CD, 3 kN

Simultaneous solution:

$$AC = \frac{3\sqrt{3} \text{ kN}}{\sqrt{3}\cos\alpha + \sin\alpha} \quad T$$

$$CD = \frac{15}{2} + \frac{-3\sqrt{3}\cos\alpha + 9\sin\alpha}{2(\sqrt{3}\cos\alpha + \sin\alpha)} \text{ kN C}$$

Joint D

AD, 60°, CD, D_x

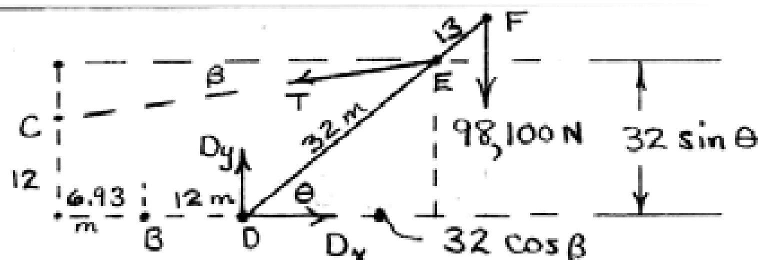
$\Sigma F_y = 0: \ AD - CD\left(\frac{1}{2}\right) = 0$

$$AD = \frac{15}{4} + \frac{-3\sqrt{3}\cos\alpha + \sin\alpha}{4(\sqrt{3}\cos\alpha + \sin\alpha)} \text{ kN T}$$

The plots below use (+) for tension and (-) for compression.

*4/140

Boom:

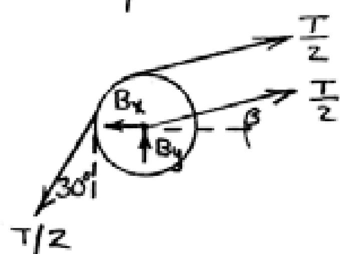


$$\beta = \tan^{-1}\left(\frac{32\sin\theta - 12}{6.93 + 12 + 32\cos\theta}\right) = \tan^{-1}\left(\frac{32\sin\theta - 12}{32\cos\theta + 18.93}\right) \quad (1)$$

$$\circlearrowleft \Sigma M_D = 0: \ (T\cos\beta)(32\sin\theta) - (T\sin\beta)(32\cos\theta) - (98{,}100)(45\cos\theta) = 0$$

So $$T = \frac{137{,}953\cos\theta}{\cos\beta\sin\theta - \sin\beta\cos\theta} \ (\text{in N}) \quad (2)$$

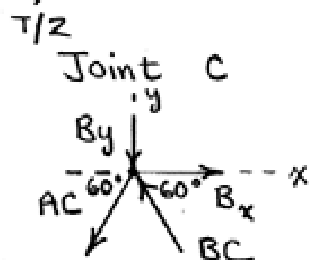
Pulley at C:



Equilibrium yields

$$\begin{cases} B_x = T\cos\beta - \frac{T}{2}\sin 30° \\ B_y = -T\sin\beta + \frac{T}{2}\cos 30° \end{cases} \quad (3)$$

Joint C

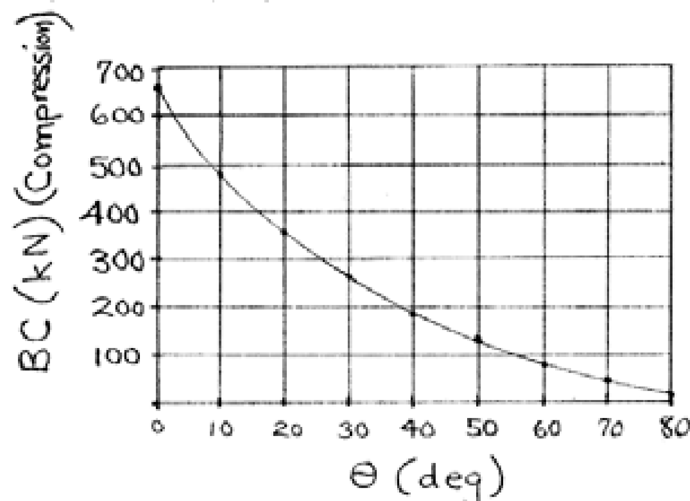


Equilibrium yields

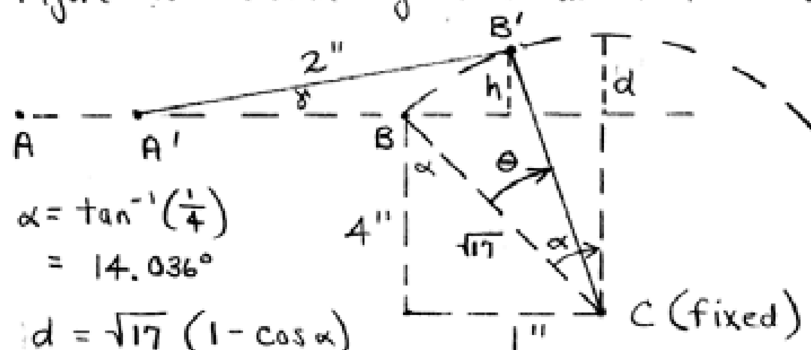
$$BC = B_x + 0.5774\,B_y \quad (4)$$

$$(\ AC = B_x - 0.5774\,B_y\ )$$

Solve Eqs. 1–4 in that order for $0 \le \theta \le 180°$.



*4/141 Geometry considerations. Note that unprimed refers to $\theta = 0$, primed to $\theta \neq 0$. Figure is reduced by a factor of 4 vertically.



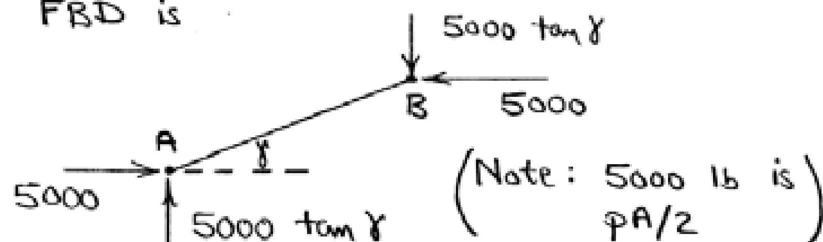
$\alpha = \tan^{-1}\left(\frac{1}{4}\right)$
$= 14.036°$

$d = \sqrt{17}\,(1 - \cos\alpha)$

$d - h = \sqrt{17}\,(1 - \cos(\alpha - \theta)) \Rightarrow h = \sqrt{17}\,[\cos(\alpha - \theta) - \cos\alpha]$

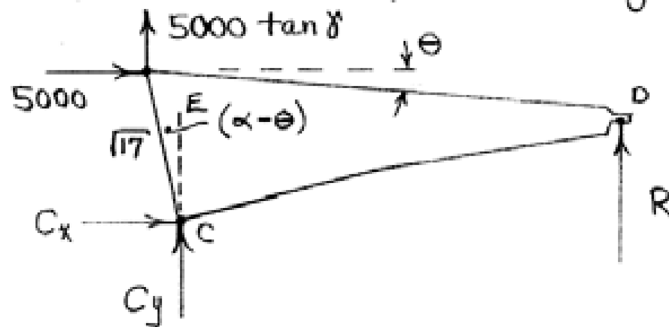
$\sin\gamma = \frac{h}{2} = \frac{\sqrt{17}}{2}\,[\cos(\alpha - \theta) - \cos\alpha]$ *

Note that AB is a two-force member. Its FBD is



(Note: 5000 lb is $pA/2$)

FBD of lower jaw, for arbitrary θ:



$\overline{CD} = \sqrt{(18)^2 + (3.75)^2} = 18.39''$

When $\theta = 0$, $\measuredangle ECD = \beta = \tan^{-1}\left(\frac{18}{3.75}\right) = 78.23°$

When $\theta \neq 0$, $\measuredangle ECD = \beta + \theta$

$\therefore$ Moment arm for force R about C is

$\overline{CD}\sin(\beta+\theta)$

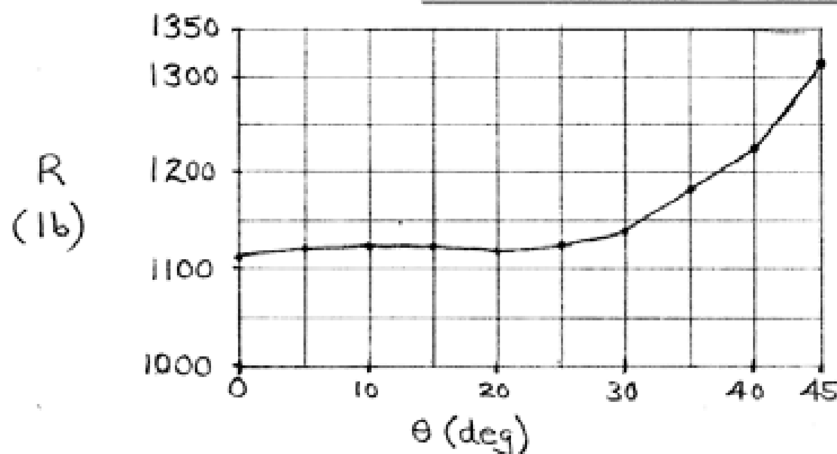
$\Sigma M_C = 0$: $\overline{CD}\sin(\beta+\theta) R - 5000\sqrt{17}\cos(\alpha-\theta)$

$- 5000\sqrt{17}\tan\gamma\sin(\alpha-\theta) = 0$

$$R = \frac{1121.233}{\sin(78.23°+\theta)}\left[\cos(\theta - 14.036°) - \tan\gamma\sin(\theta - 14.036°)\right] \text{ lb}$$

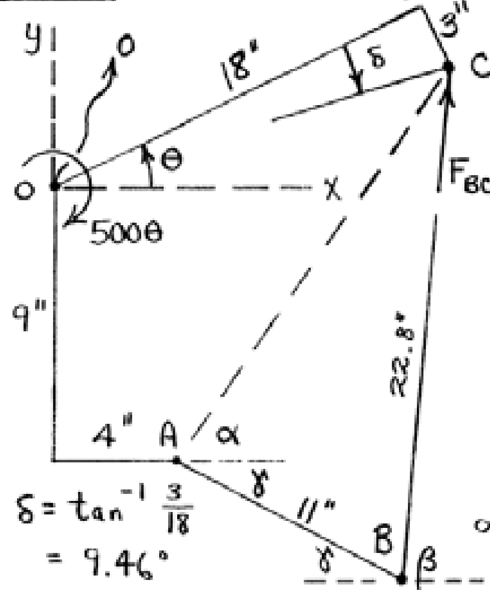
(γ given by $*$)

$R_{max} = 1314$ lb @ $\theta = 45°$



*4/142 $\overline{OC} = \sqrt{18^2 + 3^2}$; $\overline{AB} = 11''$; $\overline{BC} = \sqrt{14^2 + 17^2}$

$\overline{AC} = \{[18\cos\theta + 3\sin\theta - 4]^2 + [18\sin\theta - 3\cos\theta + 9]^2\}^{\frac{1}{2}}$



$\angle BAC = \cos^{-1}\left[\dfrac{\overline{AB}^2 + \overline{AC}^2 - \overline{BC}^2}{2\,\overline{AB}\,\overline{AC}}\right]$

$\angle ABC = \cos^{-1}\left[\dfrac{\overline{AB}^2 + \overline{BC}^2 - \overline{AC}^2}{2\,\overline{AB}\,\overline{BC}}\right]$

$\delta = \tan^{-1}\frac{3}{18} = 9.46°$

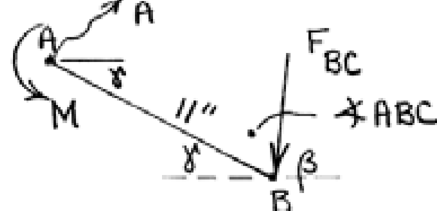
$\alpha = \tan^{-1}\left[\dfrac{18\sin\theta - 3\cos\theta + 9}{18\cos\theta + 3\sin\theta - 4}\right]$

$\gamma = \angle BAC - \alpha$; $\beta = 180° - \gamma - \angle ABC$

From above FBD of door :

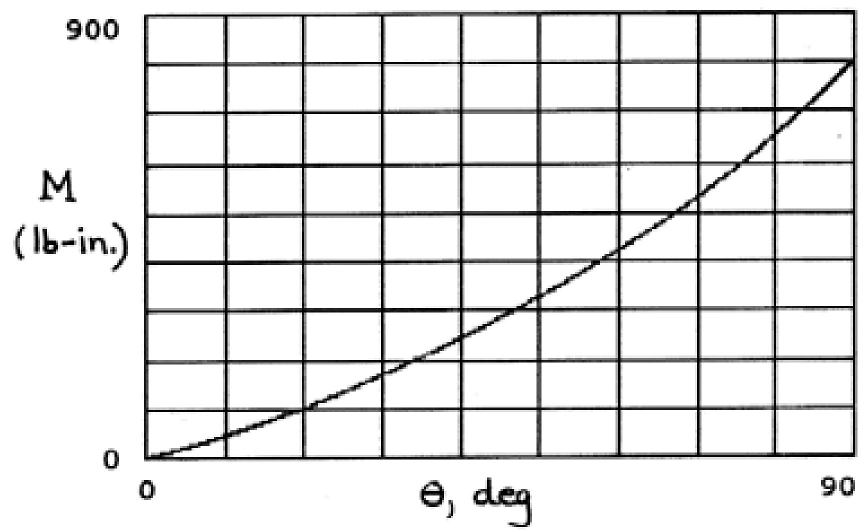
$\curvearrowleft + \Sigma M_O = 0: \ -500\theta + F_{BC}\left(\overline{OC}\sin(\beta - \theta + \delta)\right) = 0$

Then from FBD of AB:

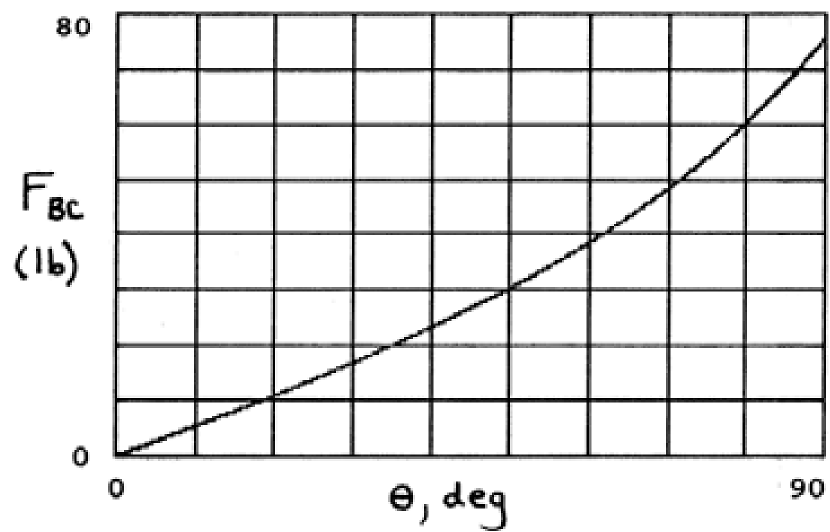


$\curvearrowleft + \Sigma M_A = 0: \ M - F_{BC}(11\sin\angle ABC) = 0$

Solve the above equations (in order) with θ varied from 0° to 90° to obtain the following plots for M and F_{BC}.

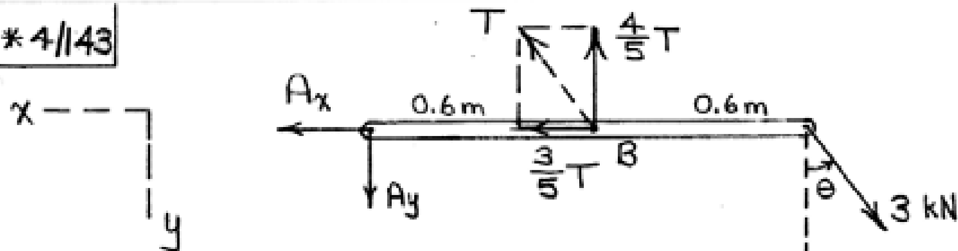


900
M
(lb-in.)
0
0
θ, deg
90



80
F_BC
(lb)
0
0
θ, deg
90

*4/143

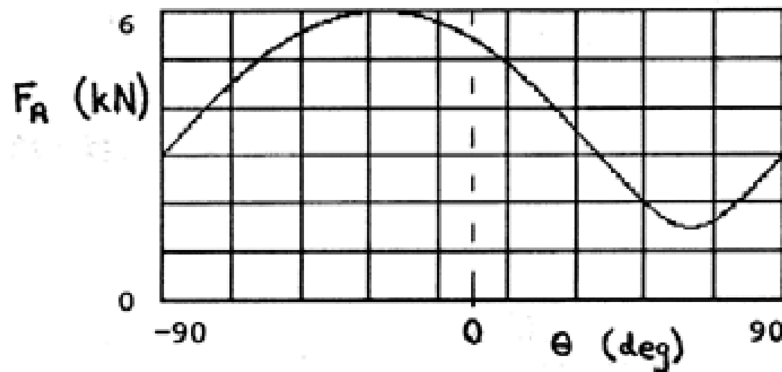


$\sum M_A = 0: \; 3\cos\theta\,(1.2) - \frac{4}{5}T(0.6) = 0, \; T = \frac{15}{2}\cos\theta$

$\sum F_x = 0: \; A_x + \frac{3}{5}\left(\frac{15}{2}\cos\theta\right) - 3\sin\theta = 0, \; A_x = 3\sin\theta - \frac{9}{2}\cos\theta$

$\sum F_y = 0: \; A_y + 3\cos\theta - \frac{4}{5}\left(\frac{15}{2}\cos\theta\right) = 0, \; A_y = 3\cos\theta$

$F_A = \sqrt{A_x^2 + A_y^2} = 3\sqrt{1 - 3\sin\theta\cos\theta + \frac{9}{4}\cos^2\theta}$



$F_{A_{max}} = 6 \text{ kN} \;@\; \theta = -26.6°$

*4/144

(Dim. in mm)

$$\overline{AC} = \sqrt{(350 - 275\cos\theta)^2 + (300 + 275\sin\theta)^2}$$

$$\measuredangle ABC = \cos^{-1}\left[\frac{\overline{AB}^2 + \overline{BC}^2 - \overline{AC}^2}{2\,\overline{AB}\,\overline{BC}}\right]$$

$$\alpha = \tan^{-1}\left[\frac{300 + 275\sin\theta}{350 - 275\cos\theta}\right]$$

$$\measuredangle CAB = \measuredangle ACB = \frac{180° - \measuredangle ABC}{2}$$

$$\delta = 180° - \alpha - \measuredangle ACB$$

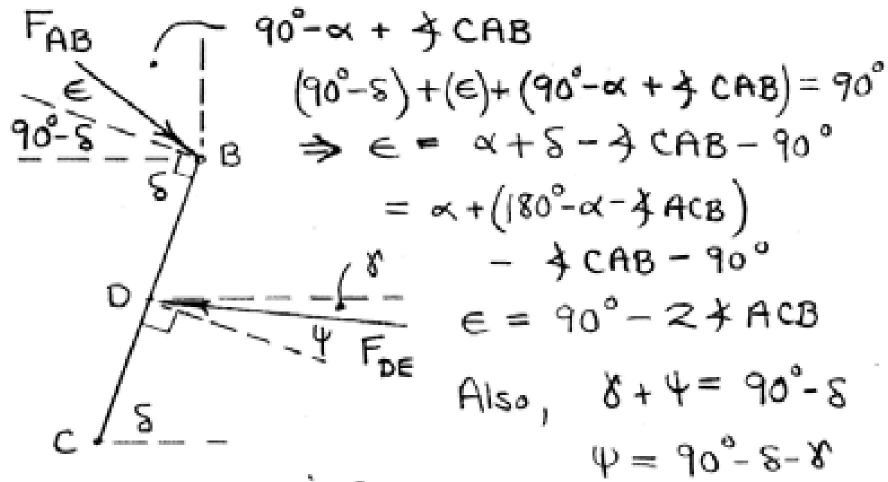
$mg = 30(9.81)$ N

$(90° - \alpha + \measuredangle CAB - \theta) = [90° - (\alpha + \theta - \measuredangle CAB)]$

$[(90° - \alpha) + \measuredangle CAB]$

$\circlearrowleft + \Sigma M_O = 0: \quad F_{AB} \cos\left[90° - (\alpha + \theta - \measuredangle CAB)\right]\overline{OA}$
$- mg\,\overline{OG}\cos\theta = 0$

$$F_{AB} = \frac{mg\,\overline{OG}\cos\theta}{\overline{OA}\sin(\alpha + \theta - \measuredangle CAB)}$$



$(90° - \delta) + (\epsilon) + (90° - \alpha + \measuredangle CAB) = 90°$

$\Rightarrow \epsilon = \alpha + \delta - \measuredangle CAB - 90°$

$= \alpha + (180° - \alpha - \measuredangle ACB)$
$- \measuredangle CAB - 90°$

$\epsilon = 90° - 2\measuredangle ACB$

Also, $\gamma + \psi = 90° - \delta$

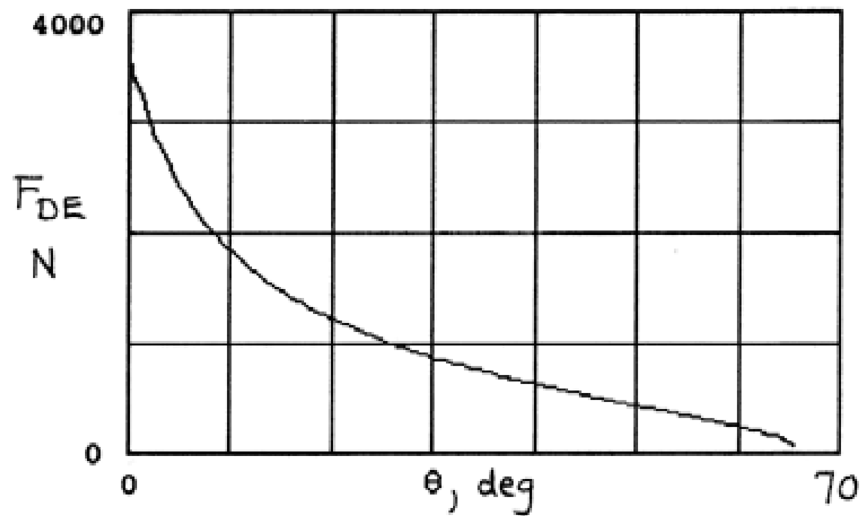
$\psi = 90° - \delta - \gamma$

$$\gamma = \tan^{-1}\left[\frac{150\sin\delta - 100}{400 - 150\cos\delta}\right]$$

$\circlearrowleft + \Sigma M_C = 0: \; -F_{AB}\cos\epsilon\,(\overline{CB}) + F_{DE}\cos\psi\,(\overline{CD}) = 0$

$$F_{DE} = \frac{F_{AB}\cos\epsilon\,(\overline{CB})}{\overline{CD}\cos\psi}$$

To find θ_{max}, set the above expression for $\overline{AC}$ to $\overline{AC} = 600$ mm and solve for $\underline{\theta_{max} = 65.9°}$. Numerically evaluate the above expressions for $0 \le \theta \le \theta_{max}$ to obtain the following plot:



$(F_{DE})_{max} = 3580$ N @ $\theta = 0$

$(F_{DE})_{min} = 0$ @ $\theta_{max} = 65.9°$ (links AB and BC are collinear and serve as (an unstable!) prop for the door)

5/1 From Table D/3, the horizontal coordinate of the centroid is

$$1 + \frac{(9-1)+(7-1)}{3} = \underline{5.67}$$

The vertical coordinate is

$$1 + \frac{9-1}{3} = \underline{3.67}$$

5/2 From Sample Problem 5/3,

$\bar{x} = 0$

$$\bar{y} = \frac{2}{3}\frac{r\sin\alpha}{\alpha} = \frac{2}{3}\frac{200\sin(120°/2)}{\frac{120°}{2}(\pi/180°)}$$

$$= \underline{110.3 \text{ mm}}$$

5/3

$\bar{x} = 0$

$$\bar{y} = -\frac{4r}{3\pi} = -\frac{4(120)}{3\pi} = \underline{-50.9\text{ mm}}$$

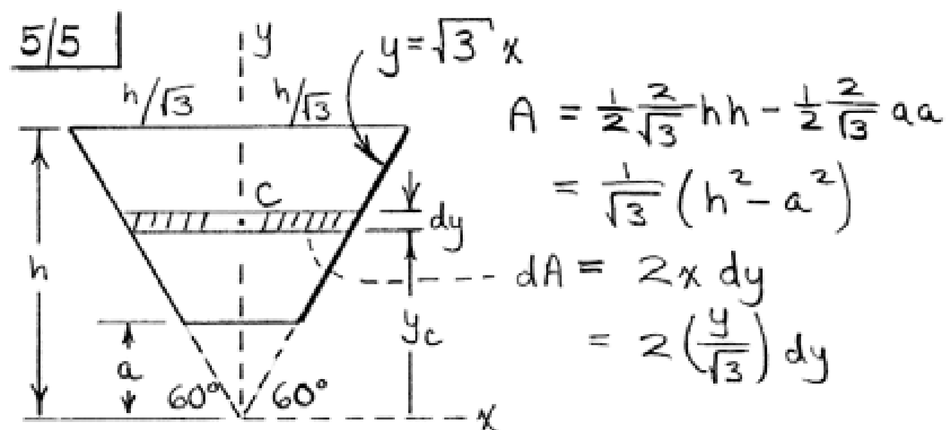
$$\bar{z} = -360/2 = \underline{-180\text{ mm}}$$

5/4

$\bar{x} = \bar{y} = -\dfrac{2r}{\pi} = -\dfrac{2(120)}{\pi} = \underline{-76.4\text{ mm}}$

$\underline{\bar{z} = -180\text{ mm}}$

5/5



$$A = \frac{1}{2}\frac{2}{\sqrt{3}}hh - \frac{1}{2}\frac{2}{\sqrt{3}}aa = \frac{1}{\sqrt{3}}\left(h^2 - a^2\right)$$

$$dA = 2x\,dy = 2\left(\frac{y}{\sqrt{3}}\right)dy$$

$$\int y_c\,dA = \int_a^h y\frac{2}{\sqrt{3}}y\,dy = \frac{2}{\sqrt{3}}\frac{y^3}{3}\Bigg|_a^h$$

$$= \frac{2}{3\sqrt{3}}\left(h^3 - a^3\right)$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{\frac{2}{3\sqrt{3}}\left(h^3 - a^3\right)}{\frac{1}{\sqrt{3}}\left(h^2 - a^2\right)} = \underline{\frac{2\left(h^3 - a^3\right)}{3\left(h^2 - a^2\right)}}$$

(For $a = 0$, $\bar{y} = \frac{2}{3}h$, the correct value.)

5/6

$y = 1 + \frac{x^3}{8}$

dA, C, y_c, y, x, dx, 0, 1, 2

$$dA = y\,dx = \left(1+\frac{x^3}{8}\right)dx$$

$$A = \int dA = \int_1^2 \left(1+\frac{x^3}{8}\right)dx$$

$$= \left(x + \frac{x^4}{32}\right)\Big|_1^2 = \frac{47}{32}$$

$$\int x_c\,dA = \int_1^2 x\left(1+\frac{x^3}{8}\right)dx$$

$$= \left(\frac{x^2}{2} + \frac{x^5}{40}\right)\Big|_1^2 = \frac{91}{40}$$

$$\int y_c\,dA = \int \frac{y}{2}\,y\,dx = \frac{1}{2}\int y^2\,dx$$

$$= \frac{1}{2}\int_1^2 \left(1+\frac{x^3}{8}\right)^2 dx = \frac{1}{2}\int_1^2 \left(1 + \frac{x^3}{4} + \frac{x^6}{64}\right)dx$$

$$= \frac{1}{2}\left(x + \frac{x^4}{16} + \frac{x^7}{448}\right)\Big|_1^2 = \frac{995}{896}$$

So

$$\bar{x} = \frac{\int x_c\,dA}{\int dA} = \frac{91/40}{47/32} = \underline{1.549}$$

$$\bar{y} = \frac{\int y_c\,dA}{\int dA} = \frac{995/896}{47/32} = \underline{0.756}$$

5/7

$$y = \left(\frac{b-a}{h}\right)x + a$$

$$dA = y\,dx$$

$$A = \int dA = \int_0^h \left[\left(\frac{b-a}{h}\right)x + a\right] dx$$

$$= \left[\frac{b-a}{h}\frac{x^2}{2} + ax\right]_0^h = \frac{h}{2}(a+b)$$

Figure labels: y, h, a, b, C, dA, x, dx

$$\int x_c\,dA = \int_0^h \left[\left(\frac{b-a}{h}\right)x^2 + ax\right] dx$$

$$= \left[\frac{b-a}{h}\frac{x^3}{3} + \frac{ax^2}{2}\right]_0^h = h^2\left(\frac{b}{3} + \frac{a}{6}\right)$$

$$\int y_c\,dA = \int \frac{y}{2}\,y\,dx = \frac{1}{2}\int_0^h y^2\,dx$$

$$= \frac{1}{2}\int_0^h \left[\left(\frac{b-a}{h}\right)^2 x^2 + 2\left(\frac{b-a}{h}\right)ax + a^2\right] dx$$

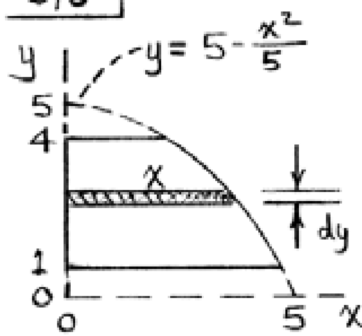
$$= \frac{1}{2}\left[\left(\frac{b-a}{h}\right)^2\frac{x^3}{3} + 2\left(\frac{b-a}{h}\right)a\frac{x^2}{2} + a^2x\right]_0^h$$

$$= \frac{h}{6}\left[a^2 + ab + b^2\right]$$

$$\bar{x} = \frac{\int x_c\,dA}{A} = \frac{h^2\left(\frac{b}{3} + \frac{a}{6}\right)}{\frac{h}{2}(a+b)} = \frac{h(a+2b)}{3(a+b)}$$

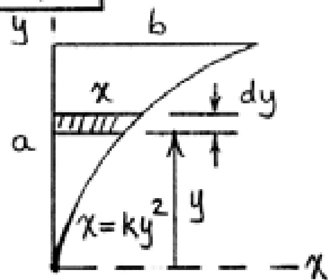
$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{\frac{h}{6}(a^2+ab+b^2)}{\frac{h}{2}(a+b)} = \frac{(a^2+ab+b^2)}{3(a+b)}$$

5/8



$$A = \int x\,dy = \int_1^4 \sqrt{5(5-y)}\,dy$$
$$= \sqrt{5}\left(-\frac{2}{3}[5-y]^{3/2}\right)_1^4 = \frac{14\sqrt{5}}{3}$$
$$\int x_c\,dA = \int_1^4 \frac{x}{2}\,x\,dy = \int_1^4 \frac{5}{2}(5-y)\,dy$$
$$= \frac{5}{2}\left(5y - \frac{y^2}{2}\right)_1^4 = \frac{75}{4}$$
$$\bar{x} = \frac{1}{A}\int x_c\,dA = \frac{75/4}{14\sqrt{5}/3} = \underline{1.797}$$
$$\int y_c\,dA = \int_1^4 y\,x\,dy = \int_1^4 y\sqrt{5(5-y)}\,dy$$
$$= \sqrt{5}\left(\frac{-2}{15}\right)(3y+10)(5-y)^{3/2}\Big|_1^4 = \frac{164\sqrt{5}}{15}$$
$$\bar{y} = \frac{1}{A}\int y_c\,dA = \frac{164\sqrt{5}/15}{14\sqrt{5}/3} = \underline{2.34}$$

5/9



$b = ka^2, \quad k = \frac{b}{a^2}, \quad x = \frac{b}{a^2} y^2$

$A = \int x\,dy = \int_0^a \frac{b}{a^2} y^2 dy$

$= \frac{b}{a^2} \frac{a^3}{3} = \frac{1}{3} ab$

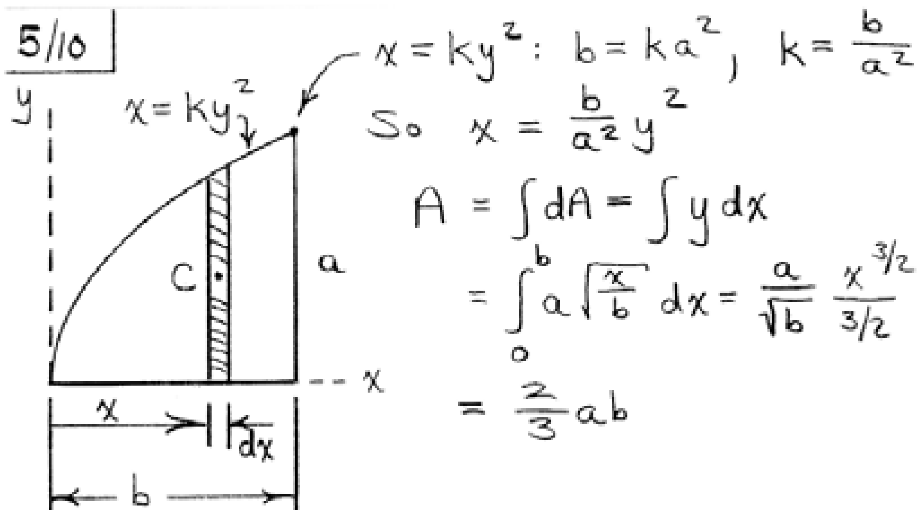
$\int x_c\,dA = \int \frac{x}{2} x\,dy = \int_0^a \frac{b^2 y^4}{2a^4} dy = \frac{ab^2}{10}$

$\bar{x} = \int x_c\,dA / A = \frac{ab^2/10}{ab/3} = \underline{\frac{3}{10} b}$

$\int y_c\,dA = \int y\,x\,dy = \int_0^a y \frac{b}{a^2} y^2 dy = \frac{ba^2}{4}$

$\bar{y} = \int y_c\,dA / A = \frac{ba^2/4}{ba/3} = \underline{\frac{3}{4} a}$

5/10



$x = ky^2$: $b = ka^2$, $k = \frac{b}{a^2}$

So $x = \frac{b}{a^2} y^2$

$$A = \int dA = \int y\,dx$$

$$= \int_0^b a\sqrt{\frac{x}{b}}\,dx = \frac{a}{\sqrt{b}} \frac{x^{3/2}}{3/2}\Big|_0^b$$

$$= \frac{2}{3}ab$$

$$\int x_c\,dA = \int_0^b \frac{a}{\sqrt{b}} x^{3/2}\,dx = \frac{a}{\sqrt{b}} \frac{x^{5/2}}{5/2}\Big|_0^b = \frac{2}{5}ab^2$$

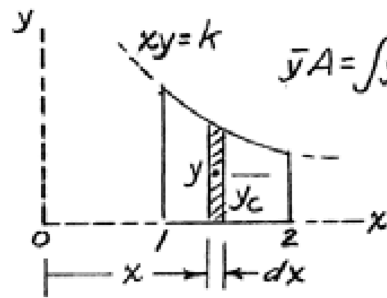
$$\int y_c\,dA = \int \frac{y}{2} y\,dx = \frac{1}{2}\int y^2\,dx$$

$$= \frac{1}{2}\int_0^b \frac{a^2}{b} x\,dx = \frac{1}{2}\frac{a^2}{b}\frac{x^2}{2}\Big|_0^b = \frac{1}{4}a^2 b$$

$$\bar{x} = \frac{\int x_c\,dA}{A} = \frac{\frac{2}{5}ab^2}{\frac{2}{3}ab} = \underline{\frac{3}{5}b}$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{\frac{1}{4}a^2 b}{\frac{2}{3}ab} = \underline{\frac{3}{8}a}$$

5/11 $dA = y\,dx$, $A = \int_1^2 \frac{k}{x} dx = k \ln x \Big]_1^2 = k \ln 2$



$\bar{y}A = \int y_c\, dA = \int_1^2 \frac{y}{2} y\,dx = \frac{1}{2}\int_1^2 \frac{k^2}{x^2} dx$

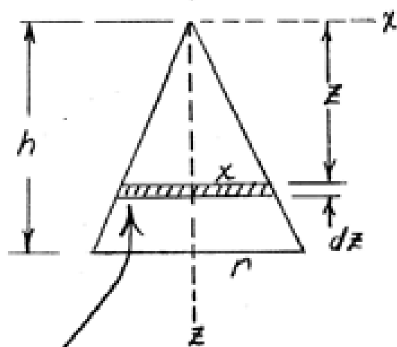
$= \frac{k^2}{2} \frac{-1}{x}\Big]_1^2 = \frac{k^2}{2}\left[-\frac{1}{2}+1\right] = \frac{k^2}{4}$

$\bar{y} = \frac{k^2/4}{k \ln 2} = \frac{k}{4 \ln 2} = \underline{0.361\,k}$

$\bar{x}A = \int x\,dA = \int_1^2 xy\,dx = \int_1^2 k\,dx = k(2-1) = k$

$\bar{x} = \frac{k}{k \ln 2} = \frac{1}{\ln 2} = \underline{1.443}$

5/12 $x = \frac{r}{h} z, \quad dV = \pi x^2 dz = \pi \frac{r^2}{h^2} z^2 dz$



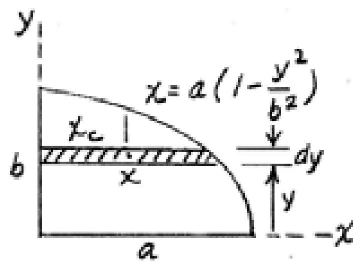
$$V = \pi \frac{r^2}{h^2} \int_0^h z^2 dz = \frac{\pi r^2 h}{3}$$

$$\int z\, dV = \pi \frac{r^2}{h^2} \int_0^h z^3 dz = \frac{\pi r^2 h^2}{4}$$

$$\bar{z} = \int z\, dV / V = \underline{3h/4}$$

(Disk-shaped element viewed edge-on.)

5/13 $dA = x\,dy$, $A = \int_0^b a\left(1 - \frac{y^2}{b^2}\right) dy = a\left[y - \frac{y^3}{3b^2}\right]_0^b = \frac{2}{3}ab$



$$\bar{x}A = \int x_c\,dA = \int_0^b \frac{x}{2}\,x\,dy$$

$$= \frac{1}{2}\int_0^b a^2\left(1 - \frac{2y^2}{b^2} + \frac{y^4}{b^4}\right) dy$$

$$= \frac{a^2}{2}\left[y - \frac{2y^3}{3b^2} + \frac{y^5}{5b^4}\right]_0^b = \frac{4}{15}a^2 b$$

$$\bar{x} = \frac{4a^2b/15}{2ab/3} = \underline{\frac{2}{5}a}$$

$$\bar{y}A = \int y\,dA = \int_0^b a\left(y - \frac{y^3}{b^2}\right) dy = a\left[\frac{y^2}{2} - \frac{y^4}{4b^2}\right]_0^b = \frac{1}{4}ab^2$$

$$\bar{y} = \frac{ab^2/4}{2ab/3} = \underline{\frac{3}{8}b}$$

5/14

$dm = \rho\, dx$ (ρ = mass per unit length)

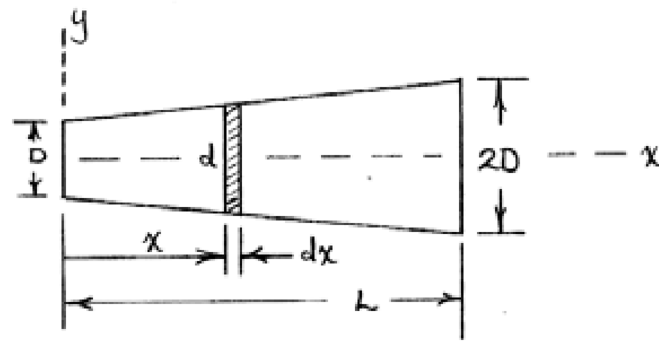
$m = \int dm = \int \rho\, dx = \int_0^{1'} \rho_0 \left(1 - \frac{x}{2}\right) dx$

$= \rho_0 \left[x - \frac{x^2}{4} \right]_0^{1'} = \frac{3}{4} \rho_0$

$\int x\, dm = \int_0^{1'} x \rho_0 \left(1 - \frac{x}{2}\right) dx = \rho_0 \left[\frac{x^2}{2} - \frac{x^3}{6} \right]_0^{1'} = \frac{\rho_0}{3}$

$\bar{x} = \frac{\int x\, dm}{\int dm} = \frac{\rho_0/3}{3\rho_0/4} = \frac{4}{9}\,'$

5/15



For constant density, $\bar{x}V = \int x\,dV$

Diameter $d = D\,(1 + x/L)$

So $dV = \frac{\pi d^2}{4}dx = \frac{\pi D^2}{4}\left(1+\frac{x}{L}\right)^2 dx$

$$V = \frac{\pi D^2}{4}\int_0^L \left(1+\frac{x}{L}\right)^2 dx = \frac{\pi D^2}{4}\left[x + \frac{x^2}{L} + \frac{x^3}{3L^2}\right]_0^L$$

$$= \frac{7}{12}\pi D^2 L$$

$$\int x\,dV = \frac{\pi D^2}{4}\int_0^L x\left(1+\frac{x}{L}\right)^2 dx = \frac{\pi D^2}{4}\left[x^2 + \frac{2x^3}{3L} + \frac{x^4}{4L^2}\right]_0^L$$

$$= \frac{17}{48}\pi D^2 L^2$$

$$\bar{x} = \frac{\frac{17}{48}\pi D^2 L^2}{\frac{7}{12}\pi D^2 L} = \underline{\frac{17}{28}L}$$

5/16

$dA = y\,dx = ae^{-bx}\,dx$

$A = \int dA = \int_0^\infty ae^{-bx}\,dx$

$= -\frac{a}{b}e^{-bx}\Big|_0^\infty = -\frac{a}{b}[0-1]$

$= a/b$

$\int x_c\,dA = a\int_0^\infty xe^{-bx}\,dx$

$= a\frac{e^{-bx}}{b^2}[-bx-1]_0^\infty$

$= -\frac{a}{b^2}\left[bxe^{-bx}+e^{-bx}\right]_0^\infty = -\frac{a}{b^2}[0+0-(0+1)]$

$= a/b^2$

$\int y_c\,dA = \int \frac{y}{2}y\,dx = \int \frac{y^2}{2}\,dx$

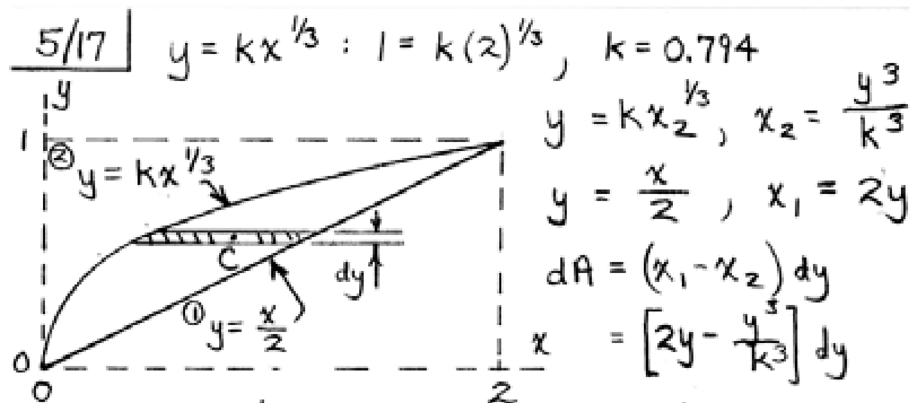
$= \frac{1}{2}\int_0^\infty a^2e^{-2bx}\,dx = \frac{a^2}{2}\frac{e^{-2bx}}{-2b}\Big|_0^\infty$

$= -\frac{a^2}{4b}[0-1] = \frac{a^2}{4b}$

$\bar{x} = \frac{\int x_c\,dA}{\int dA} = \frac{a/b^2}{a/b} = \underline{\frac{1}{b}}$

$\bar{y} = \frac{\int y_c\,dA}{\int dA} = \frac{a^2/4b}{a/b} = \underline{\frac{a}{4}}$

5/17 $y = kx^{1/3}$: $1 = k(2)^{1/3}$, $k = 0.794$



$y = kx_2^{1/3}$, $x_2 = \dfrac{y^3}{k^3}$

$y = \dfrac{x}{2}$, $x_1 = 2y$

$dA = (x_1 - x_2)\,dy$

$= \left[2y - \dfrac{y^3}{k^3}\right] dy$

$$A = \int dA = \int_0^1 \left(2y - \frac{y^3}{k^3}\right) dy = \left(y^2 - \frac{y^4}{4k^3}\right)_0^1 = 0.5$$

$$\int x_c\, dA = \int \left(\frac{x_1 + x_2}{2}\right)(x_1 - x_2)\, dy = \frac{1}{2}\int (x_1^2 - x_2^2)\, dy$$

$$= \frac{1}{2}\int_0^1 \left(4y^2 - \frac{y^6}{k^6}\right) dy = \frac{1}{2}\left[\frac{4}{3}y^3 - \frac{y^7}{7k^6}\right]_0^1 = 0.381$$

$$\int y_c\, dA = \int y\left(2y - \frac{y^3}{k^3}\right) dy = \int \left(2y^2 - \frac{y^4}{k^3}\right) dy$$

$$= \left(\frac{2y^3}{3} - \frac{y^5}{5k^3}\right)_0^1 = 0.267$$

$$\bar{x} = \frac{\int x_c\, dA}{\int dA} = \frac{0.381}{0.5} = \underline{0.762}$$

$$\bar{y} = \frac{\int y_c\, dA}{\int dA} = \frac{0.267}{0.5} = \underline{0.533}$$

5/18

$y_2 = b\left(1+\frac{x}{a}\right)$

$x = ky_1^2 = \frac{a}{b^2}y_1^2$

$$A = \int_0^a (y_2 - y_1)\,dx$$

$$= \int_0^a \left[b\left(1+\frac{x}{a}\right) - b\left(\frac{x}{a}\right)^{\frac{1}{2}}\right] dx$$

$$= b\left(x + \frac{x^2}{2a}\right) - \frac{b}{\sqrt{a}}\,\frac{2}{3}x^{3/2}\Big|_0^a$$

$$= \frac{5}{6}\,ab$$

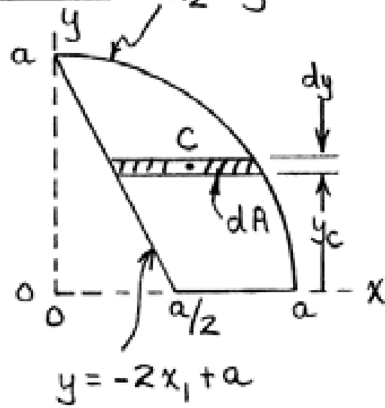
$$\int y_c\,dA = \int_0^a \left(\frac{y_1+y_2}{2}\right)(y_2-y_1)\,dx = \frac{1}{2}\int_0^a (y_2^2 - y_1^2)\,dx$$

$$= \frac{1}{2}\int_0^a \left[b^2\left(1+\frac{x}{a}\right)^2 - \frac{b^2}{a}x\right]dx$$

$$= \frac{1}{2}\left[b^2\left(x + \frac{x^2}{a} + \frac{x^3}{3a^2}\right) - \frac{b^2x^2}{2a}\right]\Big|_0^a = \frac{11}{12}ab^2$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{11ab^2/12}{5ab/6} = \underline{\frac{11}{10}\,b}$$

5/19



$$A = \frac{1}{4}\pi a^2 - \frac{1}{2}\frac{a}{2}a = \frac{a^2}{4}(\pi - 1)$$

$$dA = (x_2 - x_1)\,dy$$

$$= \left[\sqrt{a^2 - y^2} - \frac{a-y}{2}\right] dy$$

$$\int x_c\,dA = \int_0^a \left(\frac{x_1 + x_2}{2}\right)(x_2 - x_1)\,dy$$

$$= \frac{1}{2}\int_0^a \left(x_2^2 - x_1^2\right) dy$$

$$= \frac{1}{2}\int_0^a \left[(a^2 - y^2) - \left(\frac{a-y}{2}\right)^2\right] dy$$

$$= \frac{1}{2}\int_0^a \left[a^2 - y^2 - \frac{1}{4}(a^2 - 2ay + y^2)\right] dy$$

$$= \frac{1}{2}\int_0^a \left[\frac{3}{4}a^2 - \frac{5}{4}y^2 + \frac{1}{2}ay\right] dy$$

$$= \frac{1}{2}\left[\frac{3}{4}a^2 y - \frac{5}{12}y^3 + \frac{1}{4}ay^2\right]_0^a = \frac{7}{24}a^3$$

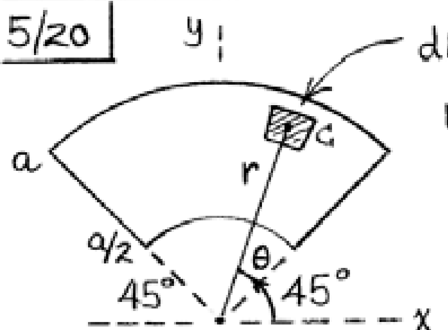
$$\int y_c\,dA = \int_0^a y\left[\sqrt{a^2 - y^2} - \frac{a-y}{2}\right] dy$$

$$= \left[-\frac{1}{3}\sqrt{(a^2 - y^2)^3} - \frac{a}{2}\frac{y^2}{2} + \frac{y^3}{6}\right]_0^a = \frac{a^3}{4}$$

$$\bar{x} = \frac{\int x_c\,dA}{A} = \frac{\frac{7}{24}a^3}{\frac{a^2}{4}(\pi - 1)} = \underline{\frac{7a}{6(\pi - 1)}}$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{a^3/4}{\frac{a^2}{4}(\pi - 1)} = \underline{\frac{a}{\pi - 1}}$$

5/20



$dA = r\,dr\,d\theta$

$A = \frac{\pi a^2}{4} - \frac{\pi}{4}\left(\frac{a}{2}\right)^2 = \frac{3}{16}\pi a^2$

$$\int y_c\,dA = \iint (r\sin\theta)\,r\,dr\,d\theta$$

$$= \int_{\pi/4}^{3\pi/4}\int_{a/2}^{a} \sin\theta\, r^2\,dr\,d\theta = \int_{\pi/4}^{3\pi/4} \sin\theta\,\frac{7}{24}a^3\,d\theta$$

$$= \frac{7}{24}a^3\left(-\cos\theta\right)\Big|_{\pi/4}^{3\pi/4} = \frac{7\sqrt{2}}{24}a^3$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{\frac{7\sqrt{2}}{24}a^3}{\frac{3}{16}\pi a^2} = \underline{\frac{14\sqrt{2}}{9\pi}a}$$

5/21

$$A = \pi\frac{ab}{4} - \frac{1}{2}a\frac{b}{2} = \frac{ab}{4}(\pi - 1)$$

$$dA = (y_2 - y_1)dx$$

$$= \left[b\sqrt{1-\frac{x^2}{a^2}} - \left(-\frac{b}{2a}x + \frac{b}{2}\right)\right]dx$$

$$= \frac{b}{a}\left[\sqrt{a^2-x^2} + \frac{x}{2} - \frac{a}{2}\right]dx$$

$$\int x_c\,dA = \int_0^a x\frac{b}{a}\left[\sqrt{a^2-x^2} + \frac{x}{2} - \frac{a}{2}\right]dx$$

$$= \frac{b}{a}\left[-\frac{1}{3}\sqrt{(a^2-x^2)^3} + \frac{x^3}{6} - \frac{ax^2}{4}\right]_0^a = \frac{1}{4}ba^2$$

$$\int y_c\,dA = \int_0^a\left(\frac{y_2+y_1}{2}\right)(y_2-y_1)\,dx = \frac{1}{2}\int_0^a\left(y_2^2 - y_1^2\right)dx$$

$$= \frac{1}{2}\int_0^a\left[b^2\left(1-\frac{x^2}{a^2}\right) - \left(-\frac{b}{2a}x + \frac{b}{2}\right)^2\right]dx$$

$$= \frac{1}{2}\int_0^a\left[b^2\left(1-\frac{x^2}{a^2}\right) - \frac{b^2}{4a^2}x^2 + \frac{b^2}{2a}x - \frac{b^2}{4}\right]dx$$

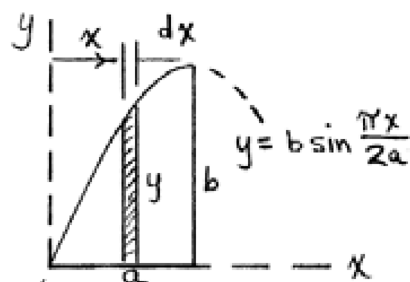
$$= \frac{1}{2}\left[b^2\left(x - \frac{x^3}{3a^2}\right) - \frac{b^2}{4a^2}\frac{x^3}{3} + \frac{b^2}{2a}\frac{x^2}{2} - \frac{b^2}{4}x\right]_0^a$$

$$= \frac{7}{24}ab^2$$

$$\bar{x} = \frac{\int x_c\,dA}{A} = \frac{\frac{1}{4}ba^2}{\frac{ab}{4}(\pi-1)} = \underline{\frac{a}{\pi-1}}$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{\frac{7}{24}ab^2}{\frac{ab}{4}(\pi-1)} = \underline{\frac{7b}{6(\pi-1)}}$$

5/22



$$A = \int y\,dx = \int_0^a b \sin\frac{\pi x}{2a}\,dx$$

$$= -\frac{2ab}{\pi}\cos\frac{\pi x}{2a}\Big|_0^a = \frac{2ab}{\pi}$$

$$\int x_c\,dA = \int_0^a xy\,dx = \int_0^a bx\sin\frac{\pi x}{2a}\,dx$$

$$= b\left(\frac{2a}{\pi}\right)^2\left[\sin\frac{\pi x}{2a} - \frac{\pi x}{2a}\cos\frac{\pi x}{2a}\right]_0^a$$

$$= 4a^2b/\pi^2$$

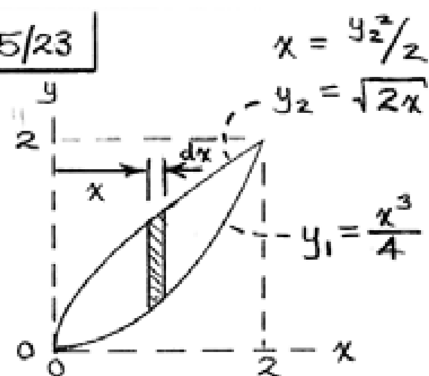
$$\bar{x} = \frac{\int x_c\,dA}{A} = \frac{4a^2b/\pi^2}{2ab/\pi} = \underline{\frac{2a}{\pi}}$$

$$\int y_c\,dA = \int_0^a \frac{y}{2}\,y\,dx = \frac{b^2}{2}\int_0^a \sin^2\frac{\pi x}{2a}\,dx$$

$$= \frac{ab^2}{\pi}\left[\frac{\pi x}{4a} - \frac{1}{4}\sin\frac{\pi x}{a}\right]_0^a = \frac{ab^2}{4}$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{ab^2/4}{2ab/\pi} = \underline{\frac{\pi b}{8}}$$

5/23



$$A = \int dA = \int_0^2 (y_2 - y_1)\,dx = \int_0^2 \left(\sqrt{2x} - \frac{x^3}{4}\right) dx$$

$$= \left(\frac{2\sqrt{2}}{3} x^{3/2} - \frac{x^4}{16}\right)_0^2 = 5/3$$

$$\int x_c\,dA = \int_0^2 x\left(\sqrt{2x} - \frac{x^3}{4}\right) dx$$

$$= \left(\frac{2\sqrt{2}}{5} x^{5/2} - \frac{x^5}{20}\right)\Big|_0^2 = 8/5$$

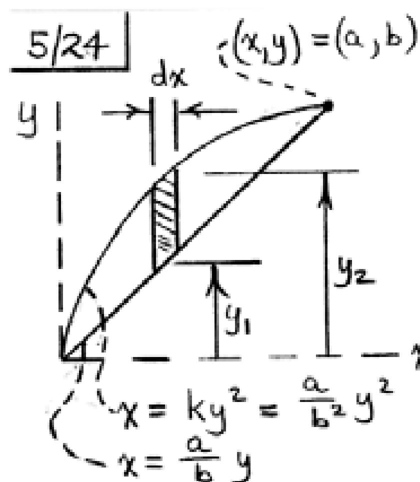
$$\int y_c\,dA = \int_0^2 \left(\frac{y_1 + y_2}{2}\right)(y_2 - y_1)\,dx = \int_0^2 \frac{1}{2}(y_2^2 - y_1^2)\,dx$$

$$= \frac{1}{2}\int_0^2 \left(2x - \frac{x^6}{16}\right) dx = \frac{1}{2}\left[x^2 - \frac{x^7}{7(16)}\right]_0^2 = 10/7$$

$$\bar{x} = \int x_c\,dA / A = \frac{8/5}{5/3} = \underline{\frac{24}{25}}$$

$$\bar{y} = \int y_c\,dA / A = \frac{10/7}{5/3} = \underline{\frac{6}{7}}$$

5/24



$$A = \int_0^a (y_2 - y_1)\,dx$$

$$= \int_0^a \left(b\sqrt{\frac{x}{a}} - x\frac{b}{a}\right)dx$$

$$= b\left[\frac{1}{\sqrt{a}}\frac{2x^{3/2}}{3} - \frac{1}{2a}x^2\right]_0^a$$

$$= \frac{ab}{6}$$

$$\int x_c\,dA = \int_0^a x(y_2 - y_1)\,dx = \int_0^a \left[\frac{b}{\sqrt{a}}x^{3/2} - \frac{b}{a}x^2\right]dx$$

$$= b\left[\frac{2x^{5/2}}{5\sqrt{a}} - \frac{x^3}{3a}\right]_0^a = \frac{a^2 b}{15}$$

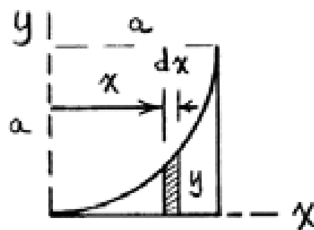
$$\bar{x} = \frac{\int x_c\,dA}{A} = \frac{ab^2/15}{ab/6} = \underline{\frac{2}{5}a}$$

$$\int y_c\,dA = \int_0^a \left(\frac{y_1 + y_2}{2}\right)(y_2 - y_1)\,dx = \frac{1}{2}\int_0^a (y_2^2 - y_1^2)\,dx$$

$$= \frac{1}{2}\int_0^a \left(\frac{xb^2}{a} - \frac{x^2 b^2}{a^2}\right)dx = \frac{1}{12}ab^2$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{ab^2/12}{ab/6} = \underline{\frac{b}{2}}$$

5/25



$$x^2 + (y-a)^2 = a^2$$

$$y = a - \sqrt{a^2 - x^2}$$

(use minus sign)

$$A = \int y\,dx = \int_0^a \left[a - \sqrt{a^2-x^2}\right] dx$$

$$= \left[ax - \frac{1}{2}\left(x\sqrt{a^2-x^2} + a^2 \sin^{-1}\frac{x}{a}\right)\right]_0^a = a^2\left(1 - \frac{\pi}{4}\right)$$

$$\int x_c\,dA = \int_0^a x y\,dx = \int_0^a \left[ax - x\sqrt{a^2-x^2}\right] dx$$

$$= \left[\frac{ax^2}{2} + \frac{1}{3}\left(a^2-x^2\right)^{3/2}\right]_0^a = a^3/6$$

$$\bar{x} = \frac{\int x_c\,dA}{A} = \frac{a^3/6}{a^2\left(1-\frac{\pi}{4}\right)} = \underline{\frac{2a}{3(4-\pi)}} = \underline{0.777a}$$

From symmetry, $\bar{y} = a - \bar{x} = a - \dfrac{2a}{3(4-\pi)} = \underline{\dfrac{10-3\pi}{3(4-\pi)}a}$

or $\bar{y} = \underline{0.223a}$

5/26 $V\bar{z} = \int z\,dV$

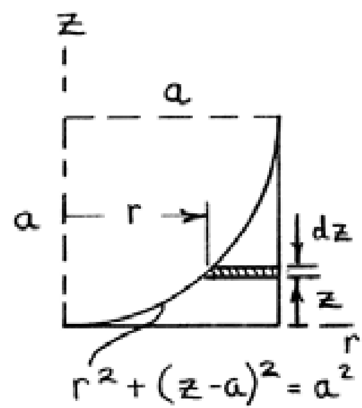
$$dV = A\,dz = \frac{\pi}{2}(a^2 - r^2)\,dz$$

$$= \frac{\pi}{2}(z-a)^2\,dz$$

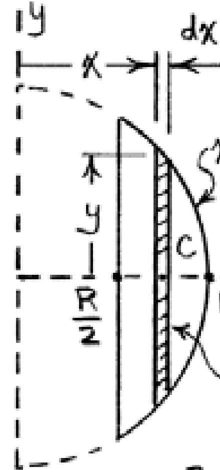
$$V = \frac{\pi}{2}\int_0^a (z-a)^2\,dz = \pi a^3/6$$

$$\int z\,dV = \int_0^a z\frac{\pi}{2}(z-a)^2\,dz = \frac{\pi a^4}{24}$$

$$\bar{z} = \frac{\pi a^4/24}{\pi a^3/6} = \underline{a/4}$$



5/27



$x^2+y^2=R^2$

(Note: Shaded element is a circular slice viewed edge-on.)

$$dV = \pi y^2 dx = \pi(R^2 - x^2)\,dx$$

$$V = \int dV = \int_{R/2}^{R} \pi(R^2 - x^2)\,dx = \pi\left[R^2 x - \frac{x^3}{3}\right]_{R/2}^{R}$$

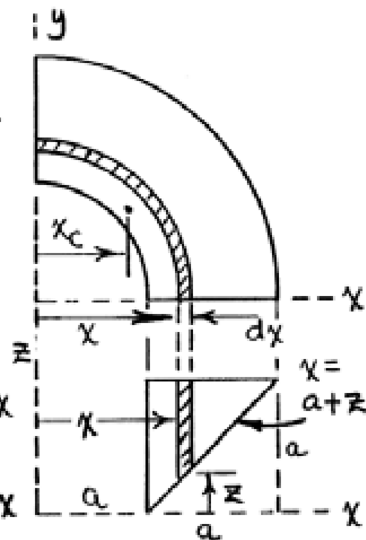
$$= \pi\left[R^3 - \frac{R^3}{3} - \left(\frac{R^3}{2} - \frac{R^3}{24}\right)\right] = \frac{5}{24}\pi R^3$$

$$\int x_c\,dV = \int_{R/2}^{R} x\,\pi(R^2 - x^2)\,dx = \pi\left[\frac{R^2x^2}{2} - \frac{x^4}{4}\right]_{R/2}^{R}$$

$$= \pi\left[\frac{R^4}{2} - \frac{R^4}{4} - \left(\frac{R^4}{8} - \frac{R^4}{64}\right)\right] = \frac{9}{64}\pi R^4$$

$$\bar{x} = \frac{\int x_c\,dV}{\int dV} = \frac{\frac{9}{64}\pi R^4}{\frac{5}{24}\pi R^3} = \underline{\frac{27}{40}R}$$

5/28 Choose elemental cylindrical shell of radius x.



$$\bar{x} = \frac{\int x_c \, dV}{\int dV}$$

From Sample Problem 5/1 or Table D/3, $x_c = \frac{2x}{\pi}$

$$dV = \frac{\pi x}{2}(a-z)dz = \frac{\pi x}{2}(2a-x)dx$$

$$\int x_c \, dV = \int_a^{2a} \frac{2x}{\pi}\frac{\pi x}{2}(2a-x)\,dx$$

$$= \int_a^{2a}(2ax^2 - x^3)\,dx = \frac{11}{12}a^4$$

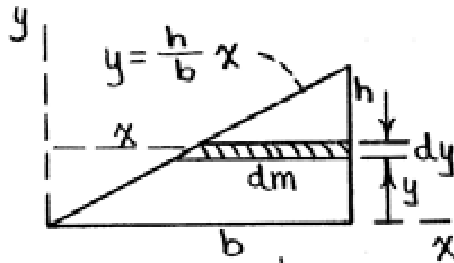
$$\int dV = \int_a^{2a}\frac{\pi x}{2}(2a-x)\,dx = \pi\int_a^{2a}\left(xa - \frac{x^2}{2}\right)dx$$

$$= \pi a^3/3$$

So $\bar{x} = \frac{11a^4/12}{\pi a^3/3} = \frac{11a}{4\pi} = \bar{y}$ by symmetry

5/29

$$dm = \rho\, dV = \rho\, dA\, t = t\rho\,(b-x)\,dy$$
$$= \left[t_0\left(\frac{y}{h}+1\right)\right]\rho\,(b-x)\,dy$$
$$= t_0\rho\left(\frac{y}{h}+1\right)\left(b-\frac{b}{h}y\right)dy$$
$$= t_0\rho b\left(1-\frac{y^2}{h^2}\right)dy$$

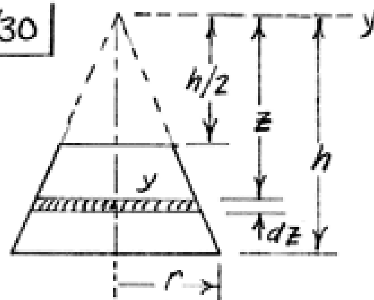


$$m = \int dm = \int_0^h t_0\rho b\left(1-\frac{y^2}{h^2}\right)dy = t_0\rho b\left[y-\frac{y^3}{3h^2}\right]_0^h$$
$$= \frac{2}{3}\rho t_0 bh$$

$$\int y_c\, dm = \int t_0\rho b\left(1-\frac{y^2}{h^2}\right)y\,dy = t_0\rho b\left[\frac{y^2}{2}-\frac{y^4}{4h^2}\right]_0^h$$
$$= \frac{1}{4}\rho t_0 bh^2$$

$$\bar{y} = \frac{\int y_c\, dm}{m} = \frac{\frac{1}{4}\rho t_0 bh^2}{\frac{2}{3}\rho t_0 bh} = \underline{\frac{3}{8}h}$$

5/30



$dV = \pi y^2 dz$ where $y = \frac{r}{h} z$

$= \pi \frac{r^2}{h^2} z^2 dz$

$V = \pi \frac{r^2}{h^2} \int_{h/2}^{h} z^2 dz = \frac{7\pi r^2 h}{24}$

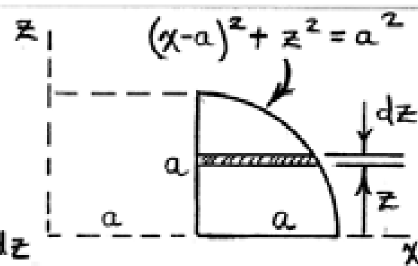
$\int z_c dV = \int_{h/2}^{h} z \pi \frac{r^2}{h^2} z^2 dz = \frac{15}{64} \pi r^2 h^2$

$\bar{z} = \int z_c dV / V = \frac{15}{64} \pi r^2 h^2 \Big/ \frac{7}{24} \pi r^2 h = \frac{45}{56} h$

$\bar{h} = h - \bar{z} = \underline{\frac{11}{56} h}$

5/31

$$\bar{z} = \frac{\int z\,dV}{V}$$



$$dV = \frac{\pi}{4}\left(x^2 - a^2\right)dz$$

$$= \frac{\pi}{4}\left[\left(a + \sqrt{a^2 - z^2}\right)^2 - a^2\right]dz$$

$$= \frac{\pi}{4}\left[a^2 - z^2 + 2a\sqrt{a^2 - z^2}\right]dz$$

$$V = \frac{\pi}{4}\int_0^a \left[a^2 - z^2 + 2a\sqrt{a^2 - z^2}\right]dz$$

$$= \frac{\pi}{4}\left[a^2 z - \frac{z^3}{3} + 2a\left(\frac{1}{2}\right)\left(z\sqrt{a^2 - z^2} + a^2 \sin^{-1}\frac{z}{a}\right)\right]_0^a$$

$$= \frac{\pi a^3}{2}\left(\frac{1}{3} + \frac{\pi}{4}\right)$$

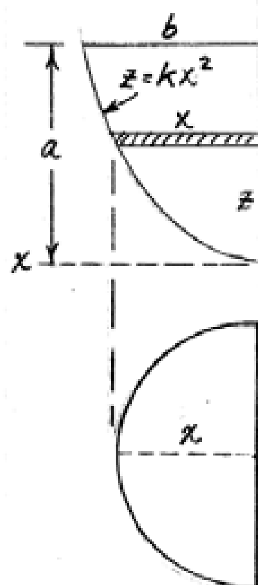
$$\int z\,dV = \frac{\pi}{4}\int_0^a \left[a^2 z - z^3 + 2az\sqrt{a^2 - z^2}\right]dz$$

$$= \frac{\pi}{4}\left[\frac{a^2 z^2}{2} - \frac{z^4}{4} - \frac{2a}{3}\sqrt{\left(a^2 - z^2\right)^3}\right]_0^a$$

$$= \frac{11}{48}\pi a^4$$

$$\bar{z} = \frac{\frac{11}{48}\pi a^4}{\frac{\pi a^3}{2}\left(\frac{1}{3} + \frac{\pi}{4}\right)} = \underline{\frac{11a}{2(4 + 3\pi)}}$$

5/32



$a = kb^2$ so $k = a/b^2$ & $z = \frac{a}{b^2}x^2$

$dV = \frac{\pi x^2}{2} dz = \frac{\pi b^2}{2a} z\, dz$

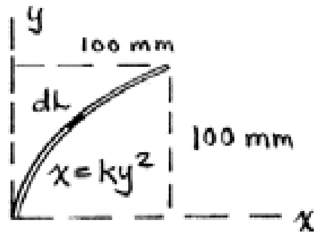
$V = \frac{\pi b^2}{2a}\int_0^a z\, dz = \frac{\pi}{4} a b^2$

$\int \bar{z}_c\, dV = \int z\, dV = \frac{\pi b^2}{2a}\int_0^a z^2 dz = \frac{\pi}{6} a^2 b^2$

$\bar{z} = \int \bar{z}_c\, dV \Big/ V = \frac{\pi}{6} a^2 b^2 \Big/ \frac{\pi}{4} a b^2 = \underline{2a/3}$

5/33

$$x = ky^2 = y^2/100\ , \quad \frac{dx}{dy} = \frac{y}{50}$$



$$L = \int dL = \int_0^{100} \sqrt{1 + (dx/dy)^2}\, dy = \int_0^{100} \sqrt{1 + \frac{y^2}{50^2}}\, dy$$

$$= \frac{1}{50}\int_0^{100} \sqrt{50^2 + y^2}\, dy = \frac{1}{50 \cdot 2}\left[y\sqrt{50^2+y^2} + 50^2 \ln\left(y + \sqrt{50^2+y^2}\right)\right]_0^{100}$$

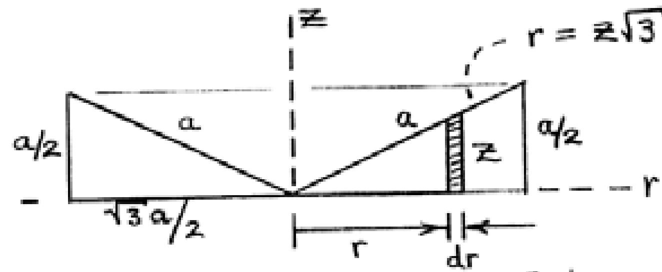
$$= 147.9 \text{ mm}$$

$$\int y_c\, dL = \frac{1}{50}\int_0^{100} y\sqrt{50^2+y^2}\, dy = \frac{1}{50}\frac{1}{3}\left(50^2+y^2\right)^{3/2}\Big|_0^{100}$$

$$= 8480 \text{ mm}^2$$

$$\bar{y} = \frac{\int y_c\, dL}{L} = \frac{8480}{147.9} = \underline{57.4 \text{ mm}}$$

5/34



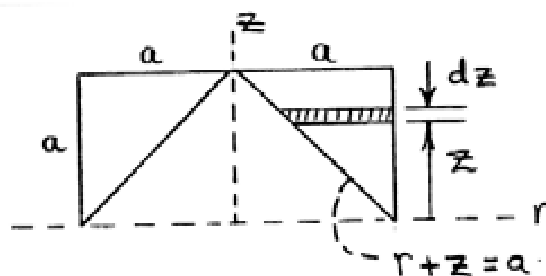
$$V = \int dV = \int_0^{\sqrt{3}a/2} 2\pi r z \, dr = \frac{2\pi}{\sqrt{3}} \int_0^{\sqrt{3}a/2} r^2 \, dr$$

$$= \frac{2\pi}{\sqrt{3}} \left(\sqrt{3}a/2\right)^3 / 3 = \frac{\pi a^3}{4}$$

$$\int z_c \, dV = \int_0^{\sqrt{3}a/2} \frac{r}{2\sqrt{3}} \frac{2\pi}{\sqrt{3}} r^2 \, dr = \frac{\pi}{3} \left[\frac{r^4}{4}\right]_0^{\sqrt{3}a/2} = \frac{3\pi a^4}{64}$$

$$\bar{z} = \frac{\int z_c \, dV}{V} = \frac{3\pi a^4/64}{\pi a^3/4} = \underline{\frac{3a}{16}}$$

5/35



$$V = \int_0^a \pi (a^2 - r^2)\,dz = \pi \int_0^a [a^2 - (a-z)^2]\,dz$$

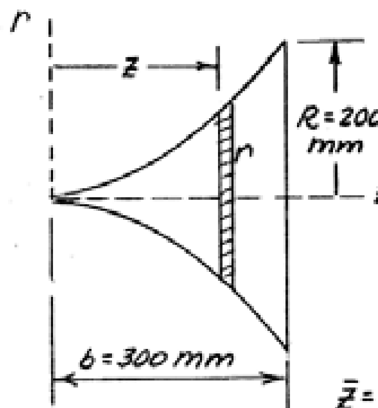
$$= \pi \int_0^a (2az - z^2)\,dz = \pi \left[az^2 - \frac{z^3}{3} \right]_0^a = \frac{2}{3}\pi a^3$$

$$\int z_c\,dV = \int_0^a \pi (2az^2 - z^3)\,dz = \pi \left[\frac{2az^3}{3} - \frac{z^4}{4} \right]_0^a$$

$$= \frac{5}{12}\pi a^4$$

$$\bar{z} = \frac{\int z_c\,dV}{V} = \frac{\frac{5}{12}\pi a^4}{\frac{2}{3}\pi a^3} = \underline{\frac{5a}{8}}$$

5/36 $r = kz^3$, $R = kb^3$, so $r = \frac{R}{b^3}z^3$



$$dV = \pi r^2 dz = \pi \frac{R^2}{b^6} z^6 dz$$

$$V = \frac{\pi R^2}{b^6}\int_0^b z^6 dz = \frac{\pi}{7}R^2 b$$

$$\int z_c dV = \int z dV = \frac{\pi R^2}{b^6}\int_0^b z^7 dz$$

$$= \frac{\pi R^2}{8} b^2$$

$$\bar{z} = \int z_c dV / V = \frac{\pi R^2 b^2}{8} \Big/ \frac{\pi}{7} R^2 b$$

$$= 7b/8 = \frac{7(300)}{8} = \underline{263 \text{ mm}}$$

▸5/37

$x^2+y^2=a^2$, $x=+\sqrt{a^2-y^2}$

$dA = 2x\,dy = 2\sqrt{a^2-y^2}\,dy$

$A=\int dA = \int_h^a 2\sqrt{a^2-y^2}\,dy$

$= 2\left(\frac{1}{2}\right)\left[y\sqrt{a^2-y^2}+a^2\sin^{-1}\frac{y}{a}\right]_h^a$

$= a^2\left(\frac{\pi}{2}-\sin^{-1}\frac{h}{a}\right)-h\sqrt{a^2-h^2}$

$\int y\,dA = \int_h^a y\,2\sqrt{a^2-y^2}\,dy = 2\left(-\frac{1}{3}\right)(a^2-y^2)^{3/2}\Big|_h^a$

$= \frac{2}{3}(a^2-h^2)^{3/2}$

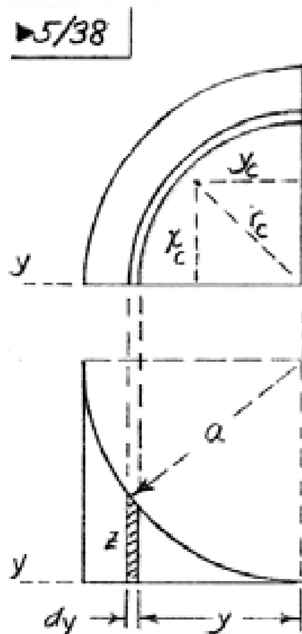
$\bar{y} = \dfrac{\int y\,dA}{\int dA} = \dfrac{\frac{2}{3}(a^2-h^2)^{3/2}}{a^2\left(\frac{\pi}{2}-\sin^{-1}\frac{h}{a}\right)-h\sqrt{a^2-h^2}}$

Special cases

$h=0$: $\bar{y} = \dfrac{\frac{2}{3}a^3}{a^2\frac{\pi}{2}} = \dfrac{4a}{3\pi}$ (the correct result)

$h=\frac{a}{4}$: $\bar{y} = \dfrac{\frac{2}{3}\left(a^2-\left(\frac{a}{4}\right)^2\right)^{3/2}}{a^2\left(\frac{\pi}{2}-\sin^{-1}\frac{1}{4}\right)-\frac{a}{4}\sqrt{a^2-\left(\frac{a}{4}\right)^2}} = \underline{0.562a}$

$h=\frac{a}{2}$: $\bar{y} = \dfrac{\frac{2}{3}\left(a^2-\left(\frac{a}{2}\right)^2\right)^{3/2}}{a^2\left(\frac{\pi}{2}-\sin^{-1}\frac{1}{2}\right)-\frac{a}{2}\sqrt{a^2-\left(\frac{a}{2}\right)^2}} = \underline{0.705a}$



From Sample Problem 5/1, for elemental shell, $x_c = y_c = r_c/\sqrt{2} = \frac{2y}{\pi}$

$$dV = \frac{\pi y}{2}(z\,dy) = \frac{\pi}{2}\left(ay - y\sqrt{a^2-y^2}\right)dy$$

$$V = \frac{\pi}{2}\int_0^a \left(ay - y\sqrt{a^2-y^2}\right)dy$$

$$= \frac{\pi}{2}\left[\frac{ay^2}{2} + \frac{1}{3}\sqrt{(a^2-y^2)^3}\right]_0^a = \frac{\pi}{2}\frac{a^3}{6} = \frac{\pi a^3}{12}$$

$$\int y_c\,dV = \frac{2}{\pi}\int_0^a \frac{\pi}{2}\left(ay^2 - y^2\sqrt{a^2-y^2}\right)dy$$

$$= \left[\frac{ay^3}{3} + \frac{y}{4}\sqrt{(a^2-y^2)^3} - \frac{a^2}{8}\left(y\sqrt{a^2-y^2} + a^2\sin^{-1}\frac{y}{a}\right)\right]_0^a$$

$$= \left[\frac{a^4}{3} + 0 - \frac{a^2}{8}\left(0 + a^2\frac{\pi}{2}\right)\right] = a^4\left[\frac{1}{3} - \frac{\pi}{16}\right]$$

$$\bar{y} = \int y_c\,dV/V = a^4\left(\frac{1}{3} - \frac{\pi}{16}\right)\Big/\frac{\pi a^3}{12} = \left(\frac{4}{\pi} - \frac{3}{4}\right)a = \bar{x}$$

$y^2 + (z-a)^2 = a^2$

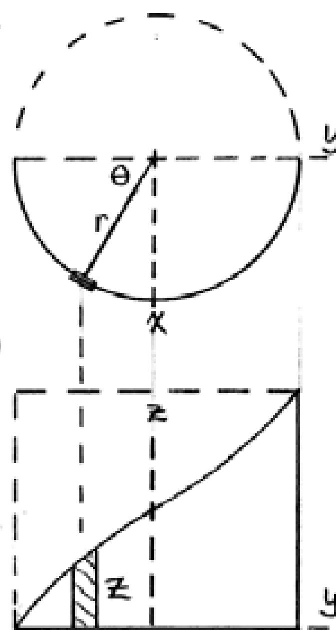
$z = a - \sqrt{a^2 - y^2}$

(Note sign)

$$\int z_c\,dV = \int \frac{z}{2}\,dV = \int_0^a \frac{a - \sqrt{a^2-y^2}}{2}\,\frac{\pi}{2}\left(ay - y\sqrt{a^2-y^2}\right)dy$$

$$= \frac{\pi a^4}{48}\;; \quad \bar{z} = \int z_c\,dV/V = \frac{\pi a^4}{48}\Big/\frac{\pi a^3}{12} = \frac{a}{4}$$

▸5/39 Let ρ = mass per unit area of shell



$$z = k\theta = \frac{h}{\pi}\theta$$

$$dm = \rho z r\,d\theta = \frac{\rho h r}{\pi}\theta\,d\theta$$

$$m = \frac{\rho h r}{\pi}\int_0^\pi \theta\,d\theta = \frac{1}{2}\rho h r \pi$$

$$m\bar{x} = \int x\,dm = \int_0^\pi r\sin\theta\,\frac{\rho h r}{\pi}\,\theta\,d\theta$$

$$= \frac{\rho r^2 h}{\pi}\int_0^\pi \theta\sin\theta\,d\theta$$

$$= \frac{\rho r^2 h}{\pi}\left(\sin\theta - \theta\cos\theta\right)_0^\pi = \rho r^2 h$$

So $\bar{x} = \dfrac{\rho r^2 h}{\rho r h \pi/2} = \underline{\dfrac{2r}{\pi}}$

$$m\bar{y} = \int y\,dm = \int(-r\cos\theta)\frac{\rho h r}{\pi}\,\theta\,d\theta$$

$$= -\frac{\rho h r^2}{\pi}\int_0^\pi \theta\cos\theta\,d\theta = -\frac{\rho h r^2}{\pi}\left(\cos\theta + \theta\sin\theta\right)_0^\pi = \frac{2\rho h r^2}{\pi}$$

So $\bar{y} = \dfrac{2\rho h r^2/\pi}{\rho h r \pi/2} = \underline{\dfrac{4r}{\pi^2}}$

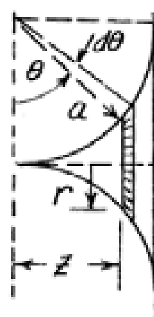
$$m\bar{z} = \int z_c\,dm = \int \frac{1}{2}\frac{h}{\pi}\theta\,\frac{\rho h r}{\pi}\,\theta\,d\theta = \frac{\rho h^2 r}{2\pi^2}\int_0^\pi \theta^2\,d\theta$$

$$= \frac{\rho h r^2}{2\pi}\left(\frac{\theta^3}{3}\right)_0^\pi = \frac{1}{6}\rho h^2 r \pi$$

So $\bar{z} = \dfrac{\rho h^2 r \pi/6}{\rho h r \pi/2} = \underline{\dfrac{1}{3}h}$

▶ 5/40 $dA = 2\pi r\, a\, d\theta = 2\pi a^2(1-\cos\theta)\, d\theta$

$$\int z\, dA = \int_0^{\pi/2} (a\sin\theta)(2\pi a^2)(1-\cos\theta)\, d\theta$$



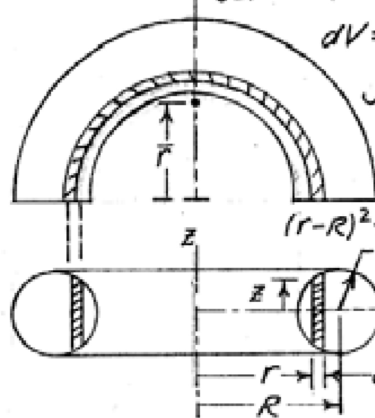
$$= 2\pi a^3 \int_0^{\pi/2} (\sin\theta - \sin\theta\cos\theta)\, d\theta$$

$$= 2\pi a^3 (1 - \tfrac{1}{2}) = \pi a^3$$

$$\int dA = 2\pi a^2 \int_0^{\pi/2} (1-\cos\theta)\, d\theta = 2\pi a^2(\tfrac{\pi}{2} - 1)$$

$$\bar{z} = \frac{\int z\, dA}{A} = \frac{\pi a^3}{2\pi a^2(\frac{\pi}{2}-1)} = \underline{\frac{a}{\pi - 2}}$$

5/41 From Sample Problem 5/1, centroidal coordinate for elemental ring is $r_c = 2r/\pi$



$$dV = \pi r(2z)\,dr = 2\pi r\sqrt{a^2-(r-R)^2}\,dr$$

$$\int r_c\,dV = 4\int_{R-a}^{R+a} r^2\sqrt{a^2-(r-R)^2}\,dr$$

$$= ① + ② + ③$$

$(r-R)^2 + z^2 = a^2$ where $u = r - R$ &

$$① = 4\int_{-a}^{a} u^2\sqrt{a^2-u^2}\,du = \frac{\pi a^4}{2}$$

$$② = 4\int_{-a}^{a} 2Ru\sqrt{a^2-u^2}\,du = 0$$

$$③ = 4\int_{-a}^{a} R^2\sqrt{a^2-u^2}\,du = 2\pi a^2R^2$$

$$\int dV = 2\pi\int_{-a}^{a}(u+R)\sqrt{a^2-u^2}\,du = 0 + 2\pi R\,\frac{\pi a^2}{2} = \pi^2a^2R$$

$$\bar{r} = \frac{\int r_c\,dV}{V} = \frac{\frac{\pi a^4}{2} + 2\pi a^2R^2}{\pi^2a^2R} = \frac{a^2+4R^2}{2\pi R}$$

▶5/42

$$dV = dr\,(r\,d\phi)(r\sin\phi\,d\theta)$$

$$= r^2\sin\phi\,d\phi\,d\theta\,dr$$

$$V = \int dV = \int_{R/2}^{R}\int_0^{\pi}\int_0^{\pi} r^2\sin\phi\,d\phi\,d\theta\,dr$$

$$= \frac{7}{12}\pi R^3$$

(Check V by

$$V = \frac{\frac{4}{3}\pi R^3}{2} - \frac{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{2} = \frac{7}{12}\pi R^3)$$

$$\int x_c\,dV = \int (r\sin\phi\sin\theta)(r^2\sin\phi\,d\phi\,d\theta\,dr)$$

$$= \int_{R/2}^{R}\int_0^{\pi}\int_0^{\pi} r^3\sin^2\phi\,\sin\theta\,d\phi\,d\theta\,dr = \frac{15}{64}\pi R^4$$

$$\bar{x} = \frac{\int x_c\,dV}{V} = \frac{\frac{15}{64}\pi R^4}{\frac{7}{12}\pi R^3} = \underline{\frac{45}{112}R}$$

(Compare to $\bar{x} = \frac{3}{8}R$ for no hole.)

Note: A hemispherical shell of radius r and thickness dr would be a better element.

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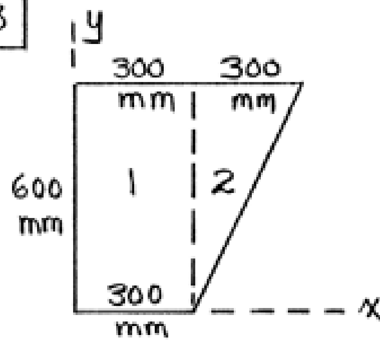

$A_1 = 18(10^4)\text{ mm}^2$, $\bar{x}_1 = 150\text{ mm}$, $\bar{y}_1 = 300\text{ mm}$

$A_2 = 9(10^4)\text{ mm}^2$, $\bar{x}_2 = 300 + \frac{1}{3}(300) = 400\text{ mm}$

$\bar{y}_2 = 400\text{ mm}$

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{18(10^4)(150) + 9(10^4)(400)}{18(10^4) + 9(10^4)} = \underline{233\text{ mm}}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{18(10^4)(300) + 9(10^4)(400)}{18(10^4) + 9(10^4)} = \underline{333\text{ mm}}$$

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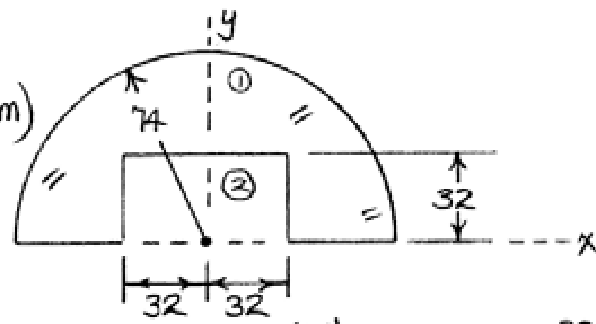
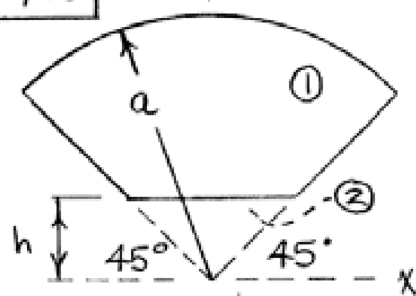
(Dim. in mm)



$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{\pi \frac{74^2}{2}\left(\frac{4(74)}{3\pi}\right) - 64(32)\left(\frac{32}{2}\right)}{\pi \frac{74^2}{2} - 64(32)}$$

$= \underline{36.2\text{ mm}}$

5/45



Circular sector (full) ①:

$$A_1 = \frac{\pi}{4}a^2$$

$$\bar{y}_1 = \frac{2}{3}a\frac{\sin 45°}{\pi/4}$$

$$= \frac{4\sqrt{2}}{3\pi}a$$

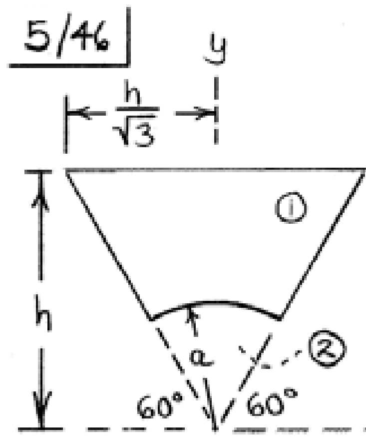
Triangular "hole" ②:

$$A_2 = \frac{1}{2}h(2h) = h^2, \quad \bar{y}_2 = \frac{2}{3}h$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{\frac{\pi}{4}a^2\left(\frac{4\sqrt{2}}{3\pi}a\right) - h^2\left(\frac{2}{3}h\right)}{\frac{\pi}{4}a^2 - h^2}$$

$$= \frac{4(\sqrt{2}a^3 - 2h^3)}{3(\pi a^2 - 4h^2)}$$

5/46



Full triangle ①:

$$A_1 = \frac{1}{2}\left(\frac{2h}{\sqrt{3}}\right)(h) = \frac{h^2}{\sqrt{3}}, \quad \bar{y}_1 = \frac{2}{3}h$$

Circular sector ②:

$$A_2 = \frac{1}{6}\pi a^2$$

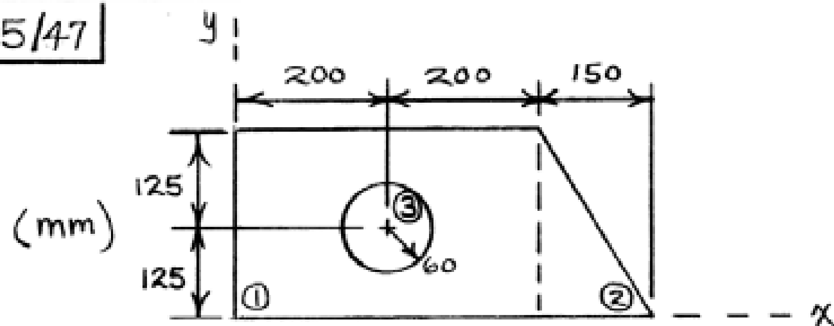
$$\bar{y}_2 = \frac{2}{3}\frac{a \sin 30^\circ}{\pi/6} = \frac{2}{\pi}a$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{\frac{h^2}{\sqrt{3}}\left(\frac{2h}{3}\right) - \frac{1}{6}\pi a^2\left(\frac{2}{\pi}a\right)}{\frac{h^2}{\sqrt{3}} - \frac{1}{6}\pi a^2}$$

$$= \frac{4h^3 - 2\sqrt{3}a^3}{6h^2 - \sqrt{3}\pi a^2}$$

{ reduces to the correct $\bar{Y} = \frac{2}{3}h$ for $a = 0$

5/47



Comp.	A mm^2	$\bar{x}$ mm	$\bar{y}$ mm	$A\bar{x}$ mm^3	$A\bar{y}$ mm^3
Rect. 1	$100(10^3)$	200	125	$20(10^6)$	$12.50(10^6)$
Triangle 2	$18.75(10^3)$	450	$250/3$	$8.44(10^6)$	$1.563(10^6)$
Circle 3	$-11.31(10^3)$	200	125	$-2.26(10^6)$	$-1.414(10^6)$
Totals	$107.4(10^3)$			$26.2(10^6)$	$12.65(10^6)$

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{26.2(10^6)}{107.4(10^3)} = \underline{244 \text{ mm}}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{12.65(10^6)}{107.4(10^3)} = \underline{117.7 \text{ mm}}$$

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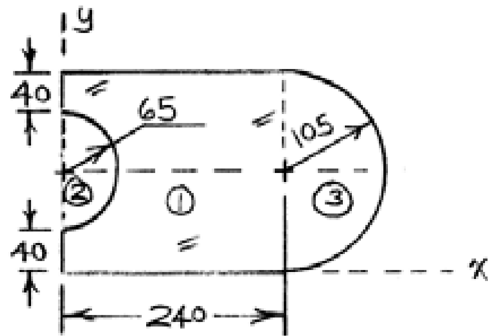


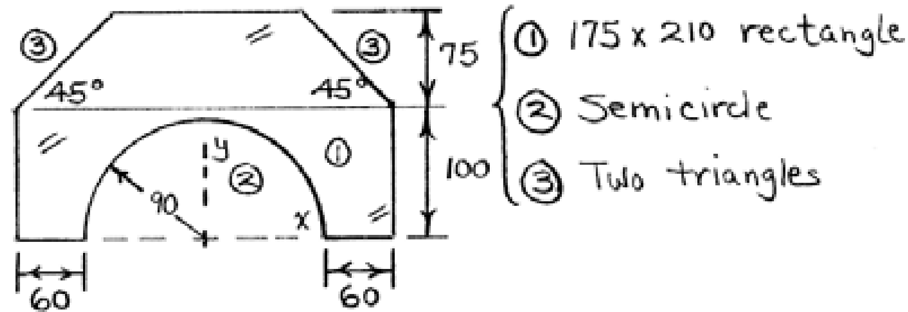
$$\bar{Y} = \frac{1}{2}\left(40 + 2(65) + 40\right) = \underline{105\ \text{mm}}$$

$$\bar{X} = \frac{210(240)\left(\frac{240}{2}\right) - \frac{\pi 65^2}{2}\frac{4(65)}{3\pi} + \frac{\pi 105^2}{2}\left(240 + \frac{4(105)}{3\pi}\right)}{(210)(240) - \frac{\pi 65^2}{2} + \frac{\pi 105^2}{2}}$$

$$= \underline{176.7\ \text{mm}}$$

5/49 (Dim. in mm)

Components:



① 175 x 210 rectangle

② Semicircle

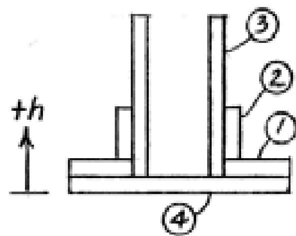
③ Two triangles

Comp.	$A\ (mm^2)$	$\bar{y}\ (mm)$	$A\bar{y}\ (mm^3)$
①	$175(210)$	$\frac{175}{2}$	3 215 625
②	$-\frac{\pi(90^2)}{2}$	$\frac{4(90)}{3\pi}$	− 486 000
③	$-2\frac{1}{2}(75)(75)$	$(100+\frac{2}{3}75)$	− 843 750

$\Sigma A = 18\,400$ $\qquad$ $\Sigma A\bar{y} = 1\,886\,000$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \underline{102.5 \text{ mm}}$$

5/50



Part	Size mm	A mm²	$\bar{h}$ mm	$A\bar{h}$ mm³
(1)	10×40	800	15	12000
(2)	10×40	800	40	32000
(3)	10×120	2400	70	168 000
(4)	10×160	1600	5	8000
Σ's		5600		220 000

$$\bar{H} = \frac{\Sigma A\bar{h}}{\Sigma A} = \frac{220\,000}{5600} = \underline{39.3\text{ mm}}$$

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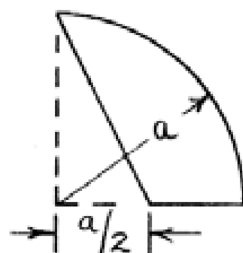

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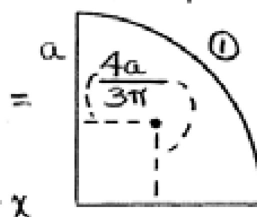

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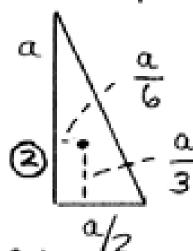


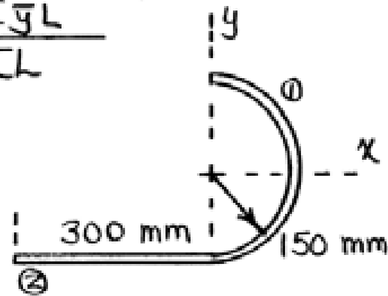
$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{\frac{\pi a^2}{4}\left(\frac{4a}{3\pi}\right) - \frac{a^2}{4}\left(\frac{a}{6}\right)}{\frac{\pi a^2}{4} - \frac{a^2}{4}}$$

$$= \frac{7a}{6(\pi-1)}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{\frac{\pi a^2}{4}\left(\frac{4a}{3\pi}\right) - \frac{a^2}{4}\left(\frac{a}{3}\right)}{\frac{\pi a^2}{4} - \frac{a^2}{4}}$$

$$= \frac{a}{\pi-1}$$

5/52 $\bar{X} = \frac{\Sigma \bar{x} L}{\Sigma L}$, $\bar{Y} = \frac{\Sigma \bar{y} L}{\Sigma L}$



$L_1 = \pi r = 150\pi$ mm

$\bar{x}_1 = \frac{2r}{\pi} = \frac{300}{\pi}$ mm

(from Sample Prob. 5/1)

$\bar{y}_1 = 0$

$L_2 = 300$ mm, $\bar{x}_2 = -150$ mm, $\bar{y}_2 = -150$ mm

So

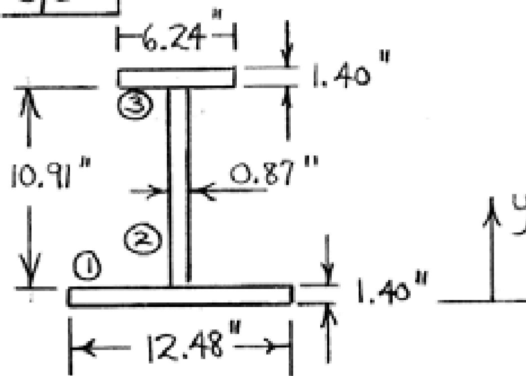
$$\bar{X} = \frac{\frac{300}{\pi}(150\pi) + 300(-150)}{150\pi + 300} = \underline{0}$$

$$\bar{Y} = \frac{\frac{300}{\pi}(0) + 300(-150)}{150\pi + 300} = \underline{-58.3 \text{ mm}}$$

5/53

$$\bar{z} = \frac{\Sigma m\bar{z}}{\Sigma m} = \frac{2(0) + 1.5(90) + 1(180)}{2 + 1.5 + 1} = \underline{70\text{ mm}}$$

5/54

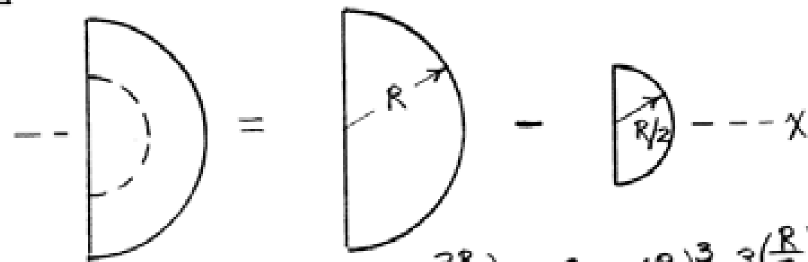


Comp.	A (in.²)	$\bar{y}$ (in.)	$A\bar{y}$ (in.³)
①	(12.48)(1.40)	$\frac{1.40}{2}$	12.23
②	(10.91)(0.87)	$1.4 + \frac{10.91}{2}$	65.1
③	(6.24)(1.40)	$1.4 + 10.91 + \frac{1.4}{2}$	113.7

$\Sigma A = 35.7$ $\Sigma A\bar{y} = 191.0$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \underline{5.35 \text{ in.}}$$

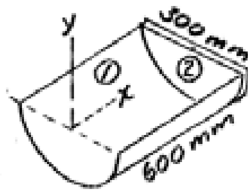
5/55



$$\bar{X} = \frac{\Sigma V x}{\Sigma V} = \frac{\frac{2}{3}\pi R^3\left(\frac{3R}{8}\right) - \frac{2}{3}\pi\left(\frac{R}{2}\right)^3 \frac{3\left(\frac{R}{2}\right)}{8}}{\frac{2}{3}\pi R^3 - \frac{2}{3}\pi\left(\frac{R}{2}\right)^3}$$

$$= \frac{45}{112}R$$

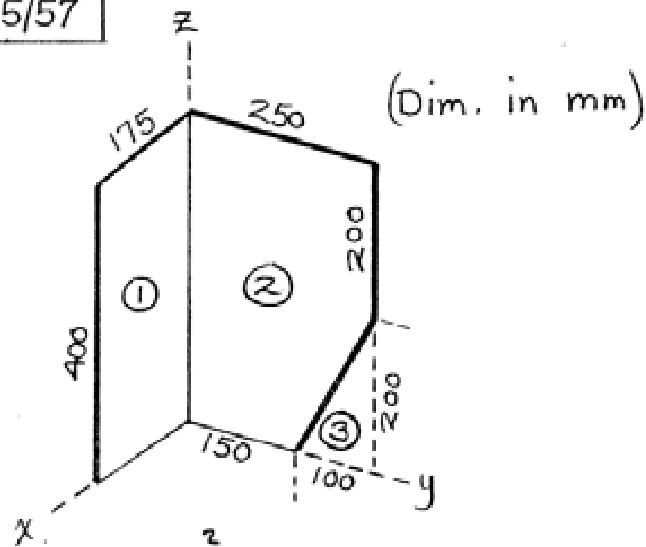
5/56



Part	A m^2	m kg	$\bar{x}$ mm	$\bar{y}$ mm	$m\bar{x}$ kg·mm	$m\bar{y}$ kg·mm
①	0.283	6.79	300	−95.5	2036	−648
②	0.0353	1.27	600	−63.7	763	−81
		8.06			2799	−729

$$\bar{X} = \frac{\Sigma m\bar{x}}{\Sigma m} = \frac{2799}{8.06} = \underline{347.4\text{ mm}}, \quad \bar{Y} = \frac{\Sigma m\bar{y}}{\Sigma m} = \frac{-729}{8.06} = \underline{-90.5\text{ mm}}$$

5/57



Comp.	A (mm^2)	$\bar{x}$ (mm)	$\bar{y}$ (mm)	$\bar{z}$ (mm)	$A\bar{x}$ (in 10^6 mm^3)	$A\bar{y}$ (in 10^6 mm^3)	$A\bar{z}$ (in 10^6 mm^3)
1	$400(175)$	$\frac{175}{2}$	0	$\frac{400}{2}$	6.13	0	14
2	$400(250)$	0	$\frac{250}{2}$	$\frac{400}{2}$	0	12.5	20
3	$-\frac{1}{2}(100)(200)$	0	$\left(150+\frac{2}{3}100\right)$	$\frac{200}{3}$	0	−2.17	−0.667
	160.000				6.13	10.33	33.3

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \underline{38.3 \text{ mm}}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \underline{64.6 \text{ mm}}$$

$$\bar{Z} = \frac{\Sigma A\bar{z}}{\Sigma A} = \underline{208 \text{ mm}}$$

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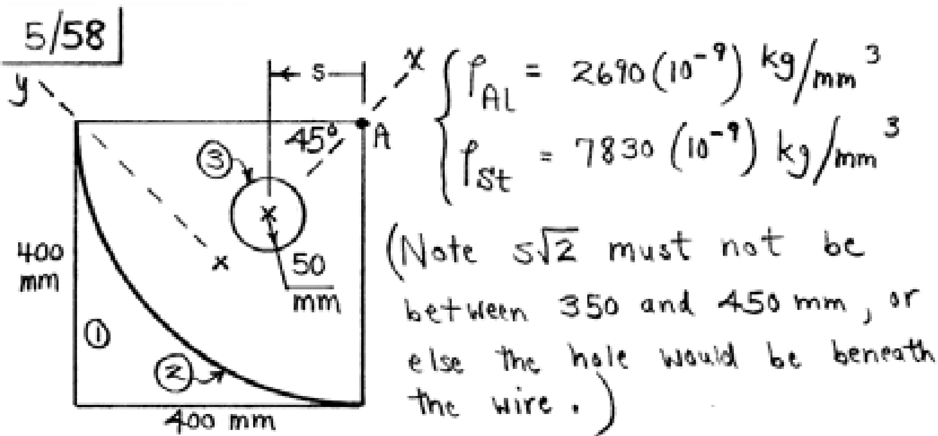


$\begin{cases} \rho_{Al} = 2690(10^{-9})\ kg/mm^3 \\ \rho_{St} = 7830(10^{-9})\ kg/mm^3 \end{cases}$

(Note $s\sqrt{2}$ must not be between 350 and 450 mm, or else the hole would be beneath the wire.)

$m_1 = (400)^2(6)(2690)(10^{-9}) = 2.58$ kg

$\bar{x}_1 = 0$

$m_2 = \frac{\pi(400)}{2}(0.5)(10^{-3}) = 0.314$ kg

$\bar{x}_2 = -\left[\frac{400 \sin\frac{\pi}{4}}{\frac{\pi}{4}} - \frac{400}{\sqrt{2}}\right] = -77.3$ mm

$m_3 = -\pi(50)^2(6)(2690)(10^{-9}) = -0.1268$ kg

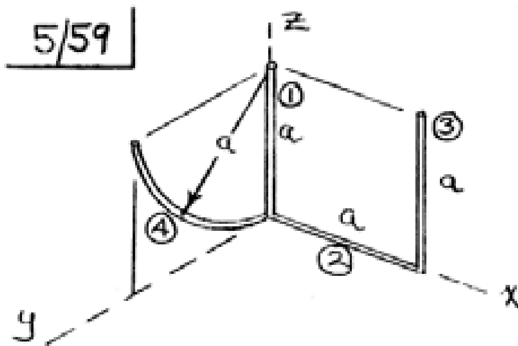
$\bar{x}_3 = \frac{400}{\sqrt{2}} - s\sqrt{2}$

$\bar{X} = \frac{\Sigma m\bar{x}}{\Sigma m} = 0 \Rightarrow \Sigma m\bar{x} = 0$

So $2.58(0) + 0.314(-77.3) - 0.1268(200\sqrt{2} - s\sqrt{2}) = 0$

Solve to obtain $s = +335$ mm ($335\sqrt{2} = 474$ mm from point A; so hole clears slender wire by 24 mm!)

5/59



(ρ = mass/length)

Comp.	m	$\bar{x}$	$\bar{y}$	$\bar{z}$	$m\bar{x}$	$m\bar{y}$	$m\bar{z}$
1	ρa	0	0	$a/2$	0	0	$\rho a^2/2$
2	ρa	$a/2$	0	0	$\rho a^2/2$	0	0
3	ρa	a	0	$a/2$	ρa^2	0	$\rho a^2/2$
4	$\rho \pi a/2$	0	$-2a/\pi$	$a(1-\frac{2}{\pi})$	0	$-\rho a^2$	$\rho a^2(\frac{\pi}{2}-1)$
Totals	$\rho a(3+\frac{\pi}{2})$				$\frac{3}{2}\rho a^2$	$-\rho a^2$	$\frac{\pi}{2}\rho a^2$

$$\bar{X} = \frac{\Sigma m\bar{x}}{\Sigma m} = \frac{\frac{3}{2}\rho a^2}{\rho a(3+\frac{\pi}{2})} = \underline{\frac{3a}{6+\pi}}$$

$$\bar{Y} = \frac{\Sigma m\bar{y}}{\Sigma m} = \frac{-\rho a^2}{\rho a(3+\pi/2)} = \underline{-\frac{2a}{6+\pi}}$$

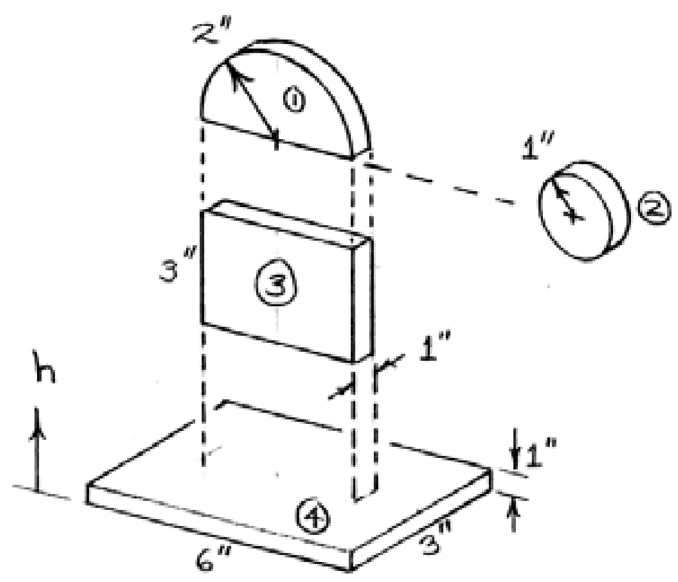
$$\bar{Z} = \frac{\Sigma m\bar{z}}{\Sigma m} = \frac{\frac{\pi}{2}\rho a^2}{\rho a(3+\pi/2)} = \underline{\frac{\pi a}{6+\pi}}$$

5/60 1 = semicircular rod
2 = two straight rods
3 = semicircular plate

Part	L, ft	A, ft²	W, lb	$\bar{x}$, in.	$\bar{z}$, in.	$W\bar{x}$	$W\bar{z}$
1	1.047	–	0.387	6	2.546	2.325	0.987
2	1	–	0.370	3	0	1.110	0
3	–	0.1745	1.396	0	1.698	0	2.370
Σ's			2.153			3.435	3.357

$\bar{X} = \frac{\Sigma W\bar{x}}{\Sigma W} = \frac{3.435}{2.153} = \underline{1.595 \text{ in.}}$ $\bar{Z} = \frac{\Sigma W\bar{z}}{\Sigma W} = \frac{3.357}{2.153} = \underline{1.559 \text{ in.}}$

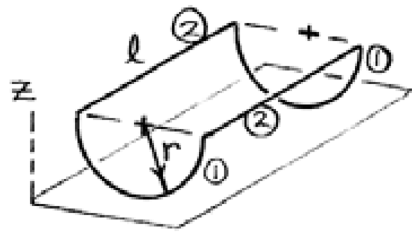
5/61



Part	V, in.^{3}	$\bar{h}$, in.	$V\bar{h}$, in.^{4}
①	6.28	4.85	30.5
②	-3.14	4	-12.57
③	12	2.5	30
④	18	0.5	9
Totals	33.1		56.9

$$\bar{H} = \frac{\Sigma V\bar{h}}{\Sigma V} = \frac{56.9}{33.1} = \underline{1.717 \text{ in.}}$$

5/62



Comp.	L	$\bar{z}$	$L\bar{z}$
1	$2(\pi r)$	$r - \frac{2r}{\pi}$	$2r^2(\pi - 2)$
2	2ℓ	r	$2\ell r$

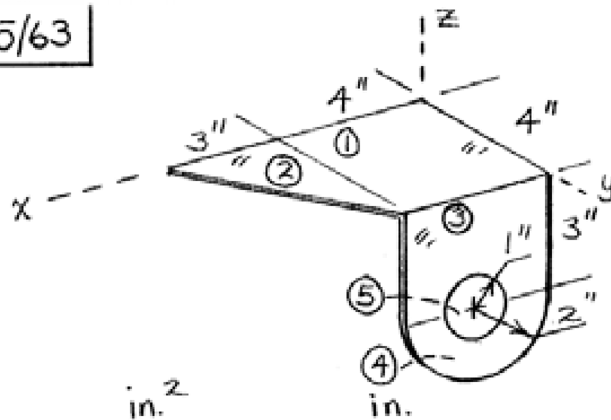
$\Sigma L = 2(\pi r + \ell)$ $\qquad$ $\Sigma L\bar{z} = 2r[r(\pi - 2) + \ell]$

Set $\bar{z} = \dfrac{\Sigma L\bar{z}}{\Sigma L} = \dfrac{2r[r(\pi-2)+\ell]}{2(\pi r+\ell)} = \dfrac{3r}{4}$

and solve for ℓ as

$$\underline{\ell = (8-\pi)r}$$

5/63



	in.²	in.			in.³		
Comp.	A	$\bar{x}$	$\bar{y}$	$\bar{z}$	$A\bar{x}$	$A\bar{y}$	$A\bar{z}$
①	16	2	2	0	32	32	0
②	6	5	4/3	0	30	8	0
③	12	2	4	−1.5	24	48	−18
④	$\frac{\pi 2^2}{2}$	2	4	$-\left(3+\frac{4(2)}{3\pi}\right)$	12.57	25.1	−24.2
⑤	$-\pi 1^2$	2	4	−3	−6.28	−12.56	9.42

$\Sigma A = 37.1$ $\qquad\qquad \Sigma = 92.3 \quad 100.6 \quad -32.8$

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{92.3}{37.1} = \underline{2.48 \text{ in.}}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{100.6}{37.1} = \underline{2.71 \text{ in.}}$$

$$\bar{Z} = \frac{\Sigma A\bar{z}}{\Sigma A} = \frac{-32.8}{37.1} = \underline{-0.882 \text{ in.}}$$

5/64

$$V_1 = \frac{\pi}{2}(30)^2(35) = 49\ 500 \text{ mm}^3$$

$$\bar{x}_1 = -\frac{4(30)}{3\pi} = -12.73 \text{ mm}$$

$$\bar{z}_1 = 17.5 \text{ mm}$$

$$V_2 = -\frac{\pi}{2}(20)^2(25) = -15\ 710 \text{ mm}^2$$

$$\bar{x}_2 = -\frac{4(20)}{3\pi} = -8.49 \text{ mm}$$

$$\bar{z}_2 = \frac{1}{2}(10+35) = 22.5 \text{ mm}$$

$$\bar{X} = \frac{\Sigma V\bar{x}}{\Sigma V} = \frac{49\ 500(-12.73) - 15\ 710(-8.49)}{49\ 500 - 15\ 710}$$

$$= \underline{-14.71 \text{ mm}}$$

$$\bar{Z}_1 = \frac{\Sigma V\bar{z}}{\Sigma V} = \frac{49\ 500\ (17.5) - 15\ 710\ (22.5)}{49\ 500 - 15\ 710}$$

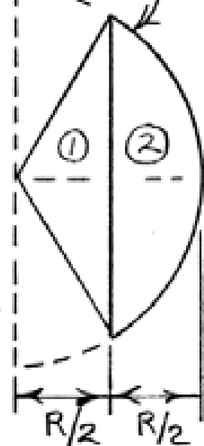
$$= \underline{15.17 \text{ mm}}$$

5/65

Interval	A_{AV} ft^2	Vol. V ft^3	$\bar{x}_{AV}$ ft	$V\bar{x}$ ft^3
0-2	1.0	2.0	1.5	3.0
2-4	3.3	6.6	3.2	21.1
4-6	6.5	13.0	5.1	66.3
6-8	10.0	20.0	7.0	140.0
8-10	12.6	25.2	9.0	226.8
10-12	14.1	28.2	11.0	310.2
12-14	14.4	28.8	13.0	374.4
14-16	13.7	27.4	15.0	411.0
16-18	12.0	24.0	17.0	408.0
18-20	9.0	18.0	19.0	342.0
20-22	5.2	10.4	21.0	218.4
22-24	1.5	3.0	22.5	67.5
		206.6		2588.7

$$\bar{X} = \frac{\Sigma V\bar{x}}{\Sigma V} = \frac{2588.7}{206.6} = \underline{12.53\ ft}$$

5/66



$x^2+y^2=R^2, \quad y^2=R^2-x^2$

At $x=\frac{R}{2}$, $y^2=R^2-\frac{R^2}{4}=\frac{3}{4}R^2$

Conical volume ①:

$$V_1=\frac{1}{3}\pi\left(\frac{3R^2}{4}\right)\left(\frac{R}{2}\right)=\frac{1}{8}\pi R^3$$

$$\bar{x}_1=\frac{3}{4}\left(\frac{R}{2}\right)=\frac{3}{8}R$$

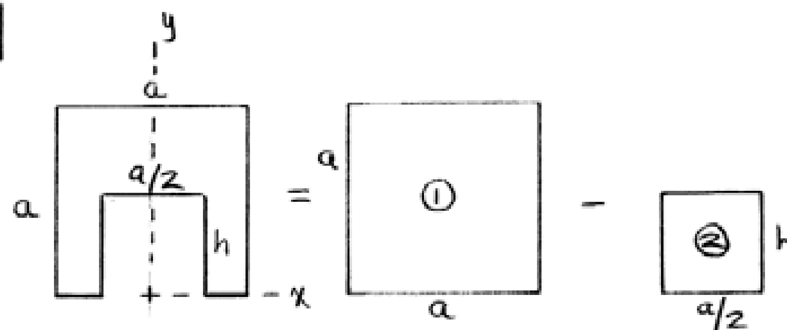
Spherical cap ②

$V_2=\frac{5}{24}\pi R^3$, $\bar{x}_2=\frac{27}{40}R$ } from solution to Prob. 5/27

$$\bar{X}=\frac{\Sigma V\bar{x}}{\Sigma V}=\frac{\frac{1}{8}\pi R^3\left(\frac{3R}{8}\right)+\frac{5}{24}\pi R^3\left(\frac{27R}{40}\right)}{\frac{1}{8}\pi R^3+\frac{5}{24}\pi R^3}$$

$$=\frac{9}{16}R \quad \left(\text{fairly close to } \frac{R}{2}!\right)$$

5/67



$A_1 = a^2$, $\bar{y}_1 = a/2$; $A_2 = -ah/2$, $\bar{y}_2 = h/2$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{a^2\left(\frac{a}{2}\right) - \frac{ah}{2}\left(\frac{h}{2}\right)}{a^2 - \frac{ah}{2}} = \frac{1}{2}\,\frac{a^2 - h^2/2}{a - h/2}$$

$$\frac{d\bar{Y}}{dh} = \frac{1}{2}\,\frac{(a - h/2)(-h) - (a^2 - h^2/2)\left(-\frac{1}{2}\right)}{(a - h/2)^2}$$

$$= \frac{1}{2}\,\frac{\frac{h^2}{4} - ah + \frac{a^2}{2}}{\left(a - \frac{h}{2}\right)^2} = 0 \text{ for max } \bar{Y}.$$

So $\frac{1}{4}(h^2 - 4ah + 2a^2) = 0$, $h = a(2 \pm \sqrt{2})$

The plus sign is rejected because h must be less than a.

Hence $h = a(2 - \sqrt{2}) = \underline{0.586a}$

(Note that $\bar{Y} = 0.586a$ for $h = 0.586a$)

5/68 Cube: $\begin{cases} V_1 = 350^3 = 42\,875\,000 \text{ mm}^3 \\ \bar{z}_1 = 175 \text{ mm} \end{cases}$

Hole: $\begin{cases} V_2 = -\pi(100)^2 h \\ \bar{z}_2 = \frac{h}{2} \end{cases}$

$$\bar{Z} = \frac{\Sigma V\bar{z}}{\Sigma V} = \frac{42\,875\,000\,(175) - \pi(100)^2 h \frac{h}{2}}{42\,875\,000 - \pi(100)^2 h}$$

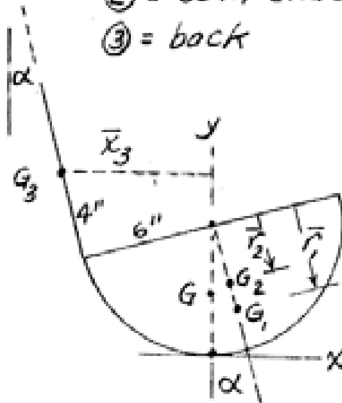
For The maximum $\bar{Z}$, set $\frac{d\bar{Z}}{dh} = 0$:

$$\frac{(42\,875\,000 - \pi\, 100^2 h)(-\pi\, 100^2 h) - \left(42\,875\,000\,(175) - \pi\, 100^2 \frac{h^2}{2}\right) \times (-\pi\, 100^2)}{(\text{denominator})} = 0$$

Set numerator equal to zero to obtain

$$\underline{h = 187.9 \text{ mm}}$$

▸5/69 ① = semicircular shell
② = both ends
③ = back



$A_1 = \pi(6)(16) = 301.6\ in^2$

$A_2 = 2(\pi \cdot 6^2/2) = 113.1\ in^2$

$A_3 = 8(16) = 128\ in^2$

$\Sigma A = 542.7\ in^2$

$\bar{r}_1 = \frac{2(6)}{\pi} = 3.82\ in$

$\bar{r}_2 = \frac{4(6)}{3\pi} = 2.55\ in.$

$\bar{x}_1 = 3.82 \sin\alpha$, $\bar{x}_2 = 2.55 \sin\alpha$

$\bar{x}_3 = -6\cos\alpha - 4\sin\alpha$

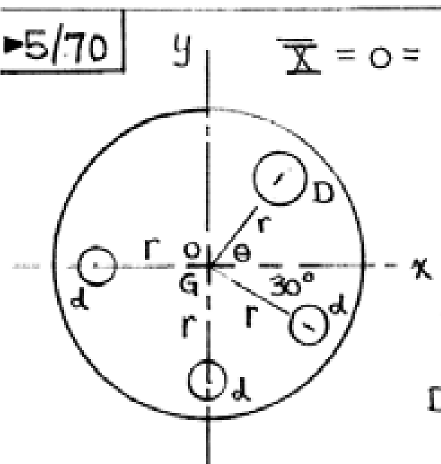
$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A}$; $0 = 301.6(3.82\sin\alpha) + 113.1(2.55\sin\alpha)$

$+ 128(-6\cos\alpha - 4\sin\alpha) = 0$

$928.0\sin\alpha = 768\cos\alpha$, $\tan\alpha = \frac{768}{928} = 0.828$

$\underline{\alpha = 39.6°}$

▸5/70



$$\overline{X} = 0 = \frac{\pi D^2}{4} r\cos\theta + \frac{\pi d^2}{4} r\cos 30° - \frac{\pi d^2}{4} r, \quad D^2\cos\theta = d^2\left(1 - \frac{\sqrt{3}}{2}\right)$$

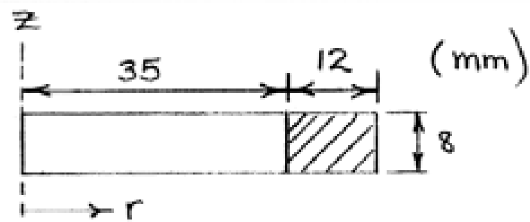
$$\overline{Y} = 0 = \frac{\pi D^2}{4} r\sin\theta - \frac{\pi d^2}{4} r - \frac{\pi d^2}{4} r\sin 30°$$

$$D^2\sin\theta = d^2\left(1 + \frac{1}{2}\right)$$

Divide : $\dfrac{\sin\theta}{\cos\theta} = \dfrac{3/2}{1 - \sqrt{3}/2}$, $\underline{\theta = 84.9°}$

$D^2 = \dfrac{3d^2/2}{\sin 84.9°}$, $\underline{D = 1.227\,d}$

5/71



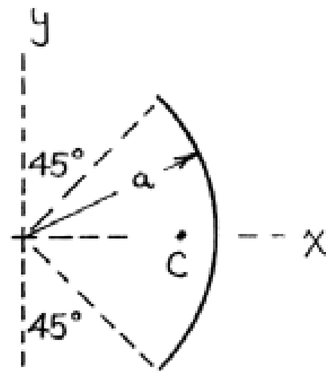
$$A = 2\pi \bar{r} L = 2\pi \left(35 + \frac{12}{2}\right)(12 + 12 + 8 + 8)$$

$$= \underline{10\ 300 \text{ mm}^2}$$

$$V = 2\pi \bar{r} A = 2\pi \left(35 + \frac{12}{2}\right)(12 \cdot 8)$$

$$= \underline{24\ 700 \text{ mm}^3}$$

5/72



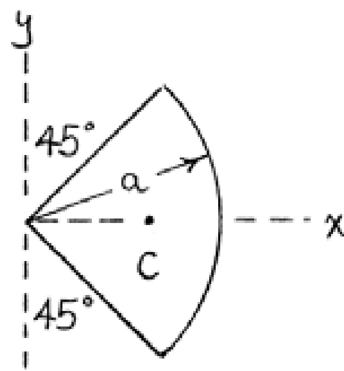
From Table D/3,

$$\bar{x} = \frac{a \sin \frac{\pi}{4}}{\pi/4} = a \frac{\sqrt{2}}{2} \frac{4}{\pi} = \frac{2\sqrt{2}}{\pi} a$$

$$A = 2\pi \bar{x} L = 2\pi \left(\frac{2\sqrt{2}}{\pi} a \right) \left(\frac{2\pi a}{4} \right)$$

$$= \underline{2\sqrt{2}\, \pi a^2}$$

5/73



From Table D/3,

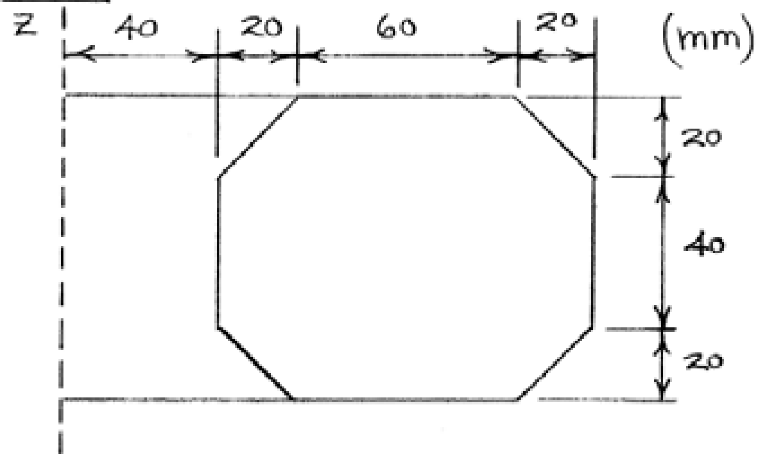
$$\bar{x} = \frac{2}{3} a \frac{\sin \pi/4}{\pi/4} = \frac{2}{3} a \left(\frac{\sqrt{2}}{2}\right)\left(\frac{4}{\pi}\right) = \frac{4\sqrt{2}}{3\pi} a$$

$$V = 2\pi \bar{x} A = 2\pi \left(\frac{4\sqrt{2}}{3\pi} a\right)\left(\frac{\pi a^2}{4}\right)$$

$$= \frac{2\pi \sqrt{2} a^3}{3}$$

5/74 $V = \theta \bar{r} A = \pi(8 + \frac{2}{3}12)\frac{1}{2}(12)(12) = \underline{3620 \text{ mm}^3}$

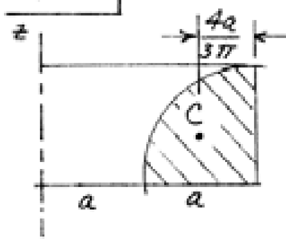
5/75



$A = 2\pi \bar{r} L = 2\pi \left[40 + 20 + \frac{60}{2}\right]\left[2(60) + 2(40) + 4 \cdot 20\sqrt{2}\right]$

$= \underline{177\ 100\ \text{mm}^2}$

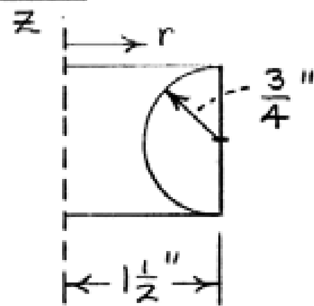
5/76



$V = \bar{r}\theta A$

$= \left(2a - \frac{4a}{3\pi}\right)\frac{\pi}{2}\frac{\pi a^2}{4} = \underline{\frac{\pi a^3}{12}(3\pi - 2)}$

5/77



$$V = 2\pi \bar{r} A = 2\pi\left[1.5 - \frac{4\left(\frac{3}{4}\right)}{3\pi}\right]\left[\frac{\pi\left(\frac{3}{4}\right)^2}{2}\right]$$

$$= \underline{6.56 \text{ in.}^3}$$

Inner area

$$A_i = 2\pi \bar{r} L = 2\pi\left[1.5 - \frac{2\left(\frac{3}{4}\right)}{\pi}\right]\left[\frac{2\pi\left(\frac{3}{4}\right)}{2}\right]$$

$$= 15.14 \text{ in.}^2$$

Outer area $A_o = 2\left(\frac{3}{4}\right)\left(2\pi \cdot 1\frac{1}{2}\right)$

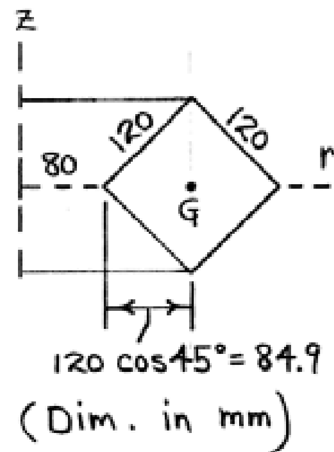
$$= 14.14 \text{ in.}^2$$

So $A = A_i + A_o = \underline{29.3 \text{ in.}^2}$

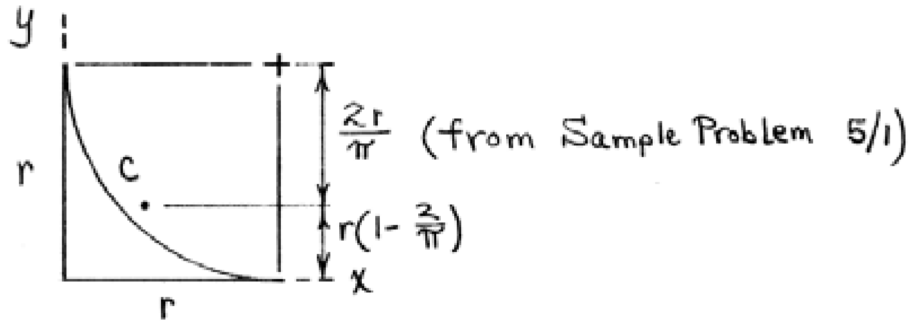
5/78

$$V = 2\pi \bar{r} A$$
$$= 2\pi(80 + 84.9)(120)^2$$
$$= \underline{14.92\,(10^6)\text{ mm}^3}$$

Surface area $A = 2\pi \bar{r} L$
$$= 2\pi(80 + 84.9)(4 \times 120)$$
$$= \underline{497\,(10^3)\text{ mm}^2}$$



5/79



"Lateral" surface area is

$$A_L = 2\pi \bar{y} L = 2\pi r\left(1-\frac{2}{\pi}\right)\left(\pi r/2\right) = \pi^2 r^2\left(1-\frac{2}{\pi}\right)$$

"Bottom" (left end) surface area is

$$A_B = \pi r^2$$

Total body surface area is

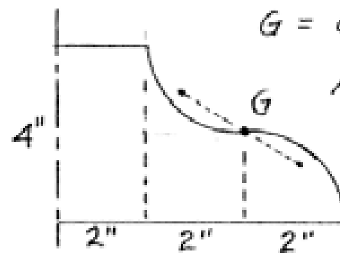
$$A = A_L + A_B = \underline{\pi r^2 [\pi - 1]}$$

5/80 | $A = 2\pi \bar{r} L + \pi d h$

$= 2\pi(8.2)34 + \pi(8)(18) = 2204 \text{ ft}^2$

No. of gal. for 2 coats $= \frac{2204}{500} \times 2 = \underline{8.82 \text{ gal}}$

5/81

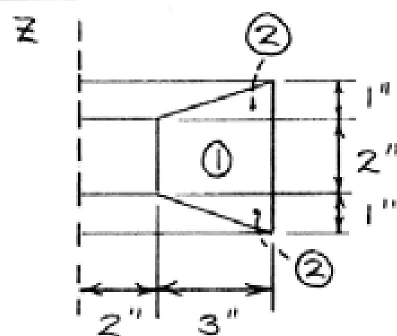


G = centroid by symmetry

$A = 2\pi \bar{r} L = 2\pi(4)\pi(2)$

$= 16\pi^2 = \underline{157.9 \text{ in.}^2}$

5/82



$$V_1 = \pi \bar{r}_1 A_1 = \pi\left(2+\frac{3}{2}\right)(3\cdot 2) = 66.0 \text{ in.}^3$$

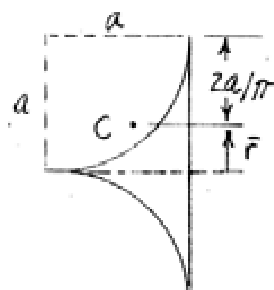
$$V_2 = 2(\pi \bar{r}_2 A_2) = 2\pi\left(5-\frac{3}{3}\right)\left(\frac{1}{2}(3)(1)\right) = 37.7 \text{ in.}^3$$

$$V = V_1 + V_2 = 103.7 \text{ in.}^3$$

$$W = \mu V = \left(168\,\frac{\text{lb}}{\text{ft}^3}\right)(103.7 \text{ in.}^3)\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$

$$= \underline{10.08 \text{ lb}}$$

5/83



$$A = 2\pi \bar{r} L$$

$$= 2\pi\left(a - \frac{2a}{\pi}\right)\frac{\pi a}{2}$$

$$= \underline{\pi a^2(\pi - 2)}$$

5/84 $m = \rho V$, where $\rho = 7830 \frac{kg}{m^3}$ (Appendix D)

$V = 2\pi \bar{r} A$, $\bar{r} A = 200(100)\left(\frac{60+160}{2}\right) - \frac{\pi(60^2)}{2}\left(60 + \frac{4(60)}{3\pi}\right)$

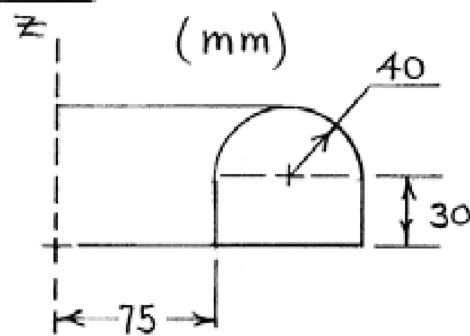
$= 1.717\,(10^6)\ mm^3$

$V = 2\pi\,(1.717 \times 10^6) = 10.79\,(10^6)\ mm^3$

or $V = 0.01079\ m^3$

$\therefore\ m = \rho V = 7830(0.01079) = \underline{84.5\ kg}$

5/85



$A_1 = \pi \bar{r} L = \pi(75+40)\left(2\cdot 30 + 80 + \pi 40\right)$

$= 96\,000 \text{ mm}^2$

End areas $A_2 = 2\left(\frac{\pi}{2}\cdot 40^2 + 80(30)\right)$

$= 9830 \text{ mm}^2$

Total area $A = A_1 + A_2 = \underline{105\,800 \text{ mm}^2}$

$V = \pi \bar{r} A = \pi(75+40)\left(30\cdot 80 + \pi 40^2/2\right)$

$= \underline{1.775(10^6) \text{ mm}^3}$

5/86 $V = 2\pi \bar{r} A$, $m = V\rho$

Where $m = 10.0$ kg, $\rho = 2.69 \times 10^3$ kg/m^3

$A = \frac{1}{2}\, 15{,}200 \times 10^{-6} = 7.600 \times 10^{-3}$ m^2

Thus $\bar{r} = \dfrac{V}{2\pi A} = \dfrac{m}{2\pi\rho A} = \dfrac{10.0}{2\pi(2.69\times10^3)(7.6\times10^{-3})}$

$= 0.0778$ m

or $\underline{\bar{r} = 77.8 \text{ mm}}$

5/87

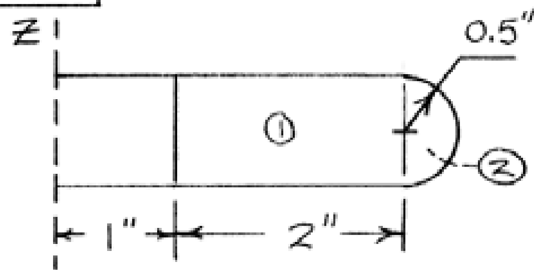
$$\bar{r}' = \frac{4r}{3\pi} = \frac{4(30)}{3\pi} = 12.73 \text{ mm}$$

$$\bar{r} = 75 + 12.73 \sin 30° = 81.4 \text{ mm}$$

$$A = \frac{\pi r^2}{2} = \frac{\pi(30)^2}{2} = 1414 \text{ mm}^2$$

$$V = \pi \bar{r} A = \pi(81.4)(1414) = \underline{361\,000 \text{ mm}^3}$$

5/88



$$V_1 = 2\pi \bar{r}_1 A_1 = 2\pi(1+1)(2\cdot 1) = 25.1 \text{ in.}^3$$

$$V_2 = 2\pi \bar{r}_2 A_2 = 2\pi\left[1+2+\frac{4(0.5)}{3\pi}\right]\left[\pi(0.5)^2/2\right]$$

$$= 7.93 \text{ in.}^3$$

$$V = V_1 + V_2 = \underline{33.1 \text{ in.}^3}$$

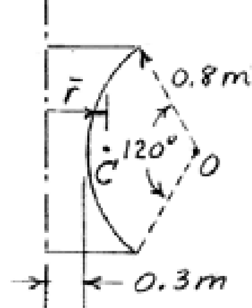
$$A_1 = 1(2\pi\cdot 1) + 2\pi(3^2 - 1^2) = 56.5 \text{ in.}^2$$

$$A_2 = 2\pi \bar{r}_2 L_2 = 2\pi\left[1+2+\frac{2(0.5)}{\pi}\right]\left[\pi(0.5)\right]$$

$$= 32.8 \text{ in.}^2$$

$$A = A_1 + A_2 = \underline{89.3 \text{ in.}^2}$$

5/89 $A = 2\pi \bar{r} L$; $L = \frac{120}{360} 2\pi(0.8) = 1.68$ m



From Sample Prob. 5/1,

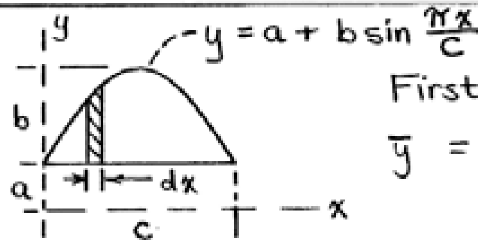
$\overline{OC} = \frac{r \sin\alpha}{\alpha}$ where $\alpha = 60°$ or $\alpha = \pi/3$ rad

$= \frac{0.8(\sqrt{3}/2)}{\pi/3} = 0.662$ m

$\bar{r} = (0.3 + 0.8 - 0.662) = 0.438$ m

so $A = 2\pi(0.438)(1.68) = \underline{4.62 \text{ m}^2}$

5/90



First, find $\bar{y}$ by $\bar{y} = \int y_c \, dA \Big/ \int dA$

$$A = \int dA = \int_0^c b\sin\frac{\pi x}{c}\,dx = -\frac{bc}{\pi}\cos\frac{\pi x}{c}\Big|_0^c = \frac{2bc}{\pi}$$

$$\int y_c \, dA = \int_0^c \left[a + \frac{b}{2}\sin\frac{\pi x}{c}\right]\left[b\sin\frac{\pi x}{c}\,dx\right]$$

$$= \int_0^c ab\sin\frac{\pi x}{c}\,dx + \int_0^c \frac{b^2}{2}\sin^2\frac{\pi x}{c}\,dx$$

$$= -ab\frac{c}{\pi}\cos\frac{\pi x}{c}\Big|_0^c + \frac{b^2}{2}\left[\frac{x}{2} - \frac{1}{4}\sin\frac{2\pi x}{c}\right]_0^c$$

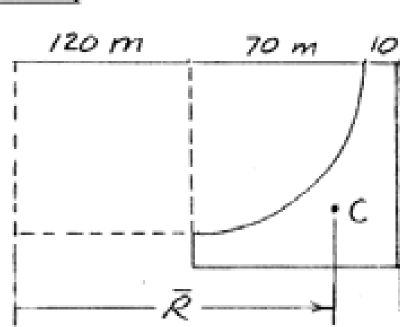
$$= \frac{2abc}{\pi} + \frac{b^2 c}{4} = bc\left[\frac{2a}{\pi} + \frac{b}{4}\right]$$

$$\therefore \bar{y} = \frac{bc\left[2a/\pi + b/4\right]}{2bc/\pi} = a + \frac{b}{8}\pi$$

$$V = 2\pi\bar{y}A = 2\pi\left(a + \frac{b}{8}\pi\right)\left(2bc/\pi\right)$$

$$= \underline{4bc\left(a + \frac{b\pi}{8}\right)}$$

5/91



Square : $A = 80^2 = 6400 \text{ m}^2$

$\frac{1}{4}$ circle : $A = \frac{1}{4}\pi(70^2)$

$= 3848 \text{ m}^2$

Net area $= 2552 \text{ m}^2$

$\bar{r}_{square} = 120 + 40 = 160 \text{ m}$

$\bar{r}_{\frac{1}{4}circle} = 120 + \frac{4(70)}{3\pi} = 149.7 \text{ m}$

$\bar{R} = \frac{\Sigma A\bar{r}}{\Sigma A} = \frac{6400(160) - 3848(149.7)}{2552}$

$= 175.5 \text{ m}$

$V = \theta \bar{R} A = \frac{\pi}{3}(175.5)(2552) = 469\ 000 \text{ m}^3$

$m = \rho V = 2.40(469\ 000) = \underline{1.126 \times 10^6 \text{ Mg}}$

5/92 | From the solution to Prob. 5/7,

$\bar{r} = 8 - \frac{2}{3}\,\frac{2(1.5)+2}{1.5+2} = 7.05 \text{ m}$

$A = \frac{2+1.5}{2}(2) = 3.5 \text{ m}^2$

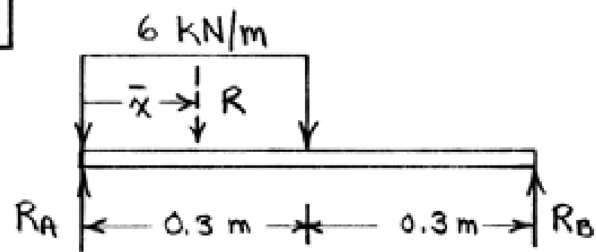
$\theta = \pi/3$

So $V = \theta \bar{r} A = \frac{\pi}{3}(7.05)(3.5) = 25.8 \text{ m}^3$

$W = \rho g V = 2400(9.81)(25.8) = 608(10^3) \text{ N}$

or $\underline{W = 608 \text{ kN}}$

5/93

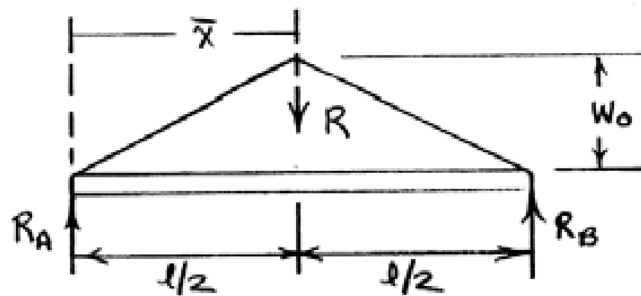


$R = 6(0.3) = 1.8 \text{ kN}$ @ $\bar{x} = \frac{1}{2}(0.3) = 0.15 \text{ m}$

$\circlearrowleft + \Sigma M_A = 0: \; R_B(0.6) - 1.8(0.15) = 0,$ $\underline{R_B = 0.45 \text{ kN}}$

$+\uparrow \Sigma F = 0: \; 0.45 - 1.8 + R_A = 0,$ $\underline{R_A = 1.35 \text{ kN}}$

5/94

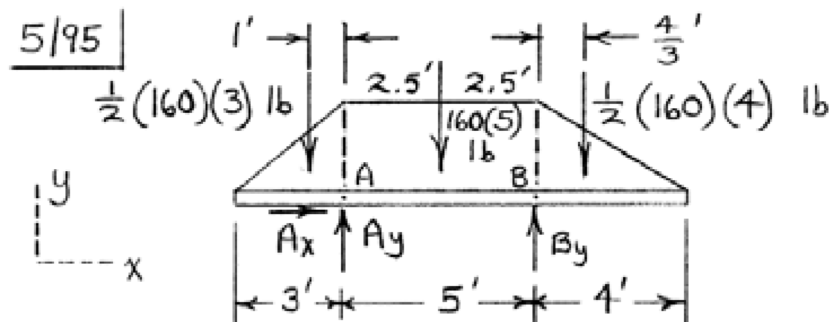


$R = 2\,\frac{1}{2}\,(w_0)(l/2) = \frac{1}{2}\,w_0 l \quad @ \ \bar{x} = \frac{l}{2}$

$\circlearrowleft + \Sigma M_A = 0: \ R_B(l) - \frac{1}{2}\,w_0 l\left(\frac{l}{2}\right) = 0, \ \underline{R_B = \frac{1}{4}\,w_0 l}$

$+\uparrow \Sigma F = 0: \ \frac{1}{4}\,w_0 l - \frac{1}{2}\,w_0 l + R_A = 0, \ \underline{R_A = \frac{1}{4}\,w_0 l}$

5/95



$\curvearrowleft + \sum M_A = 0$: $240(1) - 800(2.5) - 320(6.33) + B_y(5) = 0$

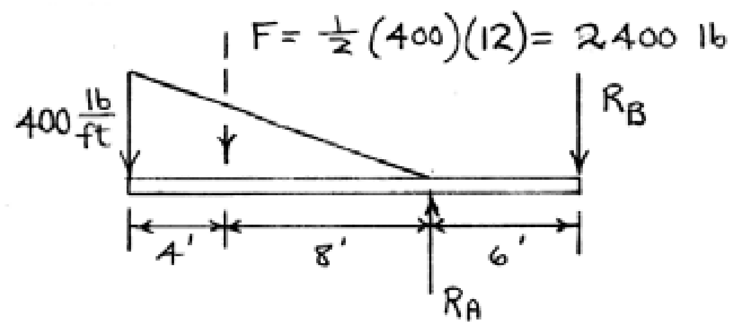
$\underline{B_y = 757 \text{ lb}}$

$\sum F_y = 0$: $A_y + 757 - 240 - 800 - 320 = 0$

$\underline{A_y = 603 \text{ lb}}$

$\sum F_x = 0 \Rightarrow \underline{A_x = 0}$

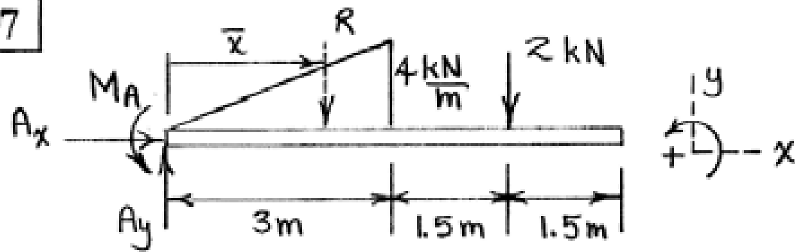
5/96



$\circlearrowleft + \Sigma M_A = 0: \quad 2400(8) - 6R_B = 0, \quad \underline{R_B = 3200 \text{ lb}}$

$+\uparrow \Sigma F = 0: \quad R_A - 2400 - 3200 = 0, \quad \underline{R_A = 5600 \text{ lb}}$

5/97



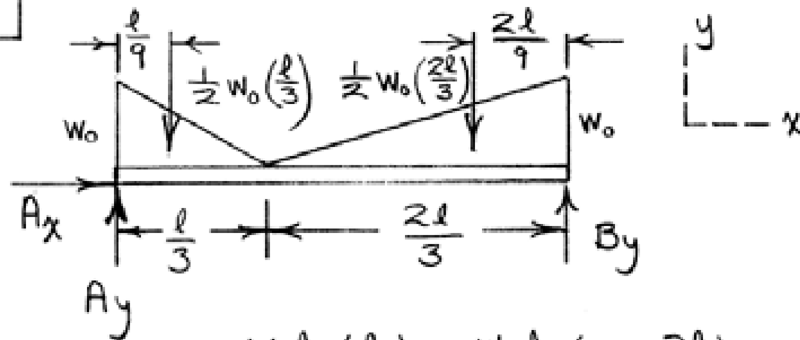
$R = \frac{1}{2}(3)(4) = 6\ kN$ @ $\bar{x} = \frac{2}{3}(3) = 2m$

$\Sigma M_A = 0: \ M_A - 6(2) - 2(4.5) = 0,$ $\underline{M_A = 21\ kN{\cdot}m}$

$\Sigma F_y = 0: \ A_y - 6 - 2 = 0,$ $\underline{A_y = 8\ kN}$

$\Sigma F_x = 0: \ \underline{A_x = 0}$

5/98



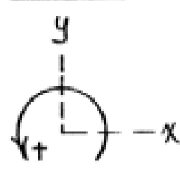
$$\circlearrowleft + \sum M_A = 0: -\frac{w_0 l}{6}\left(\frac{l}{9}\right) - \frac{w_0 l}{3}\left(l - \frac{2l}{9}\right) + B_y l = 0, \quad \underline{B_y = \frac{5}{18} w_0 l}$$

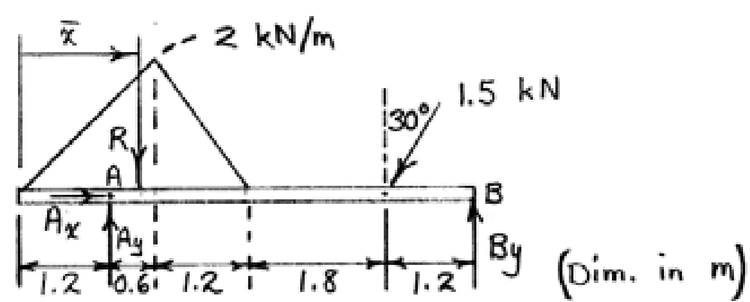
$$\sum F_y = 0: \quad A_y + \frac{5}{18} w_0 l - \frac{w_0 l}{6} - \frac{w_0 l}{3} = 0$$

$$\underline{A_y = \frac{2}{9} w_0 l}$$

$$\sum F_x = 0 \Rightarrow \underline{A_x = 0}$$

5/99





$$\begin{cases} R = \frac{1}{2}(1.2+0.6+1.2)(2) = 3 \text{ kN} \\ \bar{x} = \dfrac{(1.2+0.6)+(1.2+0.6+1.2)}{3} = 1.6 \text{ m} \end{cases}$$

$\Sigma F_x = 0: \quad A_x - 1.5 \sin 30° = 0, \quad \underline{A_x = 750 \text{ N}}$

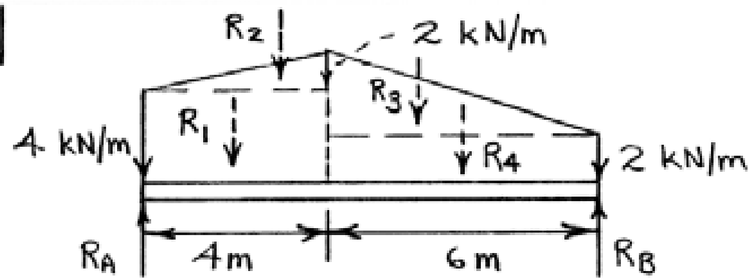
$\Sigma M_A = 0: \quad -3(1.6-1.2) - 1.5 \cos 30° (3.6) - B_y (4.8) = 0$

$\underline{B_y = 1.224 \text{ kN}}$

$\Sigma F_y = 0: \quad A_y - 3 - 1.5 \cos 30° + 1.224 = 0$

$\underline{A_y = 3.07 \text{ kN}}$

5/100



$R_1 = 4(4) = 16 \text{ kN}$, $R_2 = \frac{1}{2}(2)(4) = 4 \text{ kN}$

$R_3 = \frac{1}{2}(4)(6) = 12 \text{ kN}$, $R_4 = 2(6) = 12 \text{ kN}$

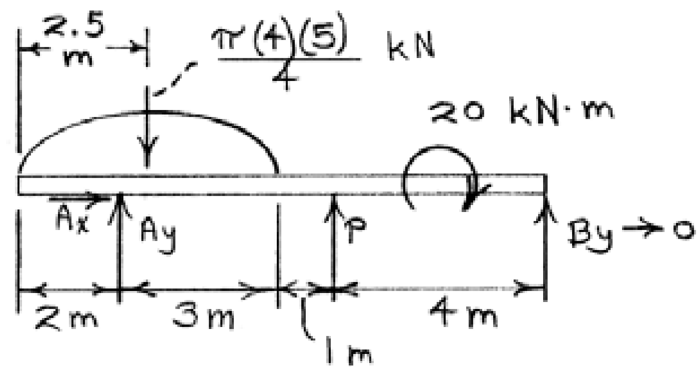
$\circlearrowleft \Sigma M_A = 0: \; 16(2) + 4\left(\frac{2}{3}4\right) + 12\left(4 + \frac{1}{3}6\right)$

$+ 12(4+3) - 10 R_B = 0$, $\underline{R_B = 19.87 \text{ kN}}$

$+\uparrow \Sigma F = 0: \; R_A + 19.87 - (16 + 4 + 12 + 12) = 0$

$\underline{R_A = 24.1 \text{ kN}}$

5/101



$$\circlearrowleft + \Sigma M_A = 0: \ -5\pi(0.5) + P(4) - 20 = 0$$

$$\underline{P = 6.96 \text{ kN}}$$

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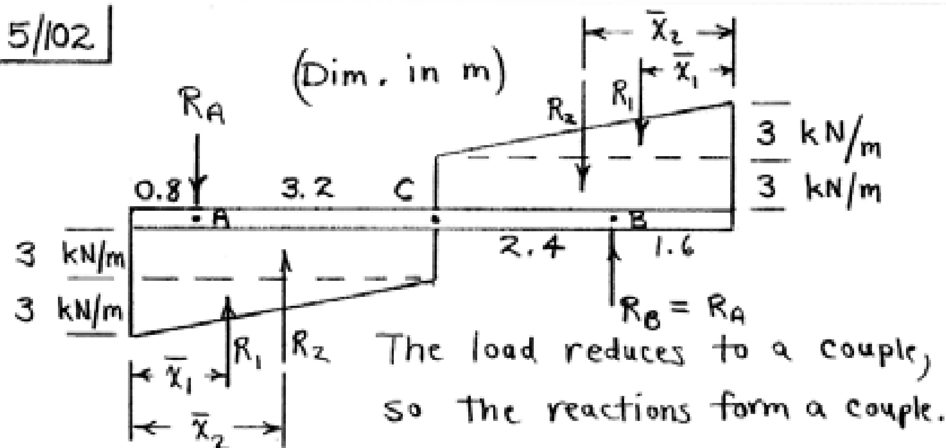


The load reduces to a couple, so the reactions form a couple.

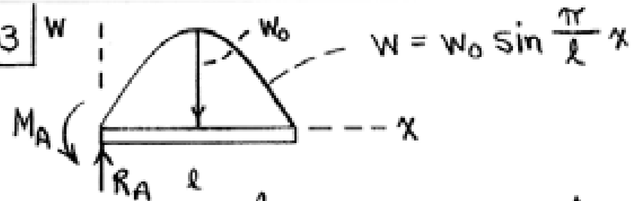
$R_1 = \frac{1}{2}(4)(3) = 6 \text{ kN}$; $\bar{x}_1 = \frac{1}{3}(4) = \frac{4}{3} \text{ m}$

$R_2 = 4(3) = 12 \text{ kN}$; $\bar{x}_2 = 4/2 = 2 \text{ m}$

$\circlearrowleft + \Sigma M_C = 0: \; R_A(3.2 + 2.4) - 12(2 + 2)$
$\quad - 6\left(2 \cdot \left(4 - \frac{4}{3}\right)\right) = 0,$ $\underline{R_A = 14.29 \text{ kN}}$

$\underline{R_B = 14.29 \text{ kN}}$

5/103



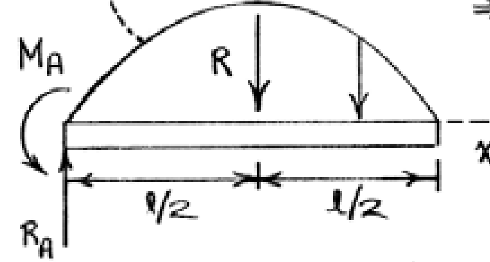
$$R = \int w\,dx = \int_0^l w_0 \sin \frac{\pi}{l} x = -w_0 \frac{l}{\pi} \cos \frac{\pi}{l} x \Big|_0^l = \frac{2 w_0 l}{\pi}$$

$\bar{x} = \frac{l}{2}$, by inspection

$\circlearrowleft + \Sigma M_A = 0: \quad M_A - \frac{2 w_0 l}{\pi}\left(\frac{l}{2}\right) = 0, \qquad \underline{M_A = \frac{w_0 l^2}{\pi}}$

$+\uparrow \Sigma F = 0: \quad R_A - \frac{2 w_0 l}{\pi} = 0, \qquad \underline{R_A = \frac{2 w_0 l}{\pi}}$

5/104



At $x = \frac{l}{2}$, $w = 0 = w_0 - k\left(\frac{l}{2}\right)^2$

$\Rightarrow k = \frac{4w_0}{l^2}$

$w = w_0\left(1 - \frac{4}{l^2}x^2\right)$

$\bar{x} = 0$, by inspection

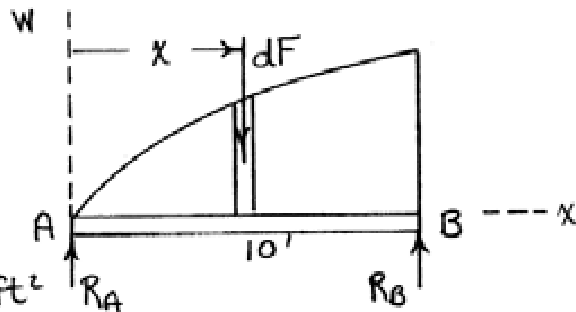
$$R = \int w\,dx = 2\int_0^{l/2} w_0\left(1 - \frac{4}{l^2}x^2\right)dx$$

$$= 2w_0\left[x - \frac{4}{3l^2}x^3\right]_0^{l/2} = \frac{2}{3}w_0 l$$

$+\uparrow \Sigma F = 0: \; R_A - \frac{2}{3}w_0 l = 0,$ $\underline{R_A = \frac{2}{3}w_0 l}$

$\circlearrowleft + \Sigma M_A = 0: \; M_A - \frac{2}{3}w_0 l\left(\frac{l}{2}\right) = 0,$ $\underline{M_A = \frac{1}{3}w_0 l^2}$

5/105



$dF = w\,dx$

$w = k_1 x - k_2 x^2$

At $x = 0$:

$\frac{dw}{dx} = k_1 = 50$ lb/ft²

At $x = 10'$, $w = 50(10) - k_2(10)^2 = 300$

$k_2 = 2$ lb/ft³

So $w = 50x - 2x^2$, $dF = (50x - 2x^2)\,dx$

$\Sigma M_A = 0$: $\int x\,dF - 10R_B = 0$

$\int_0^{10} x(50x - 2x^2)\,dx = \frac{7}{6}10^4 = 10R_B$

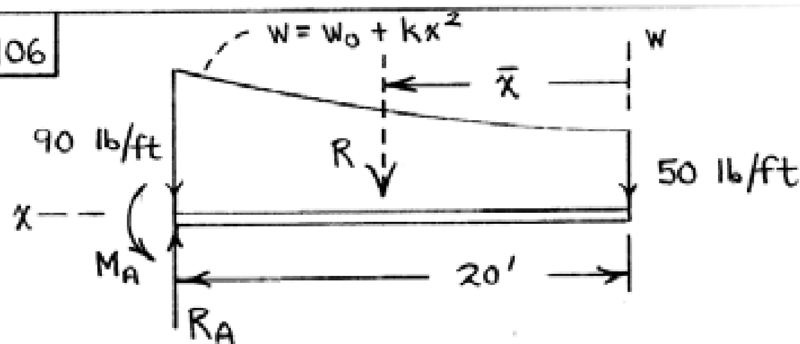
$R_B = 1167$ lb

$\Sigma F = 0$: $R_A - \int dF + 1167 = 0$

$R_A - \int_0^{10} (50x - 2x^2)\,dx + 1167 = 0$

$R_A - 1833 + 1167 = 0$, $R_A = 667$ lb

5/106



At $x = 0$, $w = 50 = w_0$

At $x = 20'$, $w = 90 = 50 + k(20)^2$, $k = \frac{1}{10}$ lb/ft^3

So $w = 50 + \frac{x^2}{10}$ (lb/ft)

$$R = \int w\,dx = \int_0^{20}\left(50 + \frac{x^2}{10}\right)dx = \left[50x + \frac{x^3}{30}\right]_0^{20} = 1267 \text{ lb}$$

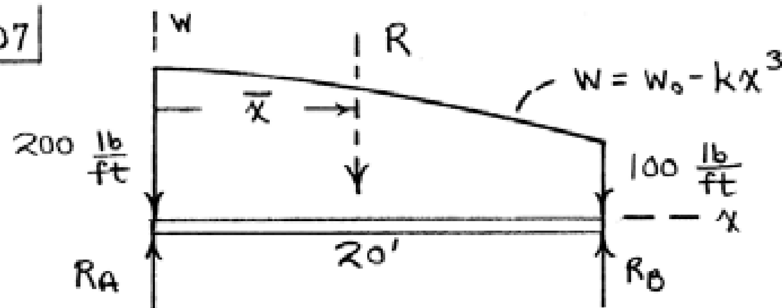
$$\bar{x} = \frac{\int x w\,dx}{R} = \frac{\int x\left(50 + \frac{x^2}{10}\right)dx}{1267} = \frac{\left[\frac{50}{2}x^2 + \frac{x^4}{40}\right]_0^{20}}{1267}$$

$= 11.05$ ft

$+\uparrow \Sigma F = 0$: $R_A - 1267 = 0$, $\underline{R_A = 1267 \text{ lb}}$

$\circlearrowleft+ \Sigma M_A = 0$: $M_A - 1267(20 - 11.05) = 0$, $\underline{M_A = 11.33(10^3) \text{ lb-ft}}$

5/107



At $x = 0$, $w = w_0 = 200$ lb/ft

At $x = 20'$, $w = 200 - k(20)^3 = 100$, $k = \frac{1}{80}\ \frac{\text{lb}}{\text{ft}^4}$

So $w = 200 - x^3/80$ (lb/ft)

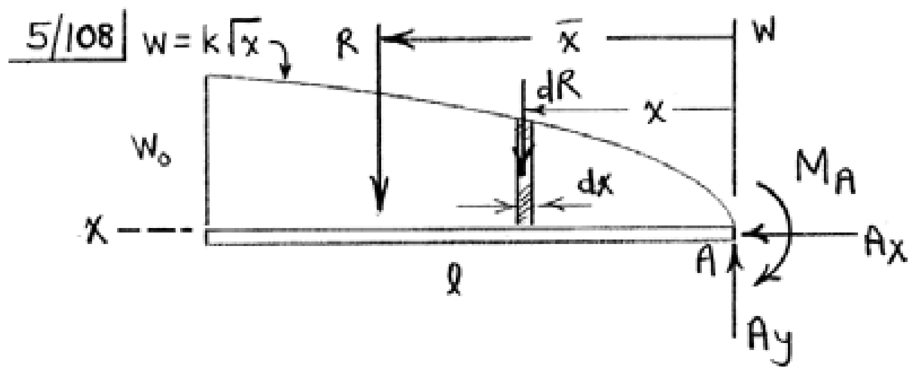
$$R = \int w\,dx = \int_0^{20}\left(200 - \frac{x^3}{80}\right)dx = \left[200x - \frac{x^4}{320}\right]_0^{20} = 3500 \text{ lb}$$

$$\bar{x} = \frac{\int xw\,dx}{R} = \frac{\int_0^{20}\left(200 - \frac{x^3}{80}\right)x\,dx}{3500} = \frac{\left[100x^2 - \frac{x^5}{400}\right]_0^{20}}{3500}$$

$$= 9.14'$$

$\circlearrowleft + \Sigma M_A = 0: \ 20R_B - 3500(9.14) = 0$, $\underline{R_B = 1600 \text{ lb}}$

$+\uparrow \Sigma F = 0: \ R_A + 1600 - 3500 = 0$, $\underline{R_A = 1900 \text{ lb}}$



$w = k\sqrt{x}$ @ left end : $w_0 = k\sqrt{l}$, $k = \frac{w_0}{\sqrt{l}}$

So $w = \frac{w_0}{\sqrt{l}}\sqrt{x}$

$$R = \int dR = \int w\,dx = \int_0^l \frac{w_0}{\sqrt{l}}\sqrt{x}\,dx = \frac{2}{3}w_0 l$$

$$\bar{x} = \frac{\int x\,w\,dx}{\int w\,dx} = \frac{\int_0^l x\,\frac{w}{\sqrt{l}}\sqrt{x}\,dx}{\frac{2}{3}w_0 l} = \frac{3}{5}l$$

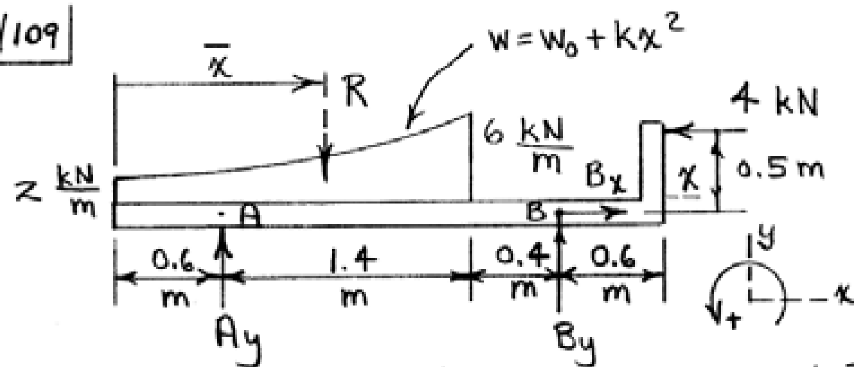
$+\uparrow \Sigma F = 0$: $A_y - \frac{2}{3}w_0 l = 0$, $\underline{A_y = \frac{2}{3}w_0 l}$

$\circlearrowleft + \Sigma M_A = 0$: $\frac{2}{3}w_0 l\left(\frac{3}{5}l\right) - M_A = 0$

$$\underline{M_A = \frac{2}{5}w_0 l^2 \quad \text{CW}}$$

$\overset{+}{\rightarrow} \Sigma F = 0 \Rightarrow \underline{A_x = 0}$

5/109



$w = 2 + kx^2 : \quad 6 = 2 + k(2)^2, \quad k = 1\ \text{kN/m}^3,$

$w = 2 + x^2$

$R = \int w\,dx = \int_0^2 (2 + x^2)\,dx = 2x + \frac{x^3}{3}\Big|_0^2 = 6.67\ \text{kN}$

$\bar{x} = \frac{\int x w\,dx}{R} = \frac{\int_0^2 (2x + x^3)\,dx}{6.67} = \frac{x^2 + \frac{x^4}{4}}{6.67}\Big|_0^2 = 1.2\ \text{m}$

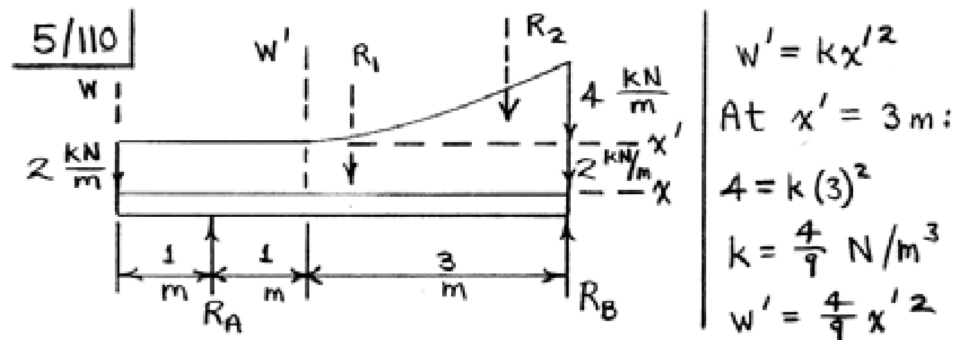
$\Sigma F_x = 0: \quad B_x - 4 = 0, \quad \underline{B_x = 4\ \text{kN}}$

$\circlearrowleft \Sigma M_B = 0: \quad 4(0.5) + 6.67(2.4 - 1.2) - A_y(1.8) = 0$

$\underline{A_y = 5.56\ \text{kN}}$

$\Sigma F_y = 0: \quad 5.56 + B_y - 6.67 = 0, \quad \underline{B_y = 1.111\ \text{kN}}$

5/110



$w' = kx'^2$

At $x' = 3$ m:

$4 = k(3)^2$

$k = \frac{4}{9}$ N/m³

$w' = \frac{4}{9}x'^2$

$$R_2 = \int_0^3 w'dx' = \int_0^3 \frac{4}{9}x'^2 = \frac{4}{9}\frac{x'^3}{3}\Big|_0^3 = 4 \text{ kN}$$

$$\bar{x}_2' = \int x'w'dx'/R_2 = \frac{1}{4}\int_0^3 \frac{4}{9}x'^3 = \frac{1}{4}\frac{4}{9}\frac{x'^4}{4}\Big|_0^3 = 2.25 \text{ m}$$

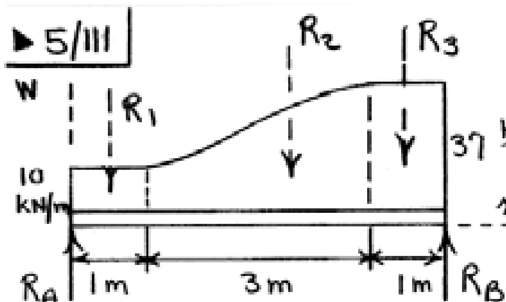
$R_1 = 2(5) = 10$ kN @ $\bar{x}_1 = 2.5$ m | $\bar{x}_2 = 4.25$ m

$$\circlearrowleft + \Sigma M_A = 0: -10(1.5) - 4(3.25) + 4R_B = 0$$

$$\underline{R_B = 7 \text{ kN}}$$

$+\uparrow \Sigma F = 0: \; 7 + R_A - 4 - 10 = 0, \quad \underline{R_A = 7 \text{ kN}}$

▶ 5/111



$w = w_0 + k_1 x + k_2 x^2 + k_3 x^3$

$w(1) = 10 = w_0 + k_1 + k_2 + k_3$

$w(4) = 37 = w_0 + k_1(4) + k_2(4)^2 + k_3(4)^3$

$\frac{dw}{dx} = k_1 + 2k_2 x + 3k_3 x^2$: $\begin{cases} 0 = k_1 + 2k_2(1) + 3k_3(1)^2 \\ 0 = k_1 + 2k_2(4) + 3k_3(4)^2 \end{cases}$

Solve simultaneously to get $w = 21 - 24x + 15x^2 - 2x^3$

$R_2 = \int w\,dx = \int_1^4 (21 - 24x + 15x^2 - 2x^3)\,dx$

$= \left[21x - 12x^2 + 5x^3 - \frac{1}{2}x^4\right]_1^4 = 70.5 \text{ kN}$

$\bar{x}_2 = \frac{1}{R_2}\int x w\,dx = \frac{1}{70.5}\int_1^4 (21 - 24x + 15x^2 - 2x^3)\,x\,dx$

$= \frac{1}{70.5}\left[\frac{21}{2}x^2 - 8x^3 + \frac{15}{4}x^4 - \frac{2}{5}x^5\right]_1^4 = 2.84 \text{ m}$

$R_1 = 10(1) = 10 \text{ kN}$ @ $\bar{x}_1 = 0.5 \text{ m}$

$R_3 = 37(1) = 37 \text{ kN}$ @ $\bar{x}_3 = 4.5 \text{ m}$

$\Sigma M_A = 0$: $5R_B - 10(0.5) - 70.5(2.84) - 37(4.5) = 0$

$\underline{R_B = 74.4 \text{ kN}}$

$\Sigma F = 0$: $74.4 - 10 - 70.5 - 37 + R_A = 0$, $\underline{R_A = 43.1 \text{ kN}}$

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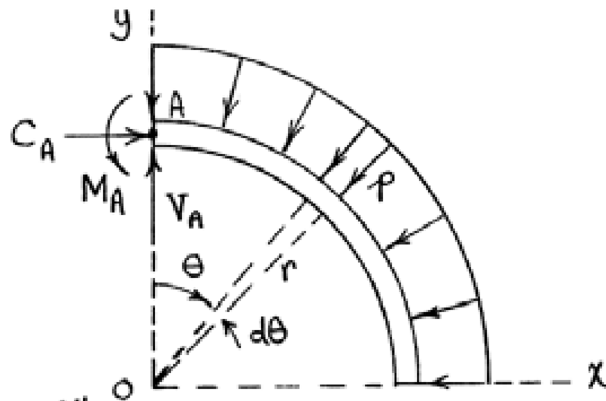

$\Sigma F_x = 0: \quad C_A - \int_0^{\pi/2} p r \, d\theta \sin\theta = 0, \qquad \underline{C_A = pr}$

$\Sigma F_y = 0: \quad V_A - \int_0^{\pi/2} p r \, d\theta \cos\theta = 0, \qquad \underline{V_A = pr}$

$\circlearrowleft \Sigma M_A = 0: \quad M_A - \int_0^{\pi/2} p r \, d\theta \, (r \sin\theta) = 0, \qquad \underline{M_A = pr^2}$

(Alternatively, $\circlearrowleft \Sigma M_O = 0: \quad M_A - C_A r = 0, M_A = pr^2$)

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P, $\frac{2l}{3}$, $\frac{l}{3}$, R_A, R_B

$\circlearrowleft + \Sigma M_A = 0: \quad R_B l - P\frac{2l}{3} = 0$

$R_B = \frac{2}{3}P$

$+\uparrow \Sigma F = 0 \Rightarrow R_A = \frac{1}{3}P$

$0 < x < \frac{2l}{3}:$

x, M, V, $\frac{P}{3}$

$+\uparrow \Sigma F = 0 \Rightarrow V = P/3$

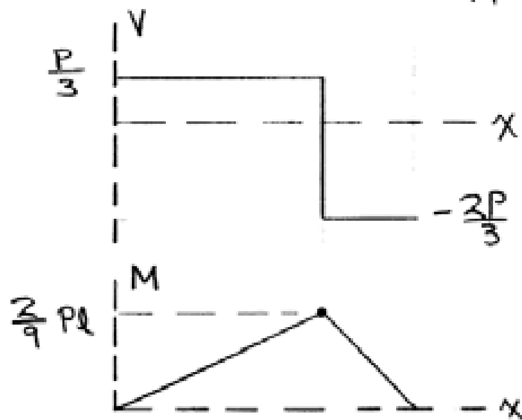
$\Sigma M = 0 \Rightarrow M = \frac{P}{3}x$

$\frac{2l}{3}$, P, x, M, V, $\frac{P}{3}$

$+\uparrow \Sigma F = 0: \quad \frac{P}{3} - P - V = 0, \quad V = -\frac{2P}{3}$

$\circlearrowleft + \Sigma M = 0: \quad M + P\left(x - \frac{2l}{3}\right) - \frac{P}{3}x = 0$

$M = \frac{2P}{3}(l - x)$



At $x = l/2$,

$V = P/3$

$M = \frac{P}{3}\left(\frac{l}{2}\right) = \frac{Pl}{6}$

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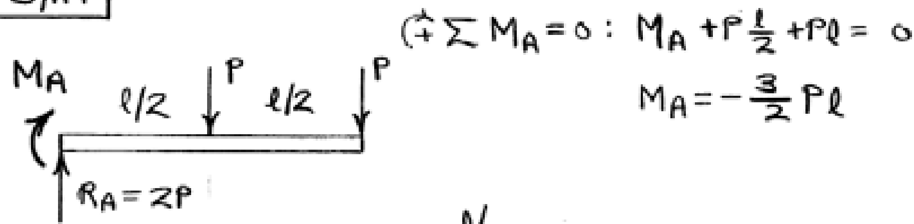
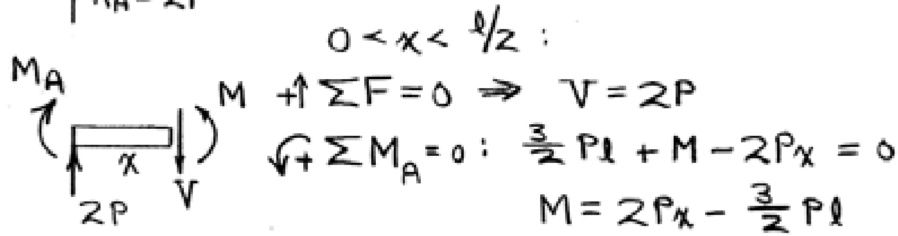


$\circlearrowleft + \Sigma M_A = 0: \quad M_A + P\frac{l}{2} + Pl = 0$

$M_A = -\frac{3}{2}Pl$

$0 < x < l/2:$

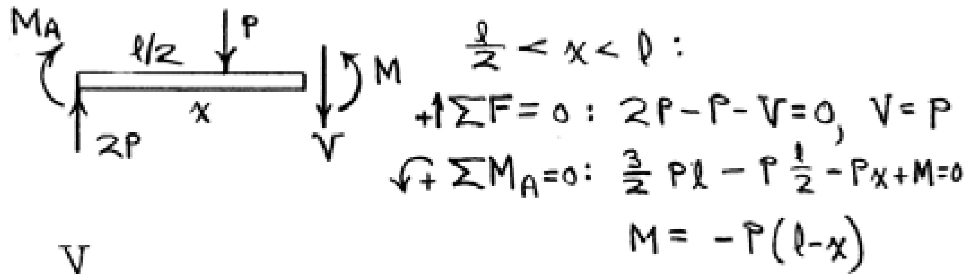


$+\uparrow \Sigma F = 0 \Rightarrow V = 2P$

$\circlearrowleft + \Sigma M_A = 0: \quad \frac{3}{2}Pl + M - 2Px = 0$

$M = 2Px - \frac{3}{2}Pl$

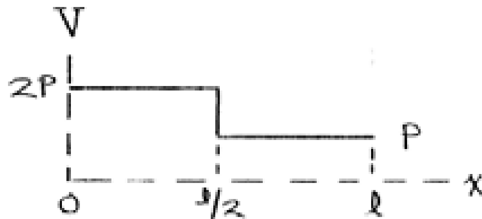
$\frac{l}{2} < x < l:$

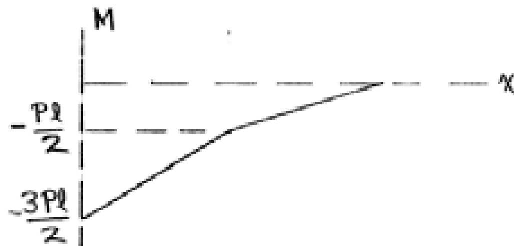


$+\uparrow \Sigma F = 0: \quad 2P - P - V = 0, \quad V = P$

$\circlearrowleft + \Sigma M_A = 0: \quad \frac{3}{2}Pl - P\frac{l}{2} - Px + M = 0$

$M = -P(l - x)$





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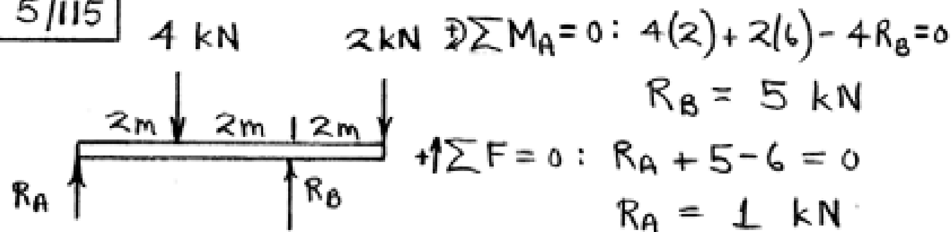
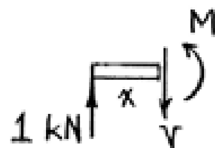


$\circlearrowleft \Sigma M_A = 0:\ 4(2) + 2(6) - 4R_B = 0$

$R_B = 5$ kN

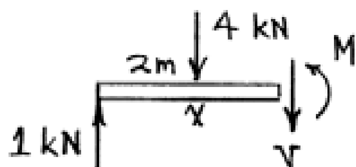
$+\uparrow \Sigma F = 0:\ R_A + 5 - 6 = 0$

$R_A = 1$ kN



$0 < x < 2$m :

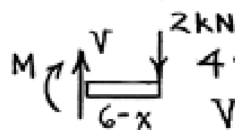
$V = 1$ kN, $M = 1x$



$2 < x < 4$ m :

$V = -3$ kN

$\circlearrowleft + \Sigma M_A = 0:\ -8 + 3x + M = 0,\ M = 8 - 3x$

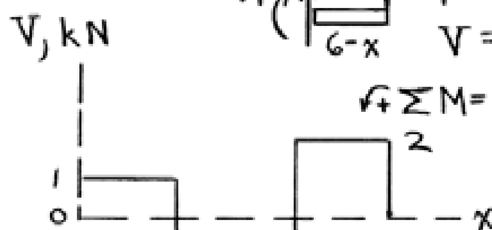


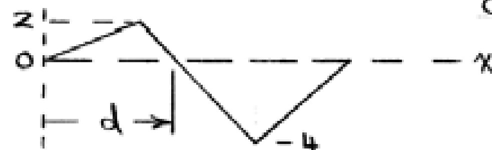
$4 < x < 6$ m :

$V = 2$ kN

$\circlearrowleft + \Sigma M = 0:\ -M - 2(6-x) = 0$

$M = -2(6-x)$





$d = 2 + \frac{1}{3}(2) = \underline{2.67\text{ m}}$

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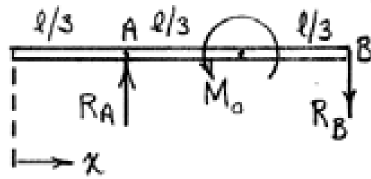


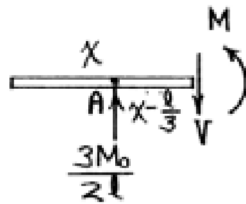
$R_A = R_B$ (load is a couple)

$\circlearrowleft + \Sigma M_A = 0: \quad M_o - R_B\left(\frac{2l}{3}\right) = 0$

$R_B = \frac{3M_o}{2l}$

$x < l/3$:

By inspection, $V = M = 0$

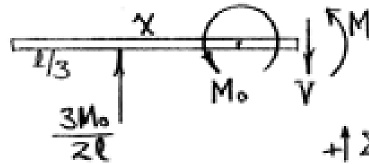


$\frac{l}{3} < x < \frac{2l}{3}$:

$+\uparrow \Sigma F = 0 \Rightarrow \quad V = \frac{3M_o}{2l}$

$\circlearrowleft + \Sigma M = 0: \quad M - \frac{3M_o}{2l}\left(x - \frac{l}{3}\right) = 0$

$M = \frac{3M_o}{2l}\left(x - \frac{l}{3}\right)$

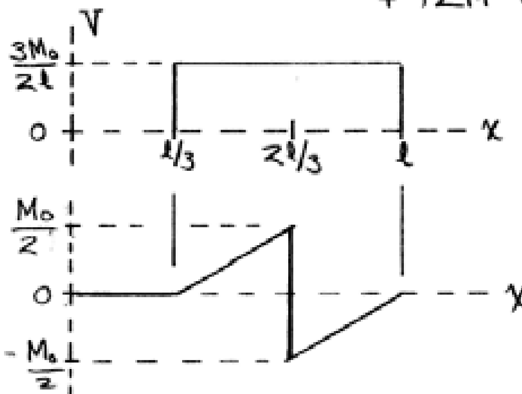


$\frac{2l}{3} < x < l$:

$+\uparrow \Sigma F = 0 \Rightarrow \quad V = \frac{3M_o}{2l}$

$+\circlearrowleft \Sigma M = 0: \ M + M_o - \frac{3M_o}{2l}\left(x - \frac{l}{3}\right) = 0$

$M = \frac{3M_o}{2}\left(\frac{x}{l} - 1\right)$



At $x = l/2$:

$V = \frac{3M_o}{2l}$

$M = \frac{3M_o}{2l}\left(\frac{l}{2} - \frac{l}{3}\right)$

$= \underline{\frac{M_o}{4}}$

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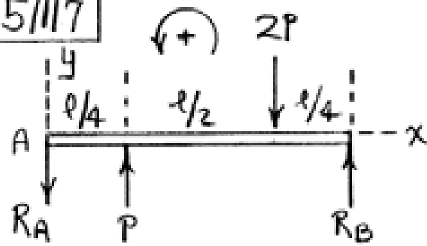


$\Sigma M_A = 0: P\left(\frac{l}{4}\right) - 2P\left(\frac{3l}{4}\right) + R_B(l) = 0$

$R_B = \frac{5P}{4}$

$\Sigma F_y = 0: -R_A + P - 2P + \frac{5}{4}P = 0$

$R_A = \frac{P}{4}$

$x < \frac{l}{4}$:

$\Sigma F_y = 0: -\frac{P}{4} - V = 0, \quad V = -\frac{P}{4}$

$\Sigma M = 0: M + \frac{P}{4}x = 0, \quad M = -\frac{P}{4}x$

$\frac{l}{4} < x < \frac{3l}{4}$:

$\Sigma F_y = 0: -\frac{P}{4} + P - V = 0, \quad V = \frac{3P}{4}$

$\Sigma M = 0: M - P\left(x - \frac{l}{4}\right) + \frac{P}{4}x = 0$

$M = \frac{P}{4}(3x - l)$

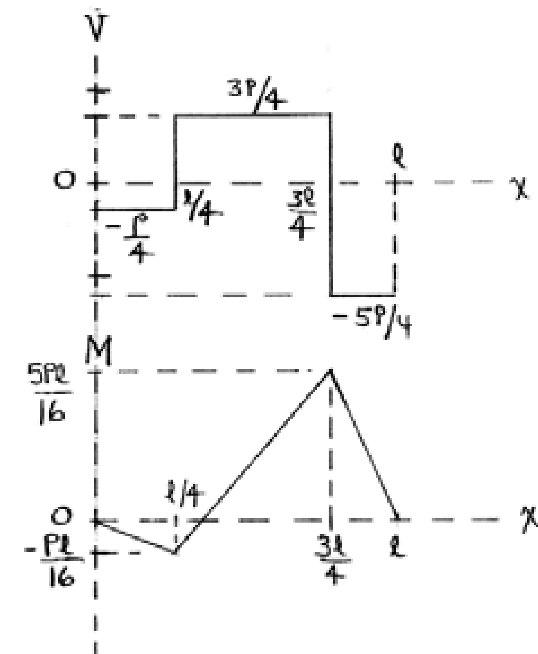
$\frac{3l}{4} < x < l$:

$\Sigma F_y = 0: -\frac{P}{4} + P - 2P - V = 0$

$V = -\frac{5P}{4}$

$\Sigma M = 0: M + 2P\left(x - \frac{3l}{4}\right) - P\left(x - \frac{l}{4}\right) + \frac{P}{4}x = 0$

$M = \frac{5}{4}P(l - x)$



(Note: $V_{max} = \frac{3P}{4}$ on $\frac{l}{4} < x < \frac{3l}{4}$.)

$M_{max} = \frac{5Pl}{16}$ @ $x = \frac{3l}{4}$

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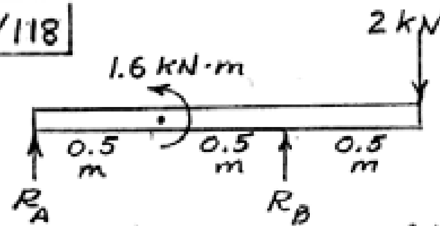
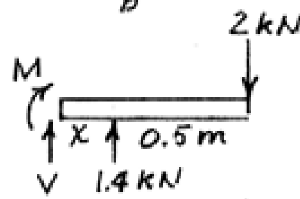


$\Sigma M_A = 0;\ 1.6 + 1.0R_B - 2(1.5) = 0$

$R_B = 1.4$ kN

$\Sigma F = 0;\ R_A = 0.6$ kN



$\Sigma F = 0;\ V + 1.4 - 2 = 0$

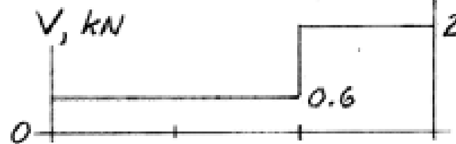
$V = 0.6$ kN

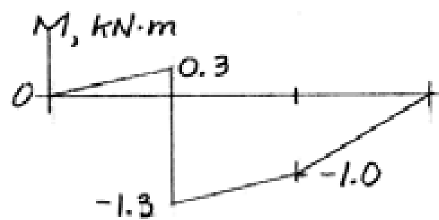
$\Sigma M = 0;\ M + 2(0.5 + x) - 1.4x = 0$

$M = -(1 + 0.6x)$

$(0 < x < 0.5\text{ m})$

(x measured to the left of B)





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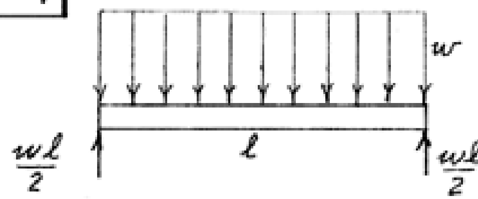
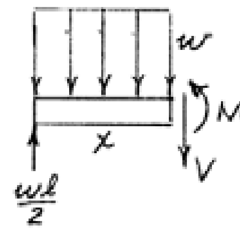




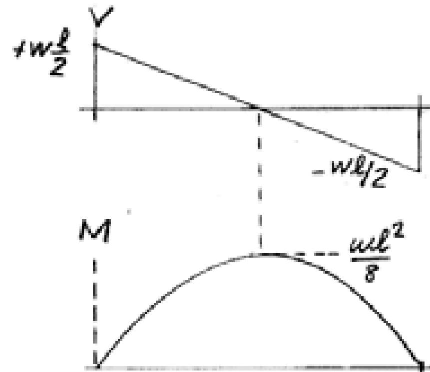
$\Sigma F = 0;\ -V - wx + \frac{wl}{2} = 0$

$V = w\left(\frac{l}{2} - x\right)$

$\Sigma M = 0;\ M + wx\left(\frac{x}{2}\right) - \frac{wlx}{2} = 0$

$M = \frac{wx}{2}(l - x)$

$M_{max} = \frac{wl^2}{8}$



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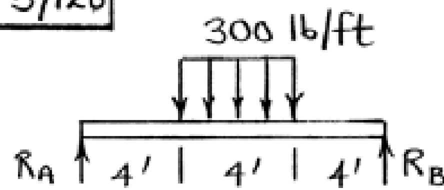
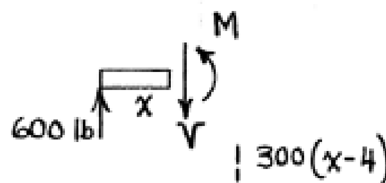


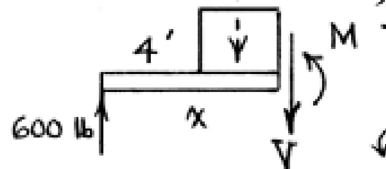
By symmetry,

$R_A = R_B = \frac{1}{2}(300)(4) = 600 \text{ lb}$



$0 < x < 4$ ft :

$V = 600$ lb, $M = 600x$



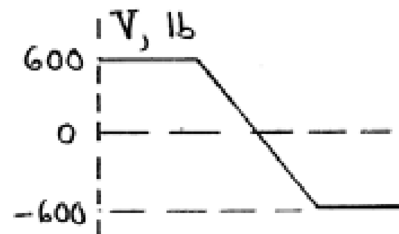
$4 < x < 8$ ft:

$+\uparrow \Sigma F = 0 : 600 - 300(x-4) - V = 0$

$V = 1800 - 300x$

$\circlearrowleft + \Sigma M = 0 : M + 300(x-4)\frac{x-4}{2}$

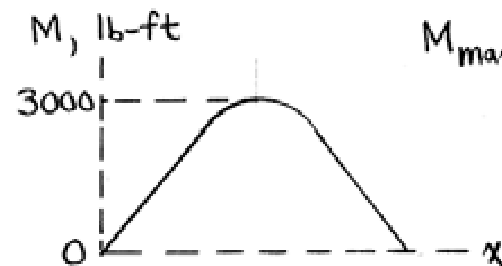
$- 600x = 0, \; M = -150x^2 + 1800x - 2400$



For M_{max},

$\frac{dM}{dx} = -300x + 1800 = 0$

$x = 6$ ft



$M_{max} = -150(6^2) + 1800(6) - 2400$

$= \underline{3000 \text{ lb-ft}}$

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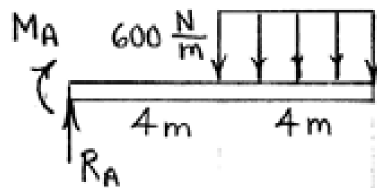
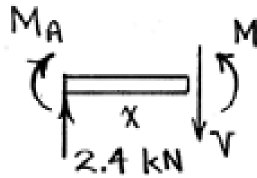


From FBD of entire beam,

$R_A = 2.4$ kN, $M_A = -14.4$ kN·m

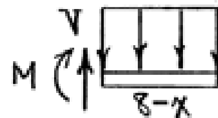


$0 < x < 4$ m :

$+\uparrow \Sigma F = 0 \Rightarrow V = 2.4$ kN

$\circlearrowleft+ \Sigma M_A = 0 : 14.4 + M - 2.4x = 0$

$M = 2.4x - 14.4$



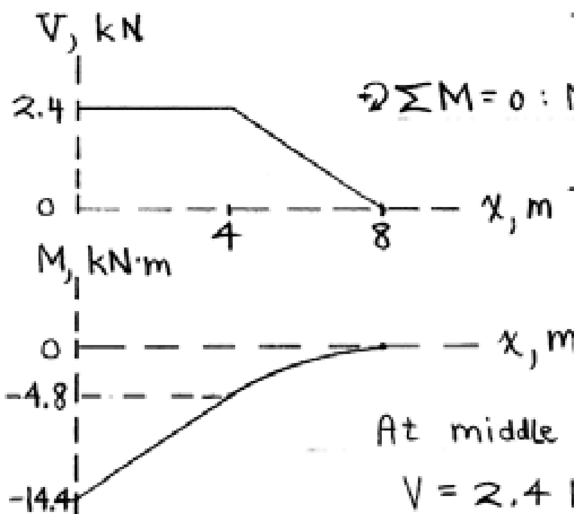
$4 < x < 8$ m :

$+\uparrow \Sigma F = 0 : V - 0.6(8-x) = 0$

$V = 4.8 - 0.6x$ kN

$\circlearrowleft+ \Sigma M = 0 : M + 0.6(8-x)\frac{8-x}{2} = 0$

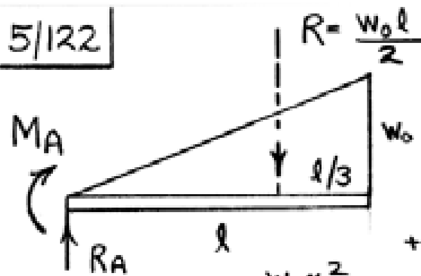
$M = -0.3(8-x)^2$ kN·m



At middle of beam:

$V = 2.4$ kN

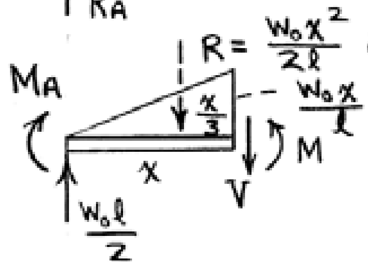
$M = -4.8$ kN·m



$\circlearrowleft + \Sigma M_A = 0: M_A + \frac{w_0 l}{2}\frac{2l}{3} = 0$

$$M_A = -\frac{w_0 l^2}{3}$$

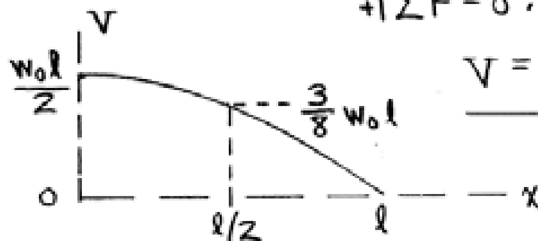
$+\uparrow \Sigma F = 0 \Rightarrow R_A = \frac{w_0 l}{2}$

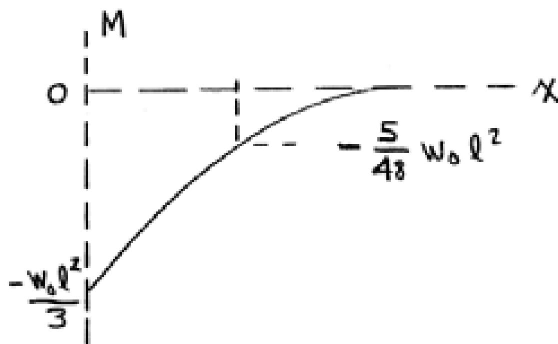


$\circlearrowright + \Sigma M = 0: M + \frac{w_0 x^2}{2l}\frac{x}{3} + \frac{w_0 l^2}{3} - \frac{w_0 l}{2}x = 0, \quad M = \frac{w_0}{6}\left(-2l^2 + 3lx - \frac{x^3}{l}\right)$

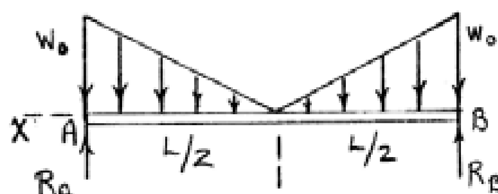
$+\uparrow \Sigma F = 0: \frac{w_0 l}{2} - \frac{w_0 x^2}{2l} - V = 0$

$$V = \frac{w_0 l}{2}\left(1 - \frac{x^2}{l^2}\right)$$





5/123 (See beam element, lower left)



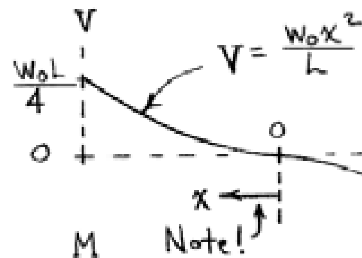
$R_A = R_B = \frac{w_0 L}{4}$

For $x = \frac{L}{2}$, $M = 0$, $V = R_A$

Element (lower left):

$+\uparrow \Sigma F = 0: \quad V - \frac{w_0 x^2}{L} = 0$

$V = \frac{w_0 x^2}{L}$

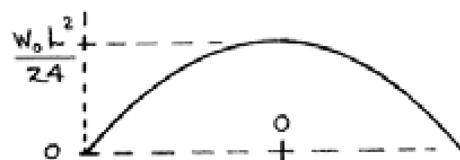


Consider $x = L/2$:

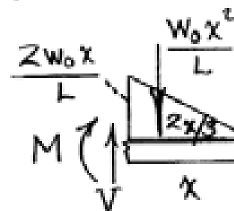
$\circlearrowleft + \Sigma M_C = 0:$

$+M_0 - \frac{w_0 L}{4}\left(\frac{L}{2}\right) + \frac{w_0 \left(\frac{L}{2}\right)^2}{L}\left(\frac{2L}{3}\right) = 0$

$M_0 = \frac{w_0 L^2}{24}$



Consider arbitrary x:



$\circlearrowleft + \Sigma M_0 = 0:$

$-M - \frac{w_0 x^2}{L} x + \frac{w_0 x^2}{L} \frac{2x}{3} + \frac{w_0 L^2}{24} = 0$

$M = \frac{w_0}{3L}\left(\frac{L^3}{8} - x^3\right)$

$M = M_{max}$ @ $x = 0$: $M_{max} = \frac{w_0}{3L}\left(\frac{L^3}{8} - 0\right)$

$= \underline{\frac{w_0 L^2}{24}}$

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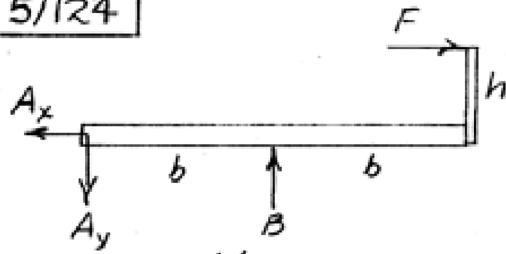
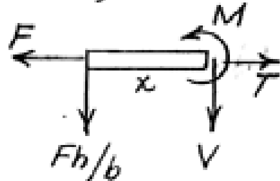


$\Sigma M_A = 0; \quad Fh - Bb = 0$

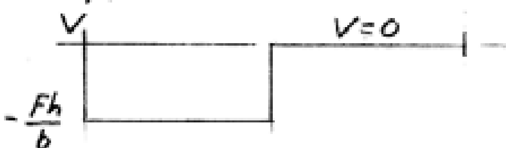
$B = Fh/b$

$\Sigma F_y = 0; \quad A_y = B = Fh/b$



$\Sigma F_y = 0; \quad V = -Fh/b$

$\Sigma M = 0; \quad M + \frac{Fh}{b} x = 0, \quad M = -\frac{Fh}{b} x$



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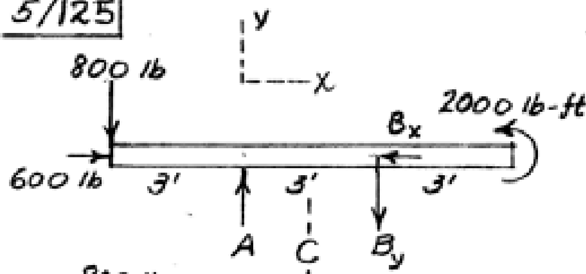
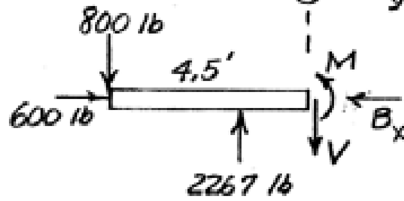




$\Sigma M_A = 0;$

$2000 - 3B_y + 3(800) = 0$

$B_y = 1467$ lb

$\Sigma F_y = 0;$

$A - 1467 - 800 = 0$

$A = 2267$ lb

$\Sigma M_C = 0;$

$M + 800(4.5) - 2267(1.5) = 0$

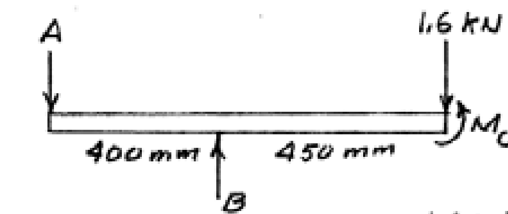
$\underline{M = -200 \text{ lb-ft}}$

$\Sigma F_y = 0;$

$2267 - 800 - V = 0$

$\underline{V = 1467 \text{ lb}}$

5/126



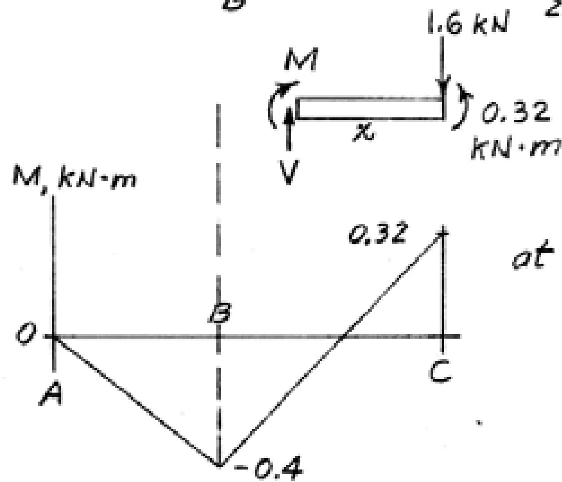
$M_C = 1.6(0.200) = 0.32$ kN·m

$\Sigma M_A = 0$; $0.4B + 0.32 - 0.85(1.6) = 0$

$B = 2.6$ kN

$\Sigma F = 0$; $A + 1.6 - 2.6 = 0$

$A = 1.0$ kN



$\Sigma M = 0$

$0.32 - 1.6x - M = 0$

$M = 0.32 - 1.6x$

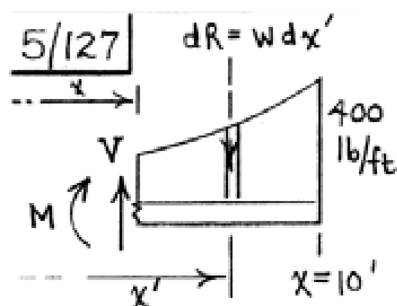
at B, $x = 0.45$ ∦

$M_B = -0.40$ kN·m

$M = 0$ when

$x = 0.32/1.6 = 0.2$ m

5/127



From $w = w_0 + kx^2 = 100 + kx^2$:

$$400 = 100 + k(10)^2$$

$$k = 3 \text{ lb/ft}^3, \quad w = 100 + 3x^2$$

$$+\uparrow \Sigma F = 0: \quad V - \int_x^{10} w\,dx' = 0$$

$$V = \int_x^{10} (100 + 3x'^2)\,dx' = 100x' + x'^3 \Big|_x^{10}$$

$$\underline{V = 2000 - 100x - x^3} \quad \text{(in lb if } x \text{ is in ft)}$$

$$\circlearrowleft \Sigma M = 0: \quad -M - \int_x^{10} (x' - x)\,w\,dx' = 0$$

$$M = -\int_x^{10} (-100x + 100x' + 3x'^3 - 3xx'^2)\,dx'$$

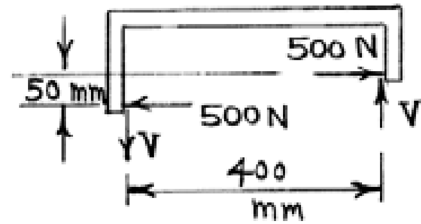
$$= -\left[-100xx' + 50x'^2 - xx'^3 + \frac{3}{4}x'^4\right]\Big|_x^{10}$$

$$= \underline{-12{,}500 + 2000x - 50x^2 - \frac{1}{4}x^4}$$

(in lb-ft if x is in ft)

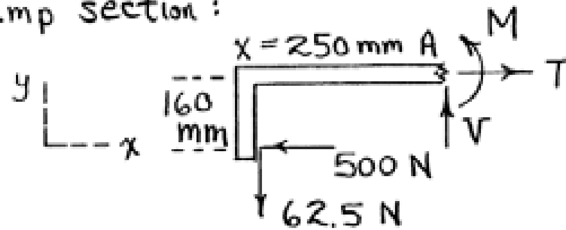
5/128

FBD of clamp:



$\Sigma M = 0: \quad 500(0.050) - V(0.400) = 0, \; V = 62.5 \text{ N}$

FBD of clamp section:



$\Sigma F_x = 0: \quad T - 500 = 0,$ $\underline{T = 500 \text{ N}}$

$\Sigma F_y = 0: \quad V - 62.5 = 0,$ $\underline{V = 62.5 \text{ N}}$

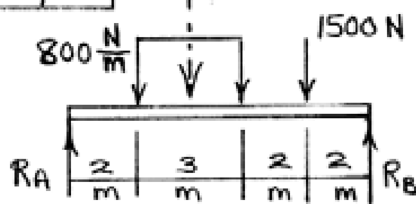
$\Sigma M_A = 0: \quad M + 62.5(0.250) - 500(0.160) = 0$

$\underline{M = 64.4 \text{ N}\cdot\text{m}}$

M is the only quantity which depends on x.

5/129

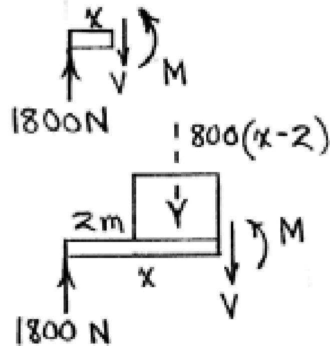
$R = 800(3) = 2400\ \text{N}$



$\Sigma M_A = 0: R_B(9) - 1500(7) - 2400(3.5) = 0,\ R_B = 2100\ \text{N}$

$\Sigma F = 0: R_A - 2400 - 1500 + 2100 = 0$

$R_A = 1800\ \text{N}$



$0 < x < 2\ \text{m}:$

$\Sigma F = 0 \Rightarrow V = 1800\ \text{N}$

$\Sigma M = 0 \Rightarrow M = 1800x$

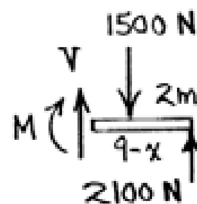
$2 < x < 5\ \text{m}:$

$\Sigma F = 0: 1800 - 800(x-2) - V = 0$

$V = 3400 - 800x$

$\Sigma M = 0: M + 800(x-2)\frac{x-2}{2} + 1800x = 0$

$M = -400x^2 + 3400x - 1600$

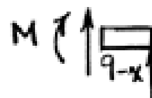


$5 < x < 7:$

$\Sigma F = 0: 2100 - 1500 + V = 0$

$V = -600\ \text{N}$

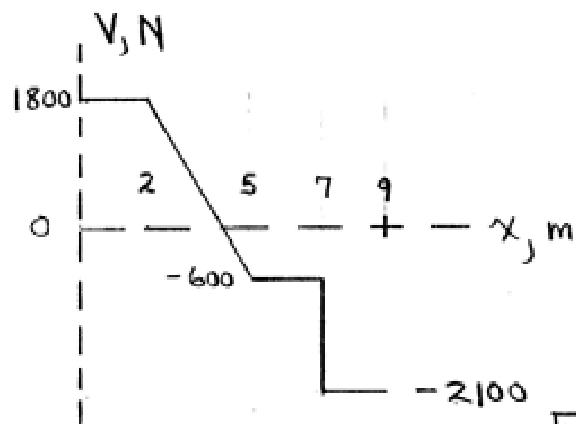
$\Sigma M = 0: -M - 1500(7-x) + 2100(9-x) = 0,\ M = 8400 - 600x$



$7 < x < 9\ \text{m}:$

$\Sigma F = 0 \Rightarrow V = -2100\ \text{N}$

$\Sigma M = 0 \Rightarrow M = 18\,900 - 2100x$



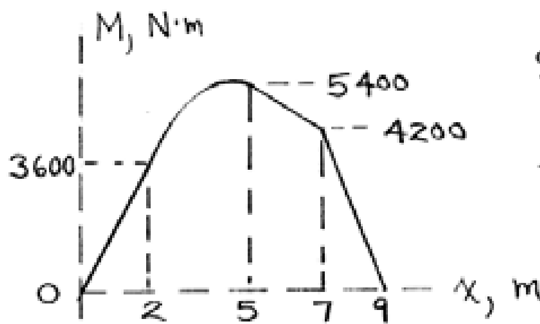
At $x = 6$ m:

$V = -600$ N

$M = 8400 - 600(6)$

$= 4800$ N·m

For M_{max},



$$\frac{dM}{dx} = 0$$

$$\frac{d}{dx}\left(-400x^2 + 3400x - 1600\right)$$

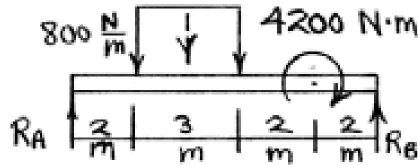
$$= -800x + 3400 = 0$$

$$x = 4.25 \text{ m}$$

$$M_{max} = -400(4.25)^2 + 3400(4.25) - 1600 = 5620 \text{ N·m}$$

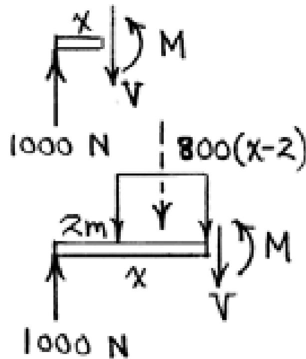
5/130

$R = 800(3) = 2400\ \text{N}$



$\Sigma M_A = 0: \ R_B(9) - 2400(3.5) - 4200 = 0, \ R_B = 1400\ \text{N}$

$\Sigma F = 0 \Rightarrow R_A = 1000\ \text{N}$



$0 < x < 2\ \text{m}:$

$\Sigma F = 0 \Rightarrow V = 1000\ \text{N}$

$\Sigma M = 0 \Rightarrow M = 1000x$

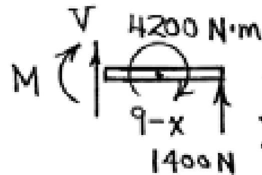
$2 < x < 5\ \text{m}:$

$\Sigma F = 0: \ 1000 - 800(x-2) - V = 0$

$V = 2600 - 800x$

$\Sigma M = 0: \ M + 800(x-2)\frac{x-2}{2} - 1000x = 0$

$M = -400x^2 + 2600x - 1600$

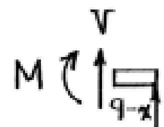


$5 < x < 7\ \text{m}:$

$\Sigma F = 0: \ V + 1400 = 0, \ V = -1400\ \text{N}$

$\Sigma M = 0: \ -M - 4200 + 1400(9-x) = 0$

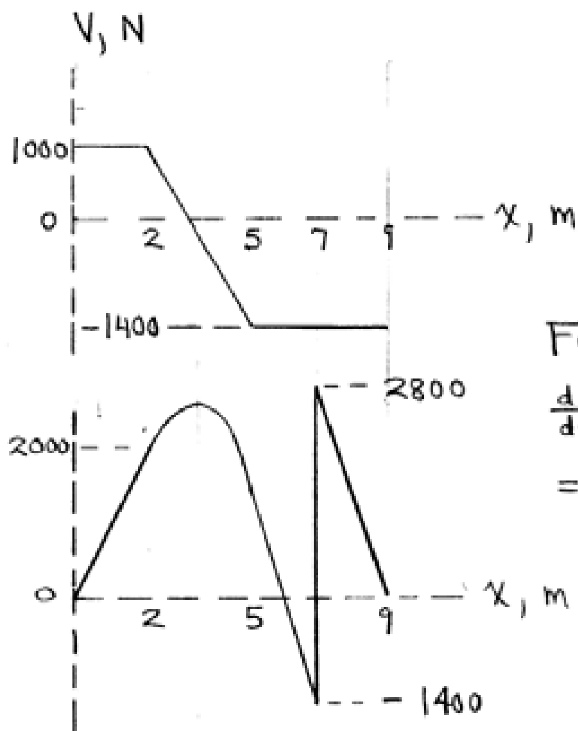
$M = 8400 - 1400x$



$7 < x < 9\ \text{m}:$

$\Sigma F = 0 \Rightarrow V = -1400\ \text{N}$

$\Sigma M = 0 \Rightarrow M = 12600 - 1400x$



At $x = 6$ m,

$V = -1400$ N

$M = 8400 - 1400(6)$

$= 0$

For M_{max}, $\frac{dM}{dx} = 0$

$\frac{d}{dx}\left(-400x^2 + 2600x - 1600\right)$

$= -800x + 2600 = 0$

$x = 3.25$ m

$M_{x=3.25} = -400(3.25)^2 + 2600(3.25) - 1600 = 2625$ N·m

So $M_{max} = 2800$ N·m

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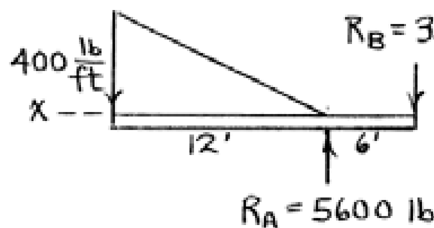
(R_A & R_B from Prob. 5/96)

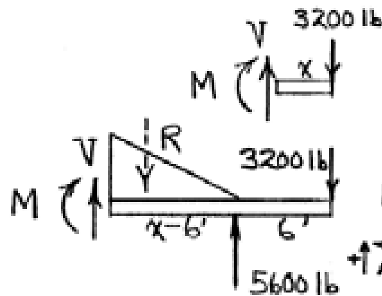


$0 < x < 6'$:

$+\uparrow \Sigma F = 0 \Rightarrow V = 3200$ lb

$\Sigma M = 0$: $M + 3200x = 0$

$M = -3200x$



$6' < x < 18'$:

$$R = \frac{1}{2}\left[\frac{100}{3}(x-6)\right]\left[x-6\right] = \frac{100}{6}(x-6)^2$$

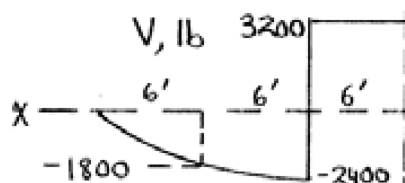
$$+\uparrow \Sigma F = 0: V + 5600 - 3200 - \frac{100}{6}(x-6)^2 = 0$$

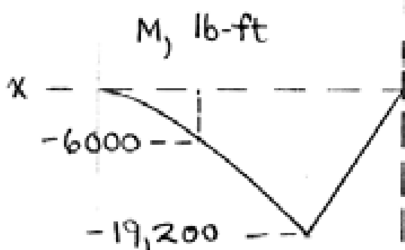
$$V = \frac{100}{6}(x-6)^2 - 2400$$

$$\Sigma M = 0: M + \frac{100}{6}(x-6)^2 \frac{x-6}{3}$$

$$+ 3200(6 + x - 6) - 5600(x-6) = 0$$

$$M = -\frac{50}{9}(x-6)^3 + 2400x - 33{,}600$$





At $x = 12'$, $V = -1800$ lb

$M = -6000$ lb-ft

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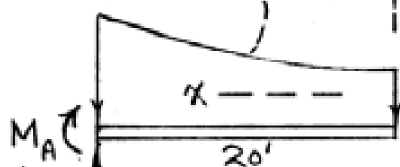
$w = 50 + x^2/10$



$\left\{ w,\ R_A, \text{ and } M_A \text{ from Prob. } 5/106 \right.$

$R_A = 1267$ lb

$M_A = -11.33(10^3)$ lb-ft

$+\uparrow \Sigma F = 0: \quad V - \int w\,dx' = 0$

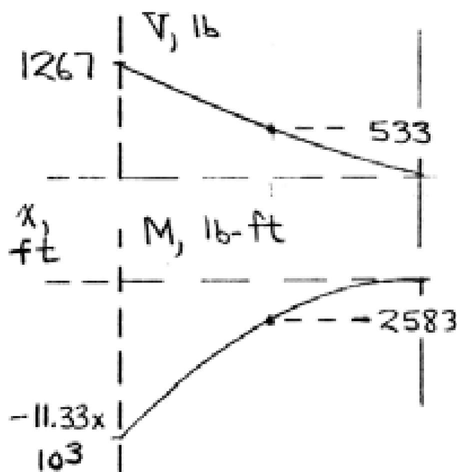
$$V = \int_0^x \left(50 + \frac{x'^2}{10}\right) dx'$$

$$= 50x + \frac{x^3}{30}$$

$$\Sigma M = 0: \quad M + \int_0^x w\,dx'(x - x'), \quad M = -\int_0^x \left(50 + \frac{x'^2}{10}\right)(x - x')\,dx'$$

$$M = -50xx' + 25x'^2 - \frac{xx'^3}{30} + \frac{x'^4}{40} \Big|_0^x$$

$$= -25x^2 - \frac{x^4}{120}$$

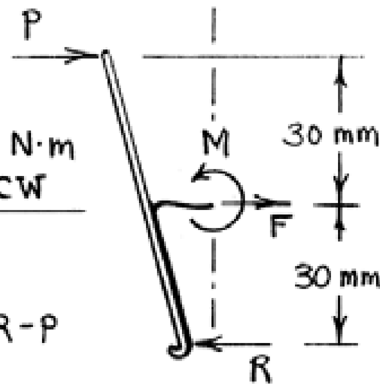


$\left\{ \text{At } x = 10', \right.$

$V = 533$ lb

$M = -2580$ lb-ft

5/133



For $P = 0$, $R = 40$ N and

$\underline{F = 40 \text{ N}}$, $M = 40(0.030) = 1.2$ N·m CCW

For $0 < P < 40$ N

$\Sigma F = 0$: $F - R + P = 0$, $F = R - P$

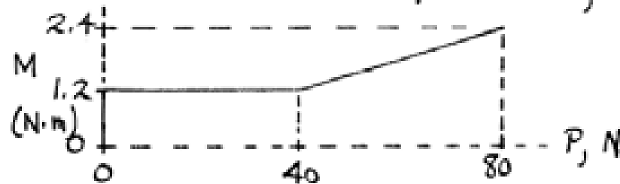
$\Sigma M_C = 0$: $\underline{M = 1.2 \text{ N·m}}$, so

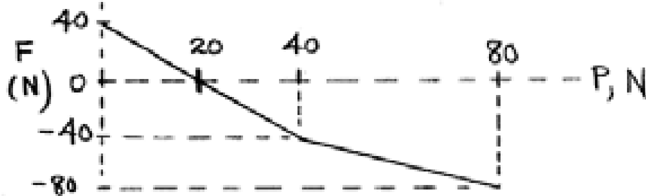
$(P + R)(0.030) - 1.2 = 0$, $P + R = 40$ N

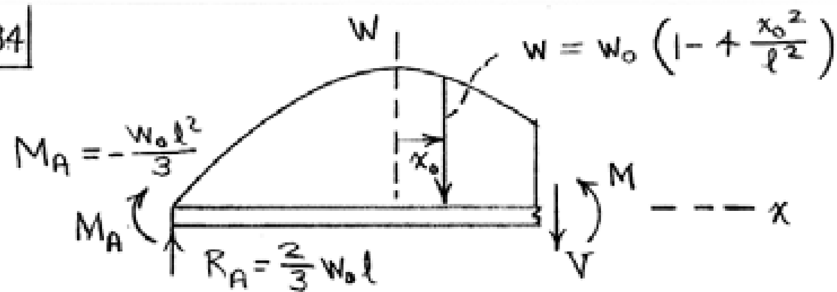
So $\underline{F = 40 - 2P}$

For $P > 40$ N, $R = 0$, $\underline{F = -P}$

$\Sigma M_C = 0$: $P(0.030) - M = 0$, $\underline{M = 0.030P}$







(R_A and M_A from Prob. 5/104)

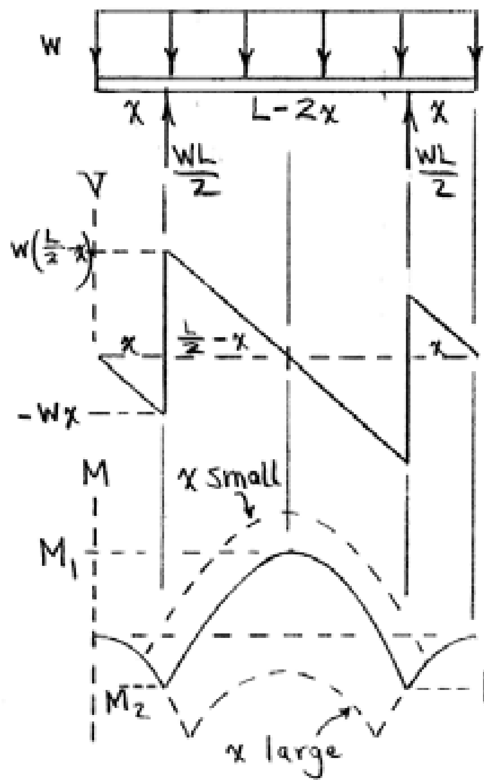
$$+\uparrow\Sigma F = 0: \quad \frac{2}{3}w_0 l - \int_{-l/2}^{x} w_0\left(1 - 4\frac{x_0^2}{l^2}\right)dx_0 - V = 0$$

$$\underline{V = w_0\left(\frac{l}{3} - x + \frac{4x^3}{3l^2}\right)}$$

$$\circlearrowleft+\Sigma M = 0: \quad M + \int_{-l/2}^{x} w_0\left(1 - 4\frac{x_0^2}{l^2}\right)(x - x_0)\,dx_0$$

$$+ \frac{w_0 l^2}{3} - \frac{2}{3}w_0 l\left(\frac{l}{2} + x\right) = 0, \quad \underline{M = w_0\left(-\frac{l^2}{16} + \frac{xl}{3} - \frac{x^2}{2} + \frac{x^4}{3l^2}\right)}$$

▶5/135 From areas under shear diagram:



$$|M_2| = \tfrac{1}{2}wx^2$$

$$|M_1| = \frac{w}{2}\left(\frac{L}{2}-x\right)^2 - \frac{1}{2}wx^2$$

$$= \frac{wL}{2}\left(\frac{L}{4}-x\right)$$

$|M|_{min}$ occurs when $|M_1| = |M_2|$. So

$$\frac{wL}{2}\left(\frac{L}{4}-x\right) = \frac{1}{2}wx^2$$

or $x^2 + Lx - \frac{L^2}{4} = 0$

$$\Rightarrow x = \frac{L}{2}\left(-1 \pm \sqrt{2}\right)$$

(−) sign : irrelevant

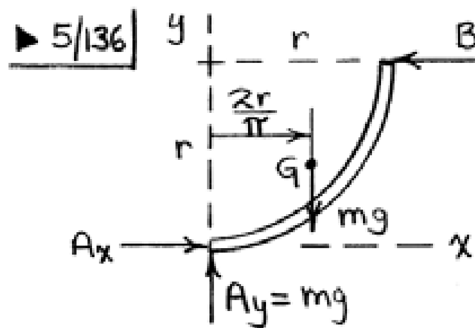
(+) sign : $\underline{x = 0.207L}$

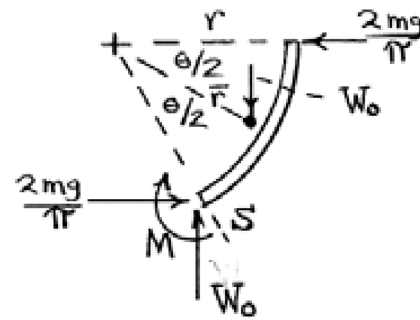
$M_{max} = |M_1|$

$$= \frac{1}{2}w\,(0.207L)^2$$

$$= \underline{0.0214wL^2}$$

▶ 5/136





As a whole : $\Sigma M_A = 0$: $Br - mg\frac{2r}{\pi} = 0$, $B = \frac{2mg}{\pi}$

Section : $W_o = \frac{\theta}{\pi/2}\, mg = \frac{2\theta}{\pi}\, mg$, $\bar{r} = r\,\frac{\sin\frac{\theta}{2}}{\theta/2}$

$\Sigma M_S = 0$: $M + \frac{2\theta}{\pi}\, mg\left(r\,\frac{\sin\frac{\theta}{2}}{\theta/2}\cos\frac{\theta}{2} - r\cos\theta\right)$

$- \frac{2mg}{\pi}\, r\sin\theta = 0$, $\underline{M = \frac{2mgr}{\pi}\,\theta\cos\theta}$

$C = W_o\cos\theta + \frac{2mg}{\pi}\sin\theta$, $\underline{C = \frac{2mg}{\pi}(\theta\cos\theta + \sin\theta)}$

$V = W_o\sin\theta - \frac{2mg}{\pi}\cos\theta$, $\underline{V = \frac{2mg}{\pi}(\theta\sin\theta - \cos\theta)}$

5/137 Given:

$$\begin{cases} 2s = 50 \text{ ft}, \quad s = 25 \text{ ft} \\ \mu = \frac{0.1}{50} = 0.002 \text{ lb/ft} \\ T = 10 \text{ lb} \end{cases}$$

$$T^2 = T_o^2 + \mu^2 s^2 : \quad 10^2 = T_o^2 + (0.002 \cdot 25)^2$$

$$T_o = 9.999875 \text{ lb}$$

Eq. 5/22: $T = T_o + \mu y$

$$10 = 9.999875 + 0.002y$$

$$\underline{y = 0.0625 \text{ ft} \quad (0.750 \text{ in.})}$$

*5/138 Eq. 5/19 at B:

$$5 = \frac{T_o}{\mu}\left[\cosh\frac{\mu(65)}{T_o} - 1\right]$$

Numerical solution with $\mu = 14(9.81) = 137.3$ N/m:

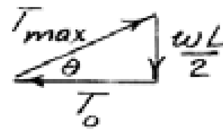
$T_o = \underline{58\,100 \text{ N} = T_A}$

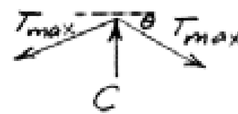
Then Eq. 5/22 gives

$T_B = 58\,100 + 137.3(5) = \underline{58\,800 \text{ N}}$

5/139 $L = 4200$ ft, $h = 470$ ft, $w = \frac{21,300}{2} = 10650$ lb/ft for each cable

$$T_o = \frac{wL^2}{8h} = \frac{10650(4200)^2}{8(470)} = \underline{50.0(10^6)\text{ lb}}$$





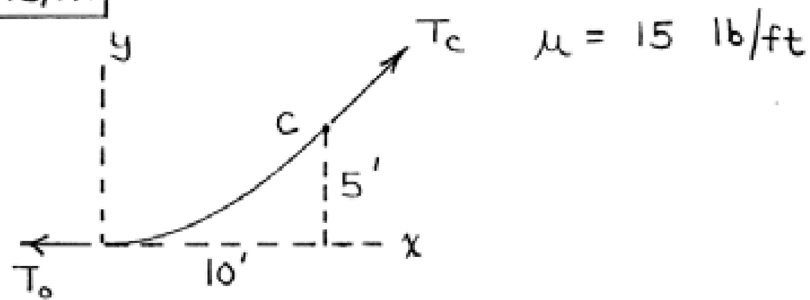
$C = 2T_{max}\sin\theta$

$= 2\left(\frac{wL}{2}\right) = wL$

$= 10\,650(4200)$

$= \underline{44.7(10^6)\text{ lb}}$

*5/140



$\mu = 15$ lb/ft

Eq. 5/19 evaluated at point C:

$$5 = \frac{T_0}{\mu} \cosh\left[\frac{10}{T_0/\mu} - 1\right]$$

Numerical solution: $\frac{T_0}{\mu} = 10.74$ ft

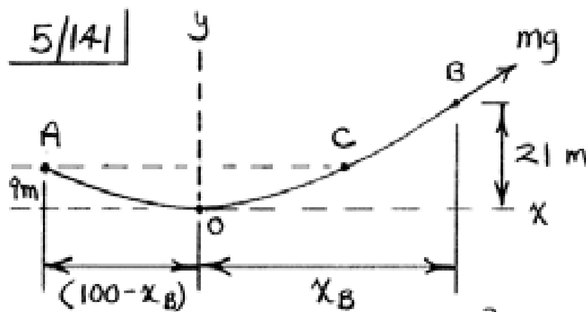
Then $T_0 = 10.74(15) = 161.1$ lb

Eq. 5/22 evaluated at C:

$T_C = 161.1 + 15(5) = \underline{236 \text{ lb}}$

Eq. 5/20 : $s = 10.74 \sinh \frac{10}{10.74} = 11.51$ ft

$L = 2s = \underline{23.0 \text{ ft}}$



Eq. 5/14 : $y = \frac{wx^2}{2T_0}$

At B : $21 = \frac{wx_B^2}{2T_0}$

At A : $9 = \frac{w(100-x_B)^2}{2T_0}$

Eliminate T_0 to obtain $x_B^2 - 350x_B + 17\,500 = 0$

Quadratic solution : $x_B = 290$ m (reject), 60.4 m

From Eq. 5/14 @ B : $21 = \frac{25(60.4)^2}{2T_0}$, $T_0 = 2170$ N

For section OB : $(mg)^2 = T_0^2 + (wx_B)^2$

$m^2(9.81)^2 = 2170^2 + (25 \cdot 60.4)^2$, $\underline{m = 270 \text{ kg}}$

Parabolic equation $y = \frac{25x^2}{2(2170)} = 0.00575x^2$

At C, $y_C = 9$ m : $9 = 0.00575x_C^2$, $x_C = 39.6$ m

$\overline{AC} = 2x_C = \underline{79.1 \text{ m}}$

*5/142 Please refer to the diagram in the solution to Prob. 5/141. Eq. 5/19 :

$$y = \frac{T_o}{\mu}\left[\cosh\frac{\mu x}{T_o} - 1\right]$$

At B : $21 = \frac{T_o}{25}\left[\cosh\frac{25x_B}{T_o} - 1\right]$

At A : $9 = \frac{T_o}{25}\left[\cosh\frac{25}{T_o}(100 - x_B) - 1\right]$

Solve these two equations numerically to obtain

$$T_o = 2240 \text{ N}, \quad x_B = 60.2 \text{ m}$$

Eq. 5/20 : $s = \frac{T_o}{\mu}\sinh\frac{\mu x}{T_o}$

At B: $s_B = \frac{2240}{25}\sinh\left(\frac{25 \cdot 60.2}{2240}\right) = 64.8 \text{ m}$

Equilibrium of section OB :

$(mg)^2 = T_o^2 + (\mu s_B)^2 : m^2(9.81)^2 = 2240^2 + (25 \cdot 64.8)^2$

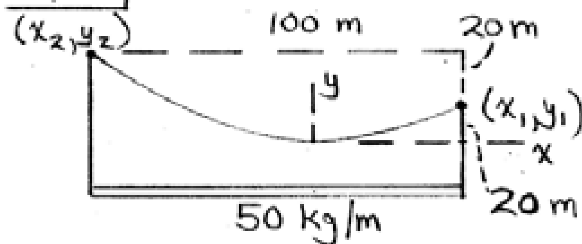
$m = 282 \text{ kg}$

Eq. 5/19 @ C : $9 = \frac{2240}{25}\left[\cosh\frac{25x_C}{2240} - 1\right]$

Numerical solution : $x_C = 39.8$ m

$\overline{AC} = 2x_C = 79.6 \text{ m}$

5/143



Eq. 5/14: $y = \frac{wx^2}{2T_0}$

$20 = \frac{50(9.81)x_1^2}{2T_0}$, $40 = \frac{50(9.81)x_2^2}{2T_0}$

Also, $x_1 + (-x_2) = 100$ m

Solve simultaneously: $x_1^2 + 200x_1 - 10\,000 = 0$

$x_1 = 41.4$ m (or -241 m)

$x_2 = x_1 - 100 = 41.4 - 100 = -58.6$ m

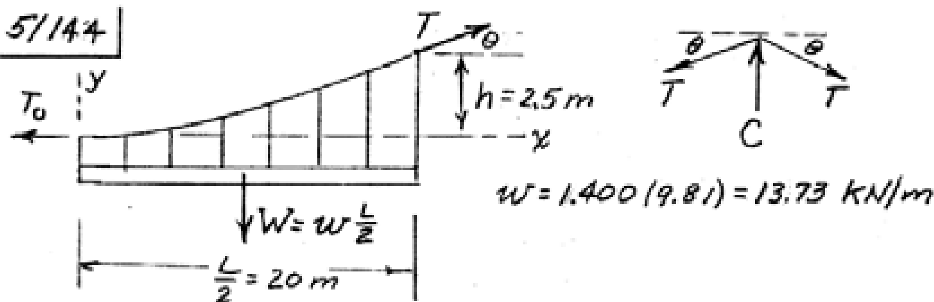
$T_0 = \frac{wx^2}{2y} = \frac{50(9.81)(41.4)^2}{2(20)} = \underline{21.0(10^3)\text{ N}}$

Maximum tension is $T_{max} = \sqrt{T_0^2 + (wx_2)^2}$

$= \sqrt{[21.0(10^3)]^2 + [50(9.81)(58.6)]^2} = 35.6(10^3)$ N

or $\underline{T_{max} = 35.6 \text{ kN}}$

5/144



$w = 1.400(9.81) = 13.73$ kN/m

From Eq. 5/15b, $T = \frac{wL}{2}\sqrt{1 + (L/4h)^2}$

$= \frac{13.73(40)}{2}\sqrt{1 + \left[\frac{40}{4(2.5)}\right]^2} = 1133$ kN

$T^2 = W^2 + T_0^2$, $T_0 = \sqrt{(1132)^2 - [(13.73)(20)]^2} = 1099$ kN

$\frac{dy}{dx} = \tan\theta = \frac{w(L/2)}{T_0} = \frac{13.73(20)}{1099} = 0.250$, $\theta = 14.04°$

$\Sigma F_y = 0$ at support; $2T\sin\theta - C = 0$

$C = 2(1133)\sin 14.04° = \underline{549 \text{ kN}}$

5/145 $w = w_o \left(1 - \frac{x}{l}\right)$

From Eq. 5/13, $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{w_o}{T_o}\left(1 - \frac{x}{l}\right)$

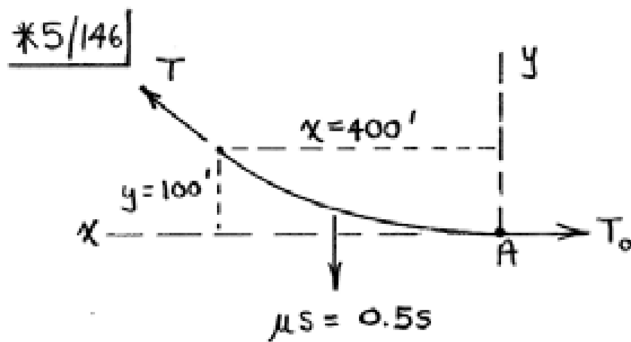
so $\frac{dy}{dx} = \frac{w_o}{T_o}\left(x - \frac{x^2}{2l}\right) + C_1$, $C_1 = 0$ since $\frac{dy}{dx} = 0$ at $x = 0$

& $y = \frac{w_o}{T_o}\left(\frac{x^2}{2} - \frac{x^3}{6l}\right) + C_2$, $C_2 = 0$ since $y = 0$ at $x = 0$

For $y = h$ & $x = l$, $T_o h = w_o\left(\frac{l^2}{2} - \frac{l^2}{6}\right)$, $T_o = \frac{w_o l^2}{3h}$

Thus $y = \frac{3hx^2}{2l^2}\left(1 - \frac{x}{3l}\right)$

*5/146



Catenary Eq. 5/19 : $y = \frac{T_0}{\mu}\left(\cosh\frac{\mu x}{T_0} - 1\right)$

$$100 = \frac{T_0}{0.5}\left(\cosh\frac{0.5(400)}{T_0} - 1\right)$$

Solve numerically to obtain $\underline{T_0 = 408 \text{ lb}}$

Parabolic Eq. 5/14 : $y = \frac{wx^2}{2T_0} \cong \frac{\mu x^2}{2T_0}$

So $T_0 \cong \frac{\mu x^2}{2y} = \frac{0.5(400)^2}{2(100)} = \underline{400 \text{ lb}}$

5/147 $w = a + bx^2$, when $x = 0$, $w = w_0$
$x = L/2$, $w = w_1$

So $a = w_0$ & $b = \frac{4}{L^2}(w_1 - w_0)$; Thus $w = w_0 + \frac{4(w_1 - w_0)}{L^2} x^2$

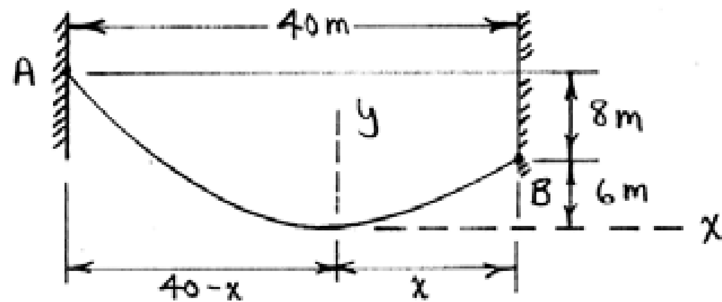
From Eq. 5/13, $\frac{dy}{dx} = \frac{1}{T_0} \int_0^x w \, dx$

$$= \frac{1}{T_0}\left[w_0 x + \frac{4(w_1 - w_0)}{L^2}\frac{x^3}{3}\right]$$

& $y = \frac{w_0 x^2}{2T_0} + \frac{w_1 - w_0}{3T_0 L^2} x^4$; Thus for $x = L/2$, $y = h$

& $h = \frac{w_0 L^2}{8T_0} + \frac{w_1 - w_0}{3T_0 L^2}\frac{L^4}{16} = \frac{L^2}{48T_0}(5w_0 + w_1)$

*5/148



Eq. 5/19 : $y = \frac{T_0}{\mu}\left(\cosh\frac{\mu x}{T_0} - 1\right)$

At B: $6 = \frac{T_0}{\mu}\left(\cosh\frac{x}{T_0/\mu} - 1\right)$ (1)

At A: $14 = \frac{T_0}{\mu}\left(\cosh\frac{x-40}{T_0/\mu} - 1\right)$ (2)

Solve Eqs. (1) & (2) numerically to obtain

$x = 16.07$ m, $\frac{T_0}{\mu} = 22.5$ m

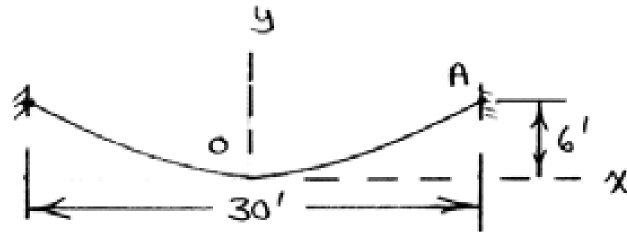
Eq. 5/20 : $s = \frac{T_0}{\mu}\sinh\frac{x}{T_0/\mu}$

At B: $s_B = 22.5 \sinh\frac{16.07}{22.5} = 17.48$ m

At A: $s_A = 22.5 \sinh\frac{40-16.07}{22.5} = 28.7$ m

$L = s_A + s_B = 28.7 + 17.48 = \underline{46.2 \text{ m}}$

*5/149



Eq. 5/19, from O to A : $6 = \frac{T_o}{\mu}\left[\cosh \frac{\mu}{T_o}(15) - 1\right]$

By numerical or graphical means, $\frac{T_o}{\mu} = 19.68$ m

Eq. 5/20: $s = \frac{L}{2} = \frac{T_o}{\mu} \sinh \frac{\mu x}{T_o}$

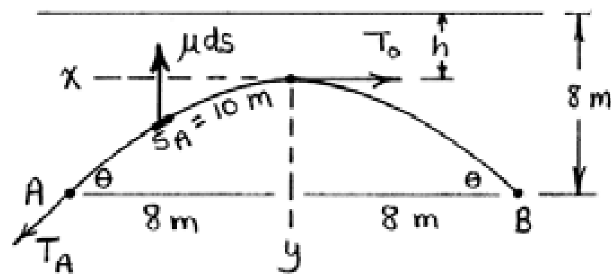
$\therefore L = 2\left(\frac{T_o}{\mu}\right) \sinh \frac{\mu}{T_o} 15$

$= 2(19.68) \sinh \frac{15}{19.68} = \underline{33.0 \text{ ft}}$

*5/150

$\mu = 560 - 100$

$= 460$ N/m



Eq. 5/20: $s = \frac{T_o}{\mu} \sinh \frac{\mu x}{T_o}$: $10 = \frac{T_o}{460} \sinh \frac{460(8)}{T_o}$

Numerical or graphical solution: $T_o = 3110$ N

Eq. 5/21 : $T = T_o \cosh \frac{\mu x}{T_o} = 3110 \cosh \frac{460(8)}{3110}$

$T_A = 5550$ N

Eq. 5/22 : $T = T_o + \mu y$ at A: $5550 = 3110 + 460y$

$y = 5.31$ m

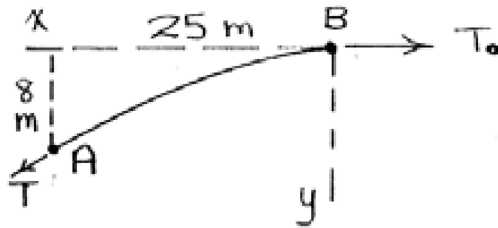
Then $h = 8 - 5.31 = \underline{2.69 \text{ m}}$

From Eq. 5/19, $\frac{dy}{dx} = \tan\theta = \sinh \frac{\mu x}{T_o}$

$\tan\theta = \sinh \frac{460(8)}{3110} = \underline{55.9°}$

*5/151

$\mu = 30$ N/m



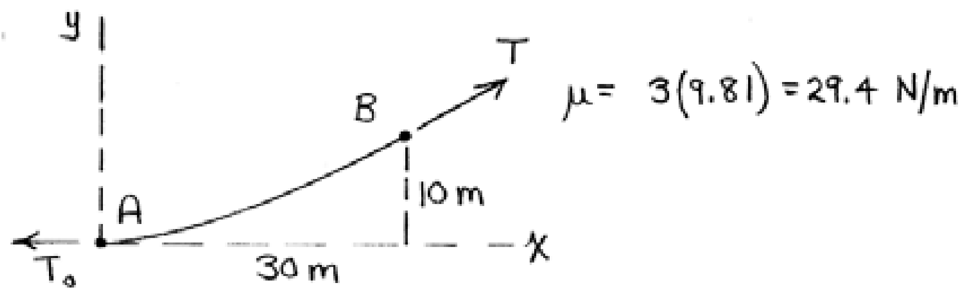
Eq. 5/19: $y = \frac{T_0}{\mu}\left[\cosh \frac{\mu x}{T_0} - 1\right]$

At A: $8 = \frac{T_0}{\mu}\left[\cosh \frac{25\mu}{T_0} - 1\right]$

Numerical solution: $\frac{T_0}{\mu} = 40.3$ m

$T_0 = 40.3(30) = \underline{1210 \text{ N}}$

*5/152



$\mu = 3(9.81) = 29.4$ N/m

Eq. 5/19: $10 = \frac{T_0}{\mu}\left[\cosh\frac{\mu 30}{T_0} - 1\right]$

Solve numerically to obtain $\frac{T_0}{\mu} = 46.6$ N

$T_0 = 46.6(29.4) = 1371$ N

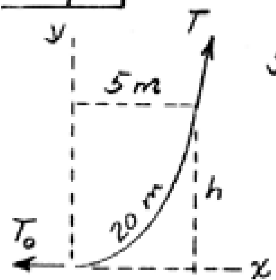
Eq. 5/22: $T = T_0 + \mu y = 1371 + 29.4(10)$

$= \underline{1665 \text{ N}}$

Eq. 5/20: $s = \frac{T_0}{\mu}\sinh\frac{\mu x}{T_0} = 46.6 \sinh\frac{30}{46.6}$

$= \underline{32.1 \text{ m}}$

∗5/153



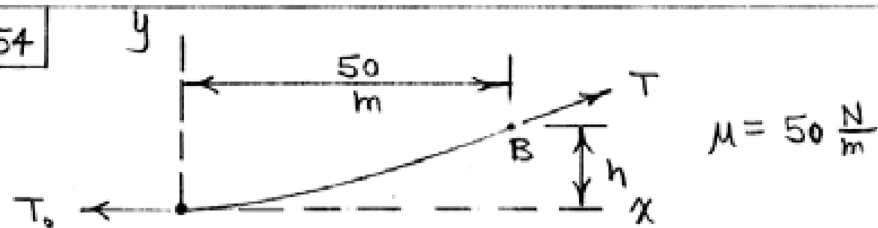
Eq. 5/20, $20 = \frac{T_0}{\mu} \sinh \frac{5\mu}{T_0}$

Solve by computer or graphically & get $T_0/\mu = 1.532$ m

Eq. 5/19, $y = 1.532\left(\cosh \frac{5}{1.532} - 1\right)$

$h = y = 1.532\,(13.09 - 1) = \underline{18.53 \text{ m}}$

*5/154



Eliminate T_0 between Eqs. 5/21 & 5/22 to get $T = (T - \mu y)\cosh\frac{\mu x}{T - \mu y}$

At B: $T = (T - 50h)\cosh\frac{50(50)}{T - 50h}$

or $T = \beta\cosh\frac{2500}{\beta}$ $\quad (\beta = T - 50h)$

For minimum T, $\frac{dT}{d\beta} = \cosh\frac{2500}{\beta} - \frac{1}{\beta}\sinh\frac{2500}{\beta} = 0$

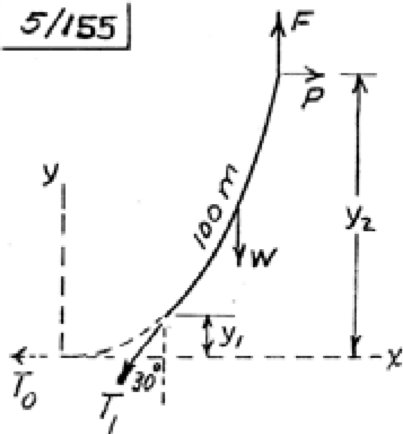
Numerically solve: $\beta = 2084$ N

$T = 2084\cosh\frac{2500}{2084} = 3772$ N

$\beta = T - 50h$: $\quad 2084 = 3772 - 50h$, $\underline{h = 33.8\text{ m}}$

(Note: $\beta = T_0$!)

5/155



$W = 0.51(9.81)(100) = 500.3 \text{ N}$

$T_1 = M/r = 400/\frac{0.5}{2} = 1600 \text{ N}$

$\Sigma F_x = 0;\ P - 1600 \sin 30° = 0$

$P = 800 \text{ N} \quad (= T_0)$

$\Sigma F_y = 0;\ F - 500.3 - 1600 \cos 30° = 0$

$F = 1886 \text{ N}$

$T = \sqrt{F^2 + P^2} = \sqrt{(1886)^2 + (800)^2}$

$= 2049 \text{ N}$

From Eq. 5/22, $T = T_0 + \mu y$, $2049 = 800 + 5.003\, y_2$

$y_2 = 249.6 \text{ m}$

Also, $1600 = 800 + 5.003\, y_1$

$y_1 = 159.9 \text{ m}$

$H = y_2 - y_1 = \underline{89.7 \text{ m}}$

*5/156 (a) Use $w = \mu = 1.2(9.81) = 11.77$ N/m

Eq. 5/14 : $y = \frac{wx^2}{2T_0}$ @ A : $2.4 = \frac{11.77(5)^2}{2T_0}$

$T_0 = 61.3$ N

So $y_P = \frac{11.77x^2}{2(61.3)} = 0.096x^2$ (see plots below)

Eq. 5/16 : $s_A = l_A\left[1 + \frac{2}{3}\left(\frac{h_A}{l_A}\right)^2 - \frac{2}{5}\left(\frac{h_A}{l_A}\right)^4 + \cdots\right]$

$= 5\left[1 + \frac{2}{3}\left(\frac{2.4}{5}\right)^2 - \frac{2}{5}\left(\frac{2.4}{5}\right)^4 + \cdots\right] = 5.66$ m

So the required length is $L_P = 2s_A = \underline{11.32 \text{ m}}$

(b) Eq. 5/9 : $y = \frac{T_0}{\mu}\left(\cosh\frac{\mu x}{T_0} - 1\right)$

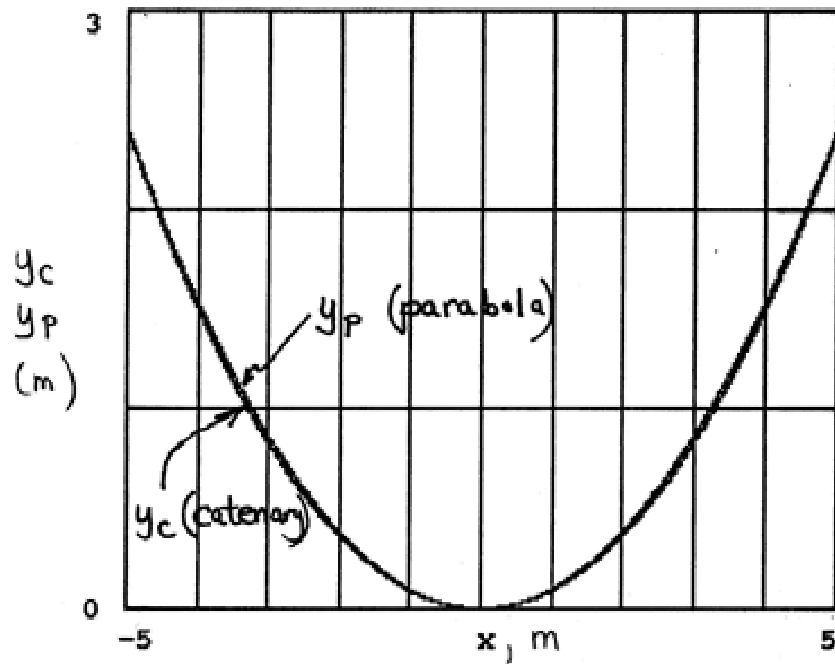
At A : $2.4 = \frac{T_0}{11.77}\left[\cosh\frac{11.77(5)}{T_0} - 1\right]$

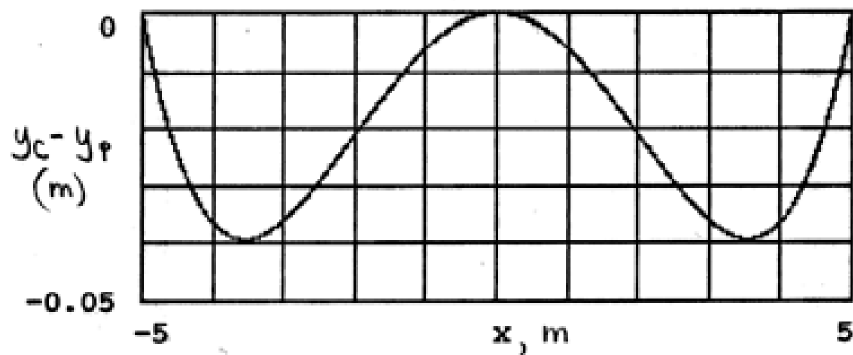
Numerical solution : $T_0 = 65.5$ N

So $y_C = 5.57\left[\cosh(0.1796x) - 1\right]$ (see plots)

Eq. 5/20 : $s_A = \frac{65.5}{11.77}\sinh\frac{11.77(5)}{65.6} = 5.70$ m

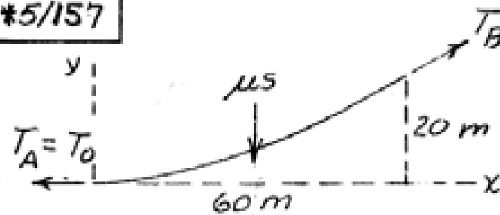
The required length is $L_C = 2s_A = \underline{11.40 \text{ m}}$





Even with an expanded vertical scale, y_p and y_c are nearly indistinguishable from each other. Note that y_p is above y_c except at the end points (A and B) and the center. So the fact that $L_c > L_p$ makes sense!

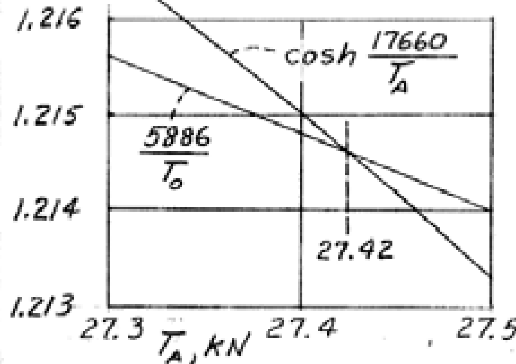
***5/157**



$\mu = (10 + 20)\,9.81 = 294$ N/m

Eq. 5/19, $y = \frac{T_0}{\mu}\left(\cosh\frac{\mu x}{T_0} - 1\right)$, $20 = \frac{T_A}{294}\left(\cosh\frac{294(60)}{T_A} - 1\right)$

$\frac{5886}{T_A} + 1 = \cosh\frac{17660}{T_A}$ solution by graphical or computer analysis $T_A = 27420$ N or $\underline{T_A = 27.4 \text{ kN}}$



Eq. 5/22, $T = T_0 + \mu y$

$T_B = 27.4 + 0.294(20)$

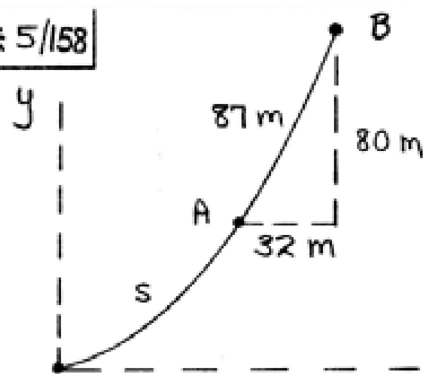
$= \underline{33.3 \text{ kN}}$

Eq. 5/20, $s = \frac{T_0}{\mu}\sinh\frac{\mu x}{T_0}$

$s = \frac{27.4(10^3)}{294}\sinh\frac{294(60)}{27.4(10^3)}$

$\underline{s = 64.2 \text{ m}}$

*5/158



$\mu = 2(9.81) = 19.62$ N/m

Eq. 5/19: $y = \frac{T_0}{\mu}\left(\cosh\frac{\mu x}{T_0} - 1\right)$

A: $y_A = \frac{T_0}{\mu}\left(\cosh\frac{\mu x_A}{T_0} - 1\right)$

B: $y_A + 80 = \frac{T_0}{\mu}\left(\cosh\frac{\mu}{T_0}(x_A + 32) - 1\right)$

Eq. 5/20: $s = \frac{T_0}{\mu}\sinh\frac{\mu x}{T_0}$

A: $s = \frac{T_0}{\mu}\sinh\frac{\mu x_A}{T_0}$

B: $s + 87 = \frac{T_0}{\mu}\sinh\frac{\mu}{T_0}(x_A + 32)$

Numerical solution: $x_A = 24.0$ m, $y_A = 12.30$ m

$\frac{T_0}{\mu} = 25.2$ m, $s = 27.8$ m

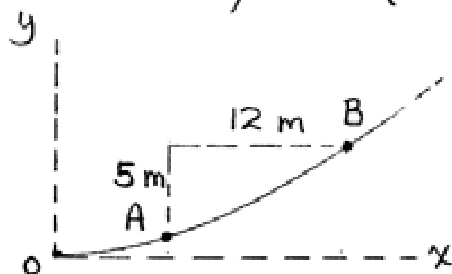
$T_0 = 25.2(19.62) = 495$ N

$T_A = T_0 + \mu y_A = 495 + 19.62(12.30) = \underline{736 \text{ N}}$

$T_B = T_0 + \mu y_B = 495 + 19.62(92.3) = \underline{2310 \text{ N}}$

*5/159

$\mu = 0.6(9.81) = 5.89$ N/m



Eq. 5/19 @ A: $y_A = \frac{T_0}{\mu}\left[\cosh\frac{\mu x_A}{T_0} - 1\right]$

Eq. 5/19 @ B: $y_A + 5 = \frac{T_0}{\mu}\left[\cosh\frac{\mu(x_A+12)}{T_0} - 1\right]$

Eq. 5/22 @ A: $200 = T_0 + \mu y_A$

Numerical solution of above three equations:

$x_A = 7.37$ m, $y_A = 0.823$ m, $T_0 = 195.2$ N

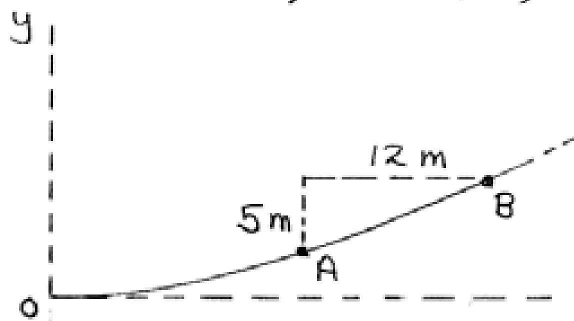
From $\frac{dy}{dx} = \sinh\frac{\mu x}{T_0}$, we have, at A

$\theta_A = \tan^{-1}\left[\sinh\frac{(5.89)(7.37)}{195.2}\right] = \underline{12.64°}$

From 5/20, $s_B - s_A = \frac{195.2}{5.89}\left[\sinh\frac{5.89(7.37+12)}{195.2} - \sinh\frac{5.89(7.37)}{195.2}\right] = \underline{13.06 \text{ m}} = L$

5/22: $T_B = 195.2 + 5.89(0.823+5) = \underline{229 \text{ N}}$

*5/160 $\mu = 0.6(9.81) = 5.89$ N/m



Eq. 5/19 @ A : $y_A = \frac{T_0}{\mu}\left[\cosh\frac{\mu x_A}{T_0} - 1\right]$

Eq. 5/19 @ B : $y_A + 5 = \frac{T_0}{\mu}\left[\cosh\frac{\mu(x_A+12)}{T_0} - 1\right]$

From Eq. 5/20

$s_B - s_A = 13.02 = \frac{T_0}{\mu}\left[\sinh\frac{\mu(x_A+12)}{T_0} - \sinh\frac{\mu x_A}{T_0}\right]$

Numerical solution of above three equations:

$x_A = 17.34$ m, $y_A = 2.63$ m, $T_0 = 339$ N

5/22: $T_A = 339 + 5.89(2.63) = \underline{355 \text{ N}}$

$\theta_A = \tan^{-1}\left[\sinh\frac{\mu x_A}{T_0}\right] = \tan^{-1}\left[\sinh\frac{5.89(17.34)}{339}\right]$

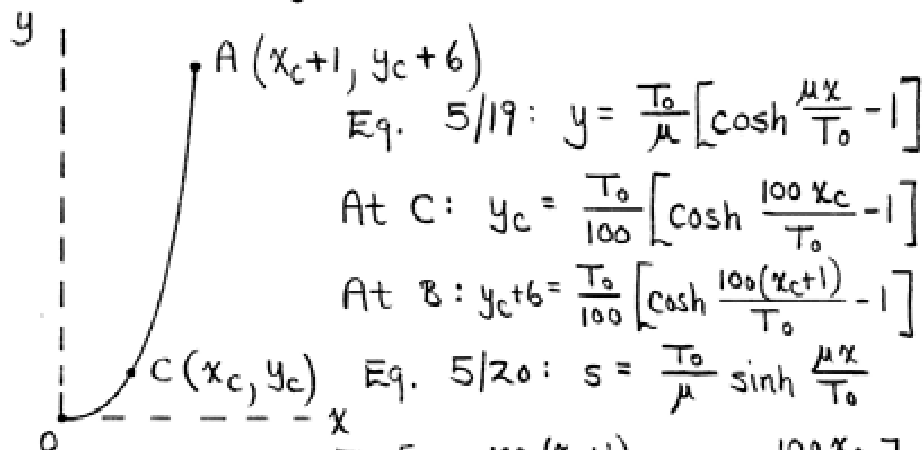
$= \underline{16.98°}$

Similarly, $T_B = 339 + 5.89(2.63+5) = \underline{384 \text{ N}}$

$\theta_B = \tan^{-1}\left[\sinh\frac{5.89(17.34+12)}{339}\right] = \underline{28.0°}$

*5/161 Architect's plan : $(T_A)_{arch} = 6(100) = 600$ N

Builder's arrangement :



Eq. 5/19 : $y = \frac{T_0}{\mu}\left[\cosh\frac{\mu x}{T_0} - 1\right]$

At C : $y_c = \frac{T_0}{100}\left[\cosh\frac{100\,x_c}{T_0} - 1\right]$

At B : $y_c + 6 = \frac{T_0}{100}\left[\cosh\frac{100(x_c+1)}{T_0} - 1\right]$

Eq. 5/20 : $s = \frac{T_0}{\mu}\sinh\frac{\mu x}{T_0}$

So $s_A - s_c = 6.1 = \frac{T_0}{100}\left[\sinh\frac{100\,(x_c+1)}{T_0} - \sinh\frac{100\,x_c}{T_0}\right]$

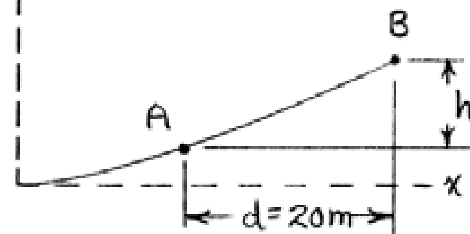
Numerical solution of three equations :

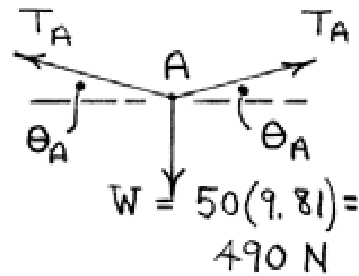
$x_c = 1.071$ m, $y_c = 1.088$ m, $T_0 = 65.5$ N

Eq. 5/22 : $T = T_0 + \mu y$, so $T_A = 65.5 + 100\,(1.088 + 6) = 774$ N

Percent increase $n = \frac{774 - 600}{600}(100) = \underline{29.0\%}$ (!)

*5/162 $\begin{cases} \mu = 1.2\,(9.81) = 11.77 \text{ N/m} \\ L = 21 \text{ m} \end{cases}$





From FBD of junction ring at A,

$+\uparrow \Sigma F = 0: \quad 2T_A \sin\theta_A - W = 0$

or $\left[T_0 + \mu y_A\right] \sin\left[\tan^{-1}\left(\sinh \frac{\mu x_A}{T_0}\right)\right] - \frac{W}{2} = 0$ (1)

Eq. 5/19 @ A : $y_A = \frac{T_0}{\mu}\left[\cosh \frac{\mu x_A}{T_0} - 1\right]$ (2)

Eq. 5/19 @ B : $y_A + h = \frac{T_0}{\mu}\left[\cosh \frac{\mu(x_A + d)}{T_0} - 1\right]$ (3)

Eq. 5/20 : $s_B - s_A = \frac{T_0}{\mu}\left[\sinh \frac{\mu(x_A + d)}{T_0} - \sinh \frac{\mu x_A}{T_0}\right] = L$ (4)

Solution of (1) – (4) with $W = 0$: $h = 5.57$ m

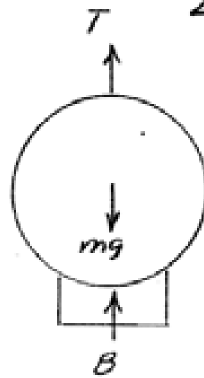
With $W \neq 0$: $h = 6.30$ m

So $\delta = 6.30 - 5.57 = \underline{0.724 \text{ m}}$

5/163

$T = 8$ kN

$mg = 6.7(9.81) = 65.7$ kN



$\Sigma F = 0$; $8 - 65.7 + B = 0$

$B = 57.7$ kN

$B = \rho g V$; $V = \dfrac{57.7}{1.03(9.81)} = \underline{5.71 \text{ m}^3}$

$\rho_{salt\ water} = 1.03(10^3) \text{ kg/m}^3$

5/164 Force on bottom = weight of water

$= \rho g V = (1000 \frac{kg}{m^3})(9.81 \frac{m}{s^2})(0.3 m)(0.7 m)(0.4 m)$

$= \underline{824 \text{ N}}$ (down, at center of bottom)

Force on front & back $= p_{av} A_f = \frac{\rho g h}{2} A_f$

$= \frac{1000(9.81)(0.4)}{2}(0.7)(0.4) = \underline{549 \text{ N}}$ (outward, at $\frac{2}{3}$ depth)

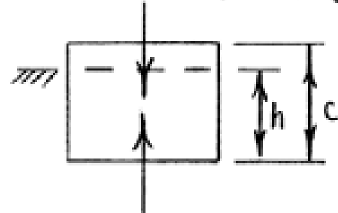
Force on each end glass $= p_{av} A_e = \frac{\rho g h}{2} A_e$

$= \frac{1000(9.81)(0.4)}{2}(0.3)(0.4) = \underline{235 \text{ N}}$ (outward, at $\frac{2}{3}$ depth)

(All side forces centered horizontally)

5/165

$W = mg = \rho_1 V g = \rho_1 abcg$



$B = \rho_2 V_{sub} g = \rho_2 abhg$

$+\uparrow \Sigma F = 0: \ \rho_2 abhg - \rho_1 abcg = 0, \quad h = \frac{\rho_1}{\rho_2} c$

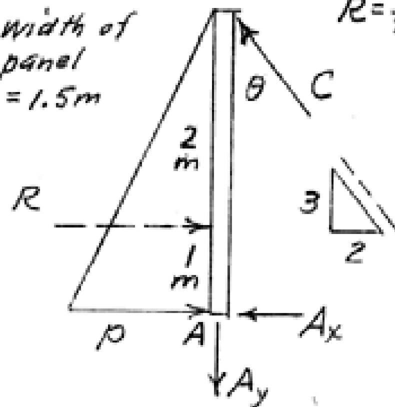
$r = \frac{h}{c} = \underline{\frac{\rho_1}{\rho_2}}$

Oak in water: $r = \frac{800}{1000} = \underline{0.8}$

Steel in mercury: $r = \frac{7830}{13\,570} = \underline{0.577}$

5/166

$p = \rho g h = 2400(9.81)(3) = 70.6 \text{ kPa}$



$\theta = \tan^{-1} 2/3 = 33.7°$

$R = \frac{1}{2} p A = \frac{1}{2}(70.6)(10^3)(3)(1.5)$

$= 158.9(10^3) \text{ N}$

$\Sigma M_A = 0;$

$(C \sin 33.7°)3 - 158.9(10^3)(1) = 0$

$C = \dfrac{158.9(10^3)}{3 \sin 33.7°} = 95.5(10^3) \text{ N}$

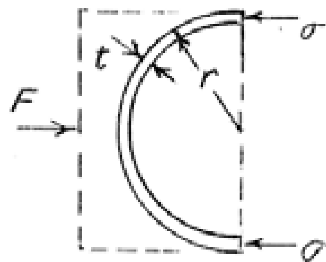
or $C = 95.5 \text{ kN}$

5/167

$F = pA = \rho g h A = \rho g h \pi r^2$

$\Sigma F = 0;\ \rho g h \pi r^2 - \sigma(\pi r^2 - \pi[r - t]^2)$

$\sigma = \frac{\rho g h r}{2t} \frac{1}{1 - t/2r}$



$= \frac{1.03(10^3)(9.81)\frac{1.500}{2}3(10^3)}{2(0.025)} \frac{1}{1 - \frac{25}{1500}}$

$= 454.7(10^6)(1.0169)$

or $\sigma = 463$ MPa

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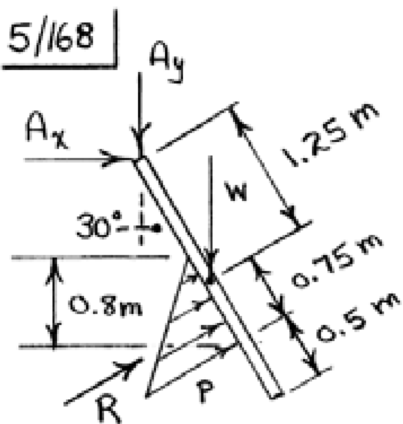

$p = \rho g h = 1000(9.81)(0.8)$

$= 7850$ Pa

$R = \frac{1}{2}(7850)\left(\frac{0.8}{\cos 30°}\right)$

$= 3620$ N/m

$\circlearrowleft + \; \Sigma M_A = 0: \quad W(1.25 \sin 30°) - 3620\left(2.5 - 0.5 - \frac{1}{3}\frac{0.8}{\cos 30°}\right)$

$= 0; \quad \underline{W = 9810 \text{ N/m}}$

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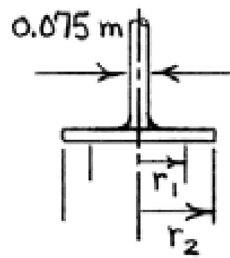


$\begin{cases} r_1 = 0.2\text{ m} \\ r_2 = 0.3\text{ m} \end{cases}$

$p = \rho g h = 1030(9.81)(0.6) = 6060\text{ Pa}$

$R = pA = 6060\,\pi\left(0.3^2 - \frac{0.075^2}{4}\right) = 1687\text{ N}$

Pressure supported by seal

$\sigma = R/\pi(r_2^2 - r_1^2) = \frac{1687}{\pi(0.3^2 - 0.2^2)}$

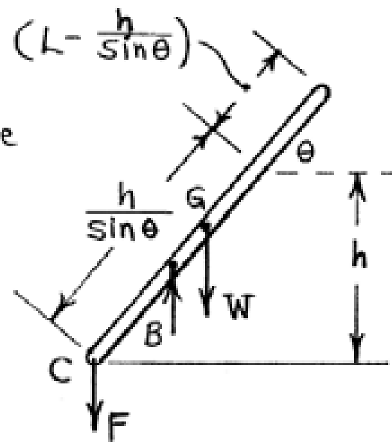
$= 10\,740\text{ Pa}$ or $\underline{10.74\text{ kPa}}$

Force to lift plunger

$P = R = 1687\text{ N}$ or $\underline{1.687\text{ kN}}$

5/170

Let A = cross-sectional area of the pole



Buoyancy force B is

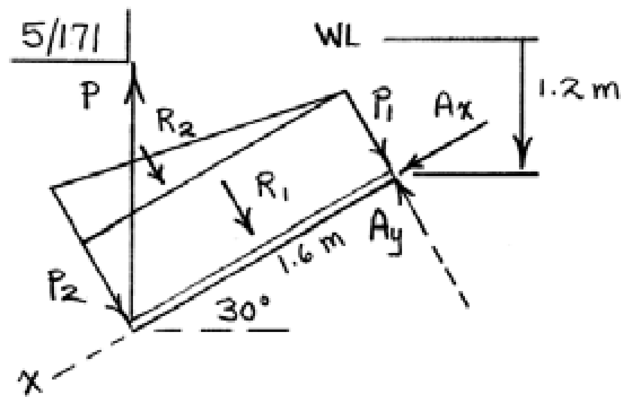
$B = \rho g \bar{V} = \rho g \frac{hA}{\sin\theta}$

Weight $W = \rho' g L A$

$$\circlearrowleft + \Sigma M_C = 0: \rho g \frac{hA}{\sin\theta}\left(\frac{1}{2}\frac{h}{\sin\theta}\cos\theta\right) - \rho' g L A \left(\frac{L}{2}\cos\theta\right) = 0$$

$$\underline{\theta = \sin^{-1}\left(\frac{h}{L}\sqrt{\frac{\rho}{\rho'}}\right)} \qquad \left(\frac{h^2\rho}{L^2\rho'} \leq 1\right)$$

5/171



$p_1 = \rho g h_1 = 1.000(9.81)(1.2) = 11.77$ kPa

$p_2 = \rho g h_2 = 1.000(9.81)(2) = 19.62$ kPa

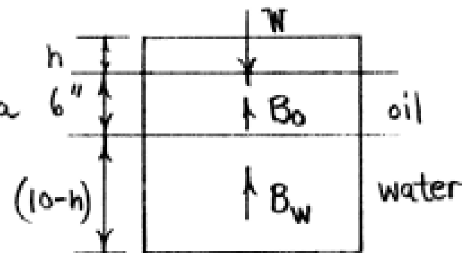
$R_1 = 11.77(1.6)(0.8) = 15.07$ kN

$R_2 = (19.62 - 11.77)\frac{1}{2}(1.6)(0.8) = 5.02$ kN

$\Sigma M_A = 0: \quad P(1.6 \cos 30°) - 15.07(0.8)$

$- 5.02\left(\frac{2}{3} 1.6\right) = 0, \qquad \underline{P = 12.57 \text{ kN}}$

5/172



Let A be the area of a horizontal slice of block

$A = \frac{16^2}{144}$ ft^2

Weight of block $W = \frac{16}{12} A (50) = 66.7A$ (lb)

Buoyancy of oil $B_o = \frac{6}{12} A (56) = 28A$ (lb)

Buoyancy of water $B_w = \frac{10-h}{12} A (64) = 5.33(10-h)A$

(in lb, h in inches)

$\Sigma F = 0: \ W - B_o - B_w = 0: \ 66.7A - 28A - 5.33(10-h)A = 0$

$h = 2.75$ in.

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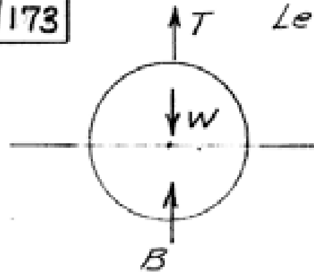


Let γ_c = wt. density of concrete $= 150\ lb/ft^3$

γ_w = wt. density of fresh water $= 62.4\ lb/ft^3$

L = length of cylinder = 6 ft

r = radius of cylinder = 2 ft

For equil., $T = W - B$

$$= \gamma_c \pi r^2 L - \gamma_w \frac{\pi r^2}{2} L$$

$$= \pi r^2 L \left(\gamma_c - \frac{1}{2}\gamma_w\right)$$

$$= \pi(2^2)(6)\left(150 - \frac{62.4}{2}\right) = \underline{8960\ lb}$$

5/174

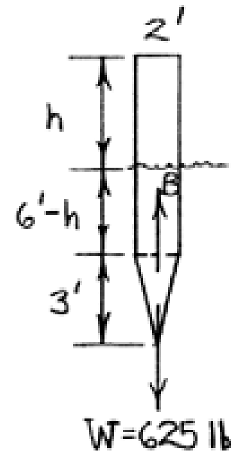
$$B = \mu V = 64\left[\pi \frac{2^2}{4}(6-h) + \frac{1}{3}\pi \frac{2^2}{4}(3)\right]$$

$$= 64\pi \frac{2^2}{4}\left[6-h+1\right]$$

$$= 64\pi\left[7-h\right]$$

$$W = B: \quad 625 = 64\pi\left[7-h\right]$$

$$\underline{h = 3.89 \text{ ft}}$$



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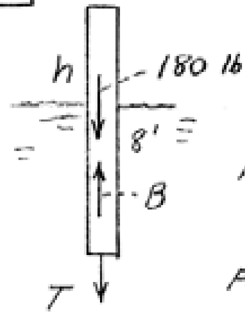

$B = \rho g V = 64\left(\frac{\pi \times 1^2}{4}[8-h]\right)$

$= 50.27(8-h)$

$\Sigma F = 0;\ T + 180 - 50.27(8-h) = 0$

For $h = 2$ ft, $T = -180 + 50.27(8-2)$

$= \underline{121.6 \text{ lb}}$

For $T = 0$, $8 - h = 180/50.27$

$\underline{h = 4.42 \text{ ft}}$

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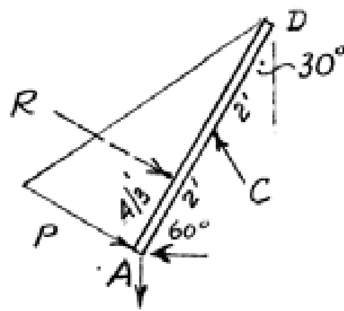

$p = \rho g h = 62.4(4\cos 30°) = 216\ lb/ft^2$

$R = \frac{p}{2}$ Area per panel

$= \frac{216}{2}(4)(2) = 865\ lb$

$\Sigma M_A = 0;\ 2C\cos 30° - 865(4/3) = 0$

$C = \frac{865(2)}{3\cos 30°} = \underline{666\ lb}$

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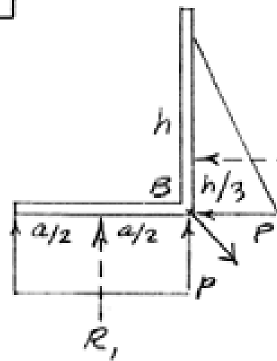

$R_1 = \rho g h a b$

$R_2 = \frac{\rho g h}{2} h b$

$\Sigma M_B = 0;\ R_1 \frac{a}{2} = R_2 \frac{h}{3}$

$\rho g h a b \frac{a}{2} = \frac{\rho g h}{2} h b \frac{h}{3}$

$h^2 = 3a^2,\ \underline{h = a\sqrt{3}}$

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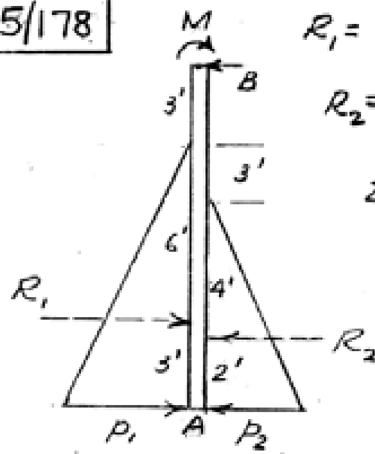

$R_1 = \frac{p_1 A_1}{2} = \frac{62.4(9)}{2}(9)(10) = 25,270 \text{ lb}$

$R_2 = \frac{p_2 A_2}{2} = \frac{64(6)}{2}(6)(10) = 11,520 \text{ lb}$

$\Sigma M_B = 0; \; M + 11,520(10) - 25,270(9) = 0$

$M = 11.22(10^4) \text{ lb-ft}$

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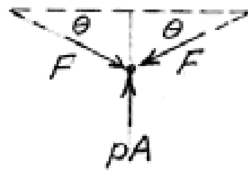
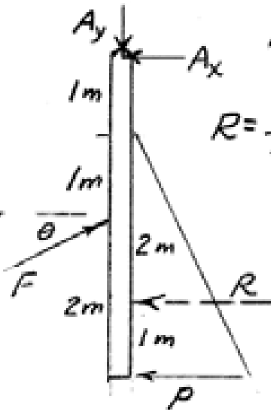
$\theta = \tan^{-1}\frac{0.5}{1} = 26.6°$





$p = \rho g h = 1.0(9.81)(3) = 29.4$ kPa

$R = \frac{p}{2}\text{Area} = \frac{29.4}{2}(3)(2) = 88.3$ kN

Gate:

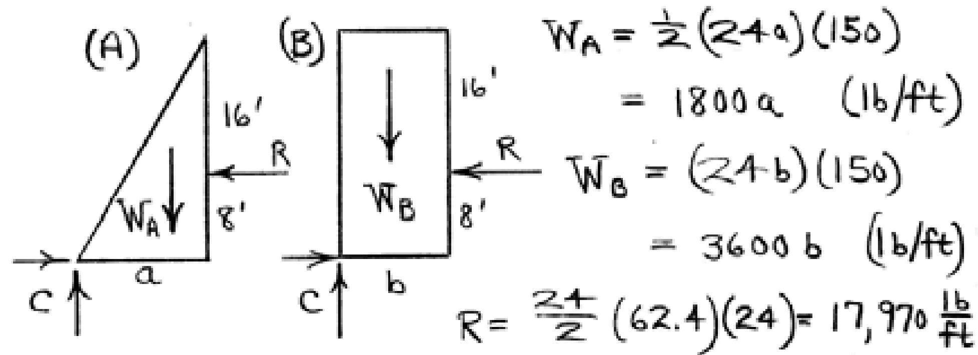
$\Sigma M_A = 0;\quad (F\cos 26.6°)2 - 88.3(3) = 0,\quad F = 148.1$ kN

Toggle:

$\Sigma F = 0;\quad pA - 2F\sin\theta = 0;\quad \frac{\pi(0.150)^2}{4}p = 2(148.1)(10^3)\sin 26.6°$

$p = 7.49(10^6)$ Pa or $\underline{p = 7.49\text{ MPa}}$

5/180 Per foot of length



$W_A = \frac{1}{2}(24a)(150)$

$= 1800a$ (lb/ft)

$W_B = (24b)(150)$

$= 3600b$ (lb/ft)

$R = \frac{24}{2}(62.4)(24) = 17{,}970 \frac{lb}{ft}$

(A) $\Sigma M_C = 0: \; 17{,}970(8) - \frac{2}{3}a(1800a) = 0$

$a = 10.95$ ft, $W_A = 1800(10.95) = 19{,}700 \frac{lb}{ft}$

(B) $\Sigma M_C = 0: \; 17{,}970(8) - \frac{b}{2}(3600b) = 0$

$b = 8.94$ ft, $W_B = 3600(8.94) = 32{,}200 \frac{lb}{ft}$

So A requires $32{,}200 - 19{,}700 = 12{,}470 \frac{lb}{ft}$ less than B.

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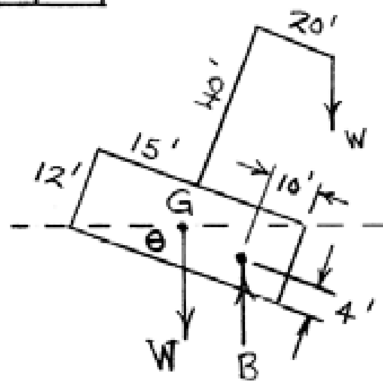

$\theta = \tan^{-1}\frac{12}{30} = 21.8°$

Moment arm of B about G

$(15-10)\cos\theta - (6-4)\sin\theta =$
3.90 ft

Moment arm of w about G

$(40+6)\sin\theta + 20\cos\theta$
$= 35.7$ ft

$B = \rho g V = 64(12)(15)(80) = 922{,}000$ lb

$\Sigma M_G = 0: \; 35.7\,w - 3.90\,(922{,}000) = 0$

$w = 100{,}800$ lb

$W = B - w = 922{,}000 - 100{,}800 = 821{,}000$ lb

or $W = \frac{821{,}000}{2240} = 366$ long tons

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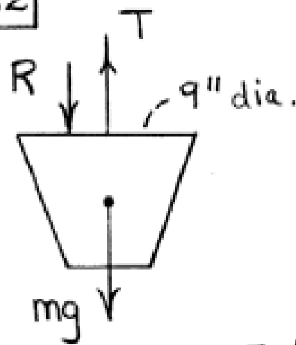


$p = \rho g h = 62.4(20) = 1248 \frac{lb}{ft^2}$

$R = pA = 1248\left[\pi\left(\frac{4.5}{12}\right)^2\right]$

$= 551 \text{ lb}$

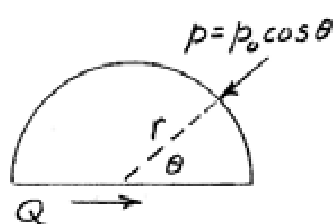
$mg = \rho V g = \rho g\left[\frac{7\pi r^2 h}{24}\right]$ (from Prob. 5/30)

$= 450\left[\frac{7\pi\,(4.5/12)^2(12/12)}{24}\right] = 58.0 \text{ lb}$

$+\uparrow \Sigma F = 0: \quad T - 551 - 58.0 = 0, \quad \underline{T = 609 \text{ lb}}$

5/183

$$Q = \int_0^{\pi} (p_0 \cos\theta)(\cos\theta)\, r\, d\theta$$



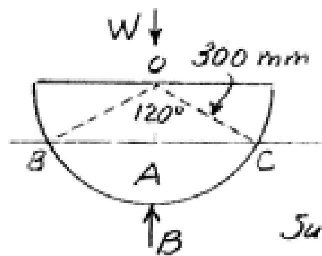
$$= p_0 r \int_0^{\pi} \cos^2\theta\, d\theta$$

$$= p_0 r \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_0^{\pi}$$

$$= p_0 r \left[\frac{\pi}{2} \right]$$

$$\underline{Q = \tfrac{1}{2} \pi r p_0}$$

5/184 Submerged area A of end = area of 120° sector minus area of triangle OBC



$$A = \frac{120}{360}\pi(0.3)^2 - (0.150)(0.3\cos 30°)$$

$$= 0.05528\ m^2$$

Submerged volume is

$$V = 0.05528(0.600) = 33.17(10^{-3})\ m^3$$

$$B = \rho g V = 1000(9.81)(33.17)(10^{-3}) = 325.4\ N$$

$$W = (26.6 + m)9.81\ N$$

For equilibrium $B = W$, so $(26.6+m)9.81 = 325.4$

$\underline{m = 6.57\ kg}$

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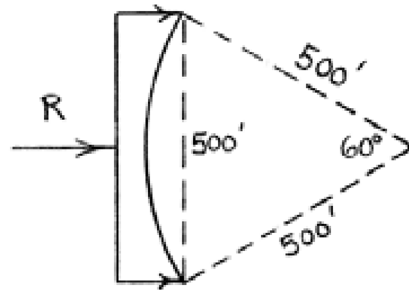
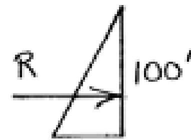
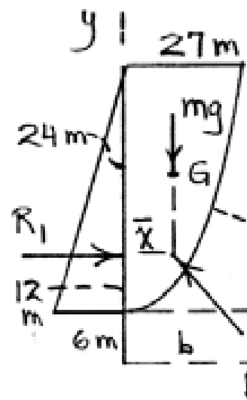




$P_{100} = \mu h = 62.4(100) = 6240 \frac{lb}{ft^2}$

$R = P_{av}(\text{Area}) = \frac{6240}{2}(500)(100) = \underline{156.0(10^6)\ lb}$

5/186 Take a vertical section of water of unit horizontal length. Let ρ be the water density in t/m^3.



$y = kx^2$: $36 = k(27)^2$, $k = \frac{4}{81}\ m^{-1}$

$\bar{x} = \frac{\int x\,dA}{\int dA}$, $dA = x\,dy = 2\frac{4}{81}x^2 dx$

$\therefore\ \bar{x} = \frac{\int_0^{27} \frac{x}{2}\frac{8}{81}x^2 dx}{\int_0^{27}\frac{8}{81}x^2 dx} = 10.12\ m$

$A = \int dA = 648\ m^2$, $mg = 648\rho g$

$R_1 = \frac{1}{2}\ 36\rho g\ (36)(1) = 648\rho g$

Resultant of mg & R_1 passes through B, so

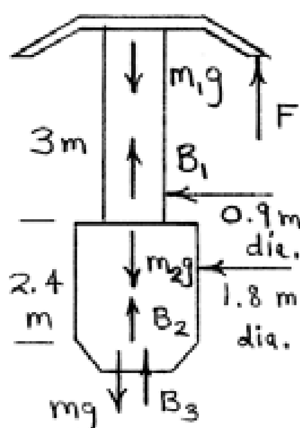
$\Sigma M_B = 0$. Thus $648\rho g\,(18) = 648\rho g\,(b - 10.12)$

$b = 28.1\ m$

5/187

$B_1 = \rho_w g V_1 = 1.03(9.81)\frac{\pi(0.9)^2}{4}(3)$

$= 19.28 \text{ kN}$



$B_2 = \rho_w g V_2 = 1.03(9.81)\frac{\pi(1.8)^2}{4}2.4$

$= 61.7 \text{ kN}$

V_3 = volume of ballast

$B_3 = \rho_w g V_3 = \rho_w g \frac{m}{\rho_L}$

$= 1.03(9.81)\frac{m}{11.37} = 0.889 m$

$F = 0.15(B_1 + B_2 + B_3) = 0.15(19.28 + 61.7 + 0.889m)$

$= 12.15 + 0.1333 m$

$(m_1 + m_2) g = 5.7(9.81) = 55.9 \text{ kN}$

$\Sigma F = 0: \; F + B_1 + B_2 + B_3 - (m_1 + m_2)g - mg = 0$

$12.15 + 0.1333m + 19.28 + 61.7 + 0.889m - 55.9 - 9.81m = 0$

$\underline{m = 4.24 \text{ Mg}}$

5/188 The gage pressure 12 m below the surface is $p = \rho g h = (1000)(9.81)(12) = 117\ 700\ N/m^2$.

(a) Cover area $A_{cov} = \pi \left(\frac{0.75}{2}\right)\left(\frac{0.5}{2}\right) = 0.295\ m^2$

Force on cover $= pA_{cov} = 34\ 700\ N$

Seal area $A_s = A_{cov} - \pi\left(\frac{0.55}{2}\right)\left(\frac{0.375}{2}\right) = 0.1325\ m^2$

$\sigma A_s = pA_{cov}$, $\sigma = \frac{34\ 700}{0.1325} = 262\ 000\ \frac{N}{m^2}$

or $\underline{\sigma = 262\ kPa}$

(b) $16\Delta T = pA_{hole} = 117\ 700\left[\pi\left(\frac{0.55}{2}\right)\left(\frac{0.375}{2}\right)\right]$

$\underline{\Delta T = 1192\ N}$

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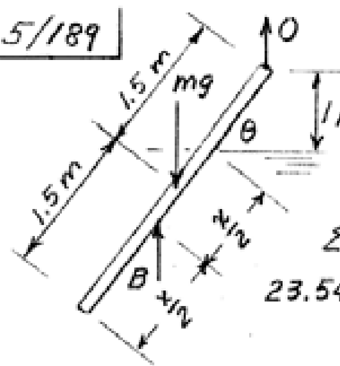


Let A = cross-sectional area of plank

$mg = 800(3)A(9.81) = 23.54(10^3)A$ N

$B = \rho_w g A x$

$= 1000(9.81)A\left(3 - \frac{1}{\sin\theta}\right)$

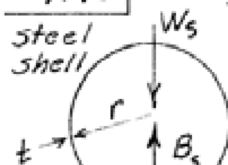
$\Sigma M_O = 0$;

$23.54(10^3)A(1.5\cos\theta) - 9.81(10^3)A\left(3 - \frac{1}{\sin\theta}\right) \cdot \left[\frac{1}{2}\left(3 - \frac{1}{\sin\theta}\right) + \frac{1}{\sin\theta}\right]\cos\theta = 0$

Simplify & get $23.54(1.5) = \frac{9.81}{2}\left(9 - \frac{1}{\sin^2\theta}\right)$

& $\sin^2\theta = 0.5556$, $\sin\theta = 0.7454$, $\underline{\theta = 48.2°}$

5/190 For equilibrium $W_S + W_L = B_S + B_L$



Lead

ρ_S = density of steel = 7.83 Mg/m^3

ρ_W = " " salt water = 1.03 Mg/m^3

ρ_L = " " lead = 11.37 Mg/m^3

$r = 1.00$ m, $t = 0.035$ m

V_L = volume of lead, m^3

m = mass of lead = $\rho_L V_L$

so $$\rho_S g\, 4\pi r^2 t + \rho_L g V_L = \rho_W g \frac{4}{3}\pi\left(r+\frac{t}{2}\right)^3 + \rho_W g V_L$$

$$V_L g(\rho_L - \rho_W) = 4\pi r^2 g\left[\frac{r}{3}\left(1+\frac{t}{2r}\right)^3 \rho_W - \rho_S t\right]$$

$$m\left(1 - \frac{\rho_W}{\rho_L}\right) = 4\pi r^2\left[\frac{r}{3}\left(1+\frac{t}{2r}\right)^3 \rho_W - \rho_S t\right]$$

$$m\left(1 - \frac{1.03}{11.37}\right) = 4\pi(1)^2\left[\frac{1}{3}\left(1+\frac{0.035}{2}\right)^3 1.03 - 7.83(0.035)\right]$$

$0.9094m = 1.1008$, $m = \underline{1.210 \text{ Mg}}$ (metric tons)

5/191 The pressure at the bottom of the 3-m wall is $p = \rho g h = 2400(9.81)(3) = 70\,600 \text{ N/m}^2$

Each tie controls an area A given by

$$pA = T, \quad A = \frac{T}{p} = \frac{6500}{70\,600} = 0.0920 \text{ m}^2$$

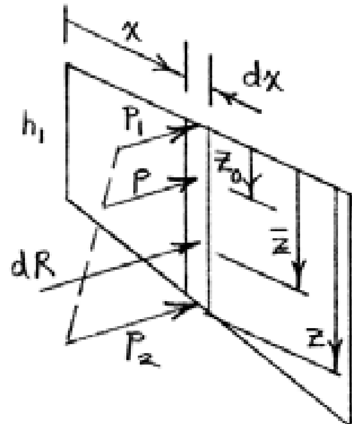
This square area has a side d given by

$$d^2 = A, \quad d = 0.303 \text{ m}$$

Using the pressure at the very bottom of the wall gives us a conservative design; a good figure for d would be $\underline{d = 0.300 \text{ m}}$.

5/192

Method I: Direct integration



ρ = water density, $p = \rho g (b + z_0)$

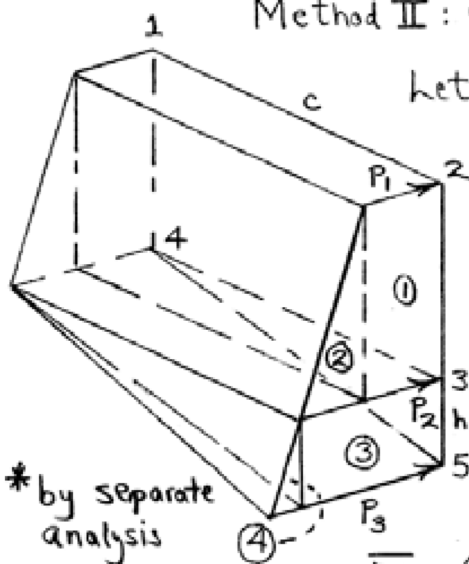
$z = h_1 + kx = f(x)$

(1) Calculate dR for elemental area $z\,dx$ & find $\bar{z}$

(2) Integrate & get $R = \int dR$

(3) $R\bar{z} = \int z\,dR$

Method II: Geometry of pressure-area volumes



Let A_a = 1-2-3-4, A_b = 3-4-5

$R_1 = p_1 A_a$, $\bar{z}_1 = h_1/2$

$R_2 = \frac{1}{2}(p_2 - p_1)A_a$, $\bar{z}_2 = \frac{2h_1}{3}$

$R_3 = p_2 A_b$, $\bar{z}_3 = h_1 + \frac{2}{3}(h_2 - h_1)$

$R_4 = \frac{1}{2}(p_3 - p_2)(h_2 - h_1)\frac{c}{3}$

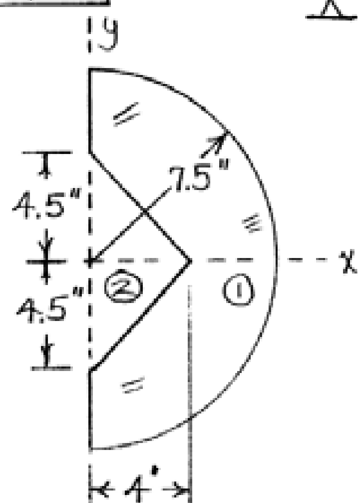
$\bar{z}_4 = \frac{1}{2}(h_2 + h_1)$ *

$R = R_1 + R_2 + R_3 + R_4$

$\bar{z} = (R_1\bar{z}_1 + R_2\bar{z}_2 + R_3\bar{z}_3 + R_4\bar{z}_4)/R$

*by separate analysis

5/193

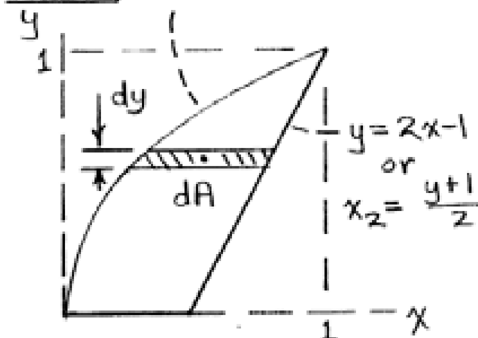


$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A}$$

$$= \frac{\pi \frac{7.5^2}{2}\left(\frac{4(7.5)}{3\pi}\right) - \frac{1}{2}(9)(4)\left(\frac{4}{3}\right)}{\pi \frac{7.5^2}{2} - \frac{1}{2}(9)(4)}$$

$$= \underline{3.66 \text{ in.}}$$

5/194 $y = x^{1/3}$ or $x_1 = y^3$



$$dA = (x_2 - x_1)\,dy$$
$$= \left(\frac{y}{2} + \frac{1}{2} - y^3\right)dy$$

$$A = \int dA = \int_0^1 \left(\frac{y}{2} + \frac{1}{2} - y^3\right)dy = \left(\frac{y^2}{4} + \frac{y}{2} - \frac{y^4}{4}\right)\Big|_0^1 = \frac{1}{2}$$

$$\int y_c\,dA = \int_0^1 \left(\frac{y^2}{2} + \frac{y}{2} - y^4\right)dy = \left(\frac{y^3}{6} + \frac{y^2}{4} - \frac{y^5}{5}\right)\Big|_0^1 = \frac{13}{60}$$

$$\int x_c\,dA = \int_0^1 \left(\frac{x_1 + x_2}{2}\right)(x_2 - x_1)\,dy = \frac{1}{2}\int_0^1 (x_2^2 - x_1^2)\,dy$$

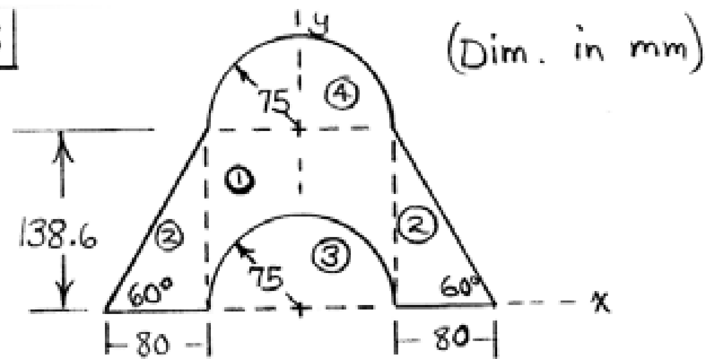
$$= \frac{1}{2}\int_0^1 \left(\frac{y^2}{4} + \frac{y}{2} + \frac{1}{4} - y^6\right)dy = \frac{1}{2}\left(\frac{y^3}{12} + \frac{y^2}{4} + \frac{y}{4} - \frac{y^7}{7}\right)\Big|_0^1$$

$$= \frac{37}{168}$$

$$\bar{y} = \int x_c\,dA \Big/ \int dA = \frac{13/60}{1/2} = \underline{13/30}$$

$$\bar{x} = \int y_c\,dA \Big/ \int dA = \frac{37/168}{1/2} = \underline{37/84}$$

5/195

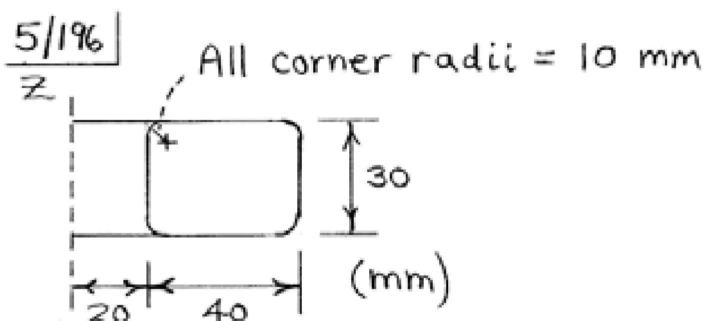


(Dim. in mm)

Comp.	A (mm^2)	$\bar{y}$ (mm)	$A\bar{y}$ (mm^3)
①	$150(138.6)$	$138.6/2$	1 440 000
②	$2\frac{1}{2}(80)(138.6)$	$\frac{1}{3}(138.6)$	512 000
③	$-\pi\frac{75^2}{2}$	$\frac{4(75)}{3\pi}$	− 281 250
④	$\pi\frac{75^2}{2}$	$\left(138.6+\frac{4(75)}{3\pi}\right)$	1 505 565

$\Sigma A = 31\,870$ $\qquad$ $\Sigma A\bar{y} = 3.18(10^6)$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \underline{99.7 \text{ mm}}$$



$$V = 2\pi \bar{r} A = 2\pi \left(20 + \frac{40}{2}\right)\Big[40(30) - 4(10^2)$$

$$= + \pi(10^2)\Big] = \underline{280\,000 \text{ mm}^3}$$

$$A = 2\pi \bar{r} L = 2\pi \left(20 + \frac{40}{2}\right)\Big[2(20) + 2(10) + 2\pi 10\Big]$$

$$= \underline{30\,900 \text{ mm}^2}$$

5/197

For circular arc, $\bar{y} = \dfrac{b \sin 30°}{\pi/6} = 3b/\pi$

$\bar{z} = b$ (See Samp. Prob. 5/1)

$$\bar{Y} = \frac{\Sigma \bar{y} L}{\Sigma L} = \frac{b(0) + 2b\left(\frac{b}{2}\cos 30°\right) + \frac{\pi}{3}b\left(\frac{3b}{\pi}\right)}{b + 2b + \frac{\pi}{3}b}$$

$$= \underline{0.461\, b}$$

$$\bar{Z} = \frac{\Sigma \bar{z} L}{\Sigma L} = \frac{b\left(\frac{b}{2}\right) + 2b(b) + \frac{\pi}{3}b(b)}{b + 2b + \frac{\pi}{3}b}$$

$$= \underline{0.876\, b}$$

5/198 Same radial distribution as for semi-circular area, so $\bar{z} = \frac{4r}{3\pi}$, or by integration,

$z = \frac{r}{h}x$

r, z, z_c, dL, dx, x

$z_c = \frac{2z}{\pi}$ from Sample Prob. 5/1

$$dA = \pi z \, dL = \pi \frac{r}{h} x \frac{\sqrt{r^2+h^2}}{h} dx$$

$$\int z_c \, dA = \int \frac{2z}{\pi} dA$$

$$= \int_0^h \frac{2r^2}{h^3} \sqrt{r^2+h^2} \; x^2 dx$$

$$= \frac{2}{3} r^2 \sqrt{r^2+h^2}$$

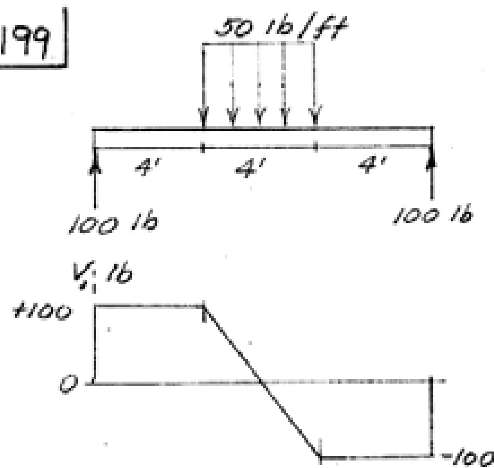
$$A = \int dA = \frac{1}{2}\pi r \sqrt{r^2+h^2} \quad \text{so} \quad \bar{z} = \int z_c \, dA / A = \underline{4r/3\pi}$$

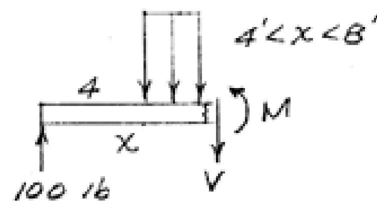
$$\int x \, dA = \int_0^h \frac{\pi r \sqrt{r^2+h^2}}{h^2} x^2 dx = \frac{\pi r \sqrt{r^2+h^2}}{3} h,$$

$$\bar{x} = \int x \, dA / A = \underline{\frac{2}{3} h}$$

This result can also be seen by inspection since the area is composed of elemental triangular areas each of which has the same x-centroidal coordinate of $2h/3$.

5/199



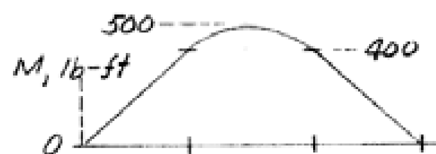


$\Sigma F = 0;\ V + 50(x-4) - 100 = 0$

$V = 50(6-x)$

$\Sigma M_V = 0;\ M + 50\frac{(x-4)^2}{2} - 100x = 0$

$M = 25(-x^2 + 12x - 16)$

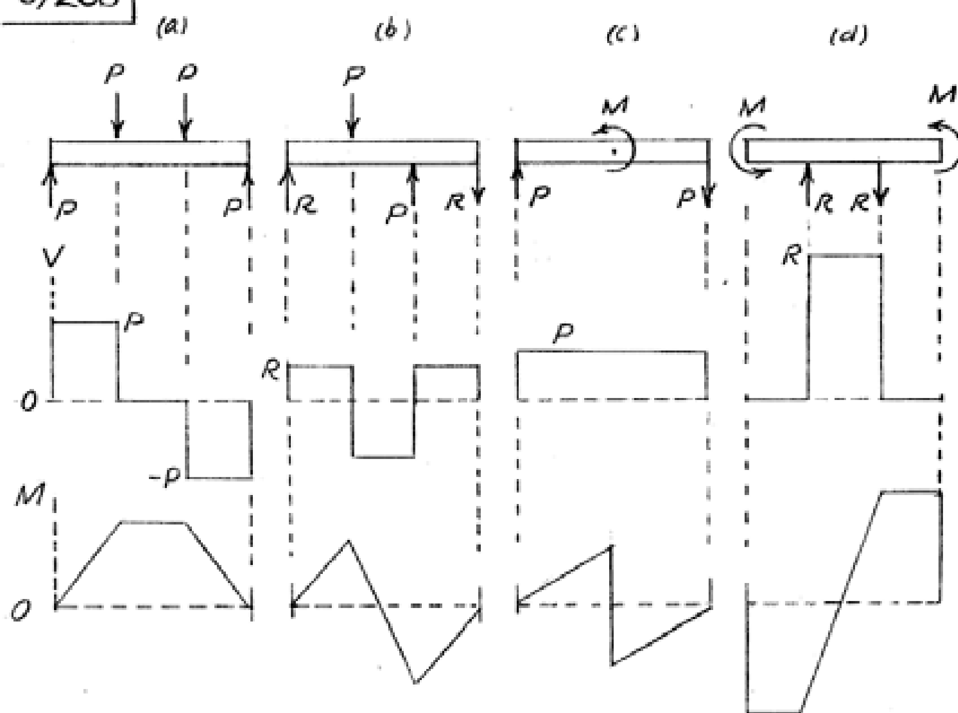


Set $\frac{dM}{dx} = 0$ to get

$M_{max} = 500$ lb-ft

@ $x = 6$ ft

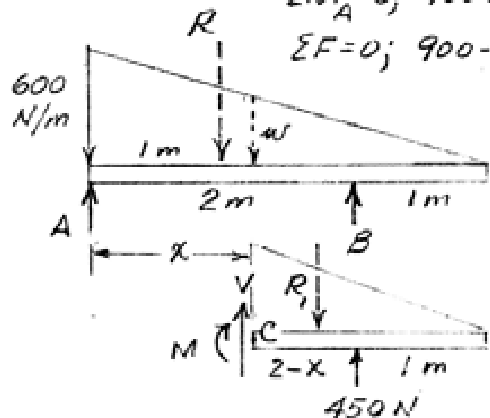
5/200



5/201 $R = \frac{600}{2}(3) = 900\ N$;

$\Sigma M_A = 0$; $900(1) - 2B = 0$, $B = 450\ N$

$\Sigma F = 0$; $900 - 450 - A = 0$, $A = 450\ N$



$R_1 = \frac{w}{2}(2 - x + 1) = \frac{3-x}{2} w$

where $w = \frac{3-x}{3} 600$ N/m

So $R_1 = 100(3-x)^2$ N

$\Sigma M_C = 0$;

$450(2-x) = 100(3-x)^2 \frac{3-x}{3} + M$

$M = 450(2-x) - \frac{100}{3}(3-x)^3 \qquad 0 \le x \le 2$ m

$\frac{dM}{dx} = -450 + 100(3-x)^2 = 0$ for max or min

$3 - x = \pm\sqrt{4.5}$, $\quad x = 3(1 \pm 1/\sqrt{2}) = \underline{0.879\ m}$ (or 5.12)

$M_{max} = 450(2 - 0.879) - \frac{100}{3}(3 - 0.879)^3 = \underline{186.4\ N \cdot m}$

5/202

$R = 2.8(10^3)$ N

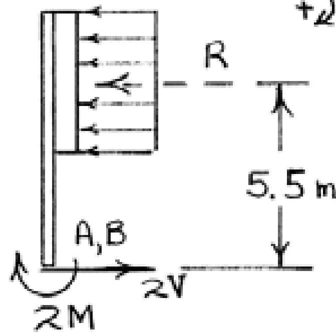
The resultant force on the sign is $R = pA = 1.4(10)^3 \frac{1}{2}(2)(2) = 2800$ N and passes through the centroid of the pressure-area prism.

By inspection,

$$\begin{cases} V = \underline{2.8 \text{ kN}} \\ M = 2.8\left(3 - \frac{2}{3}\right) = \underline{6.53 \text{ kN}\cdot\text{m}} \\ T = 2.8\left(\frac{2}{3}\right) = \underline{1.867 \text{ kN}\cdot\text{m}} \end{cases}$$

5/203

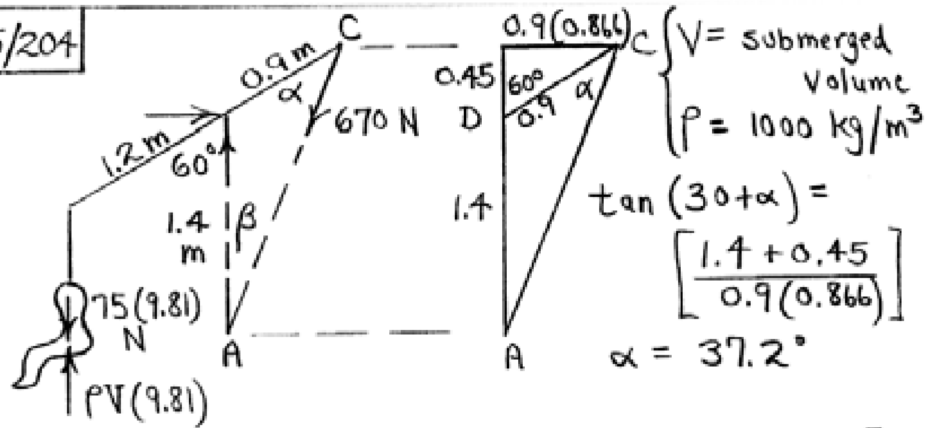
$R = pA = 1.4(10^3)(6)(4) = 33.6(10^3)$ N



$\circlearrowleft + \Sigma M_{AB} = 0 : 2M - 33.6(10^3)(5.5) = 0$

$M = 92.4(10^3)$ N·m

or $\underline{M = 92.4 \text{ kN·m}}$



$\Sigma M_D = 0: \; 670(0.9)\sin 37.2° - [75(9.81) - \rho V(9.81)] \times 1.2 \sin 60° = 0$

$\rho V = 39.3 \text{ kg}$

$V = \frac{39.3}{\rho} = \frac{39.3}{1000} = \underline{0.0393 \text{ m}^3}$

5/205

$$S = 2\int_0^{L/2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx = 2\int_0^{L/2} \sqrt{1+\left(\frac{wx}{T_0}\right)^2}\, dx$$

$$= 2\left(\frac{1}{2}\right)\frac{T_0}{w}\left[\frac{wx}{T_0}\sqrt{1+\left(\frac{wx}{T_0}\right)^2} + \ln\left(\frac{wx}{T_0} + \sqrt{1+\left(\frac{wx}{T_0}\right)^2}\right)\right]_0^{L/2}$$

$$= \frac{L}{2}\sqrt{1+\left(\frac{wL}{2T_0}\right)^2} + \frac{T_0}{w}\ln\left(\frac{wL}{2T_0} + \sqrt{1+\left(\frac{wL}{2T_0}\right)^2}\right)$$

From Eq. 5/14 $\frac{w}{2T_0} = \frac{y}{x^2} = \frac{50}{(50)^2} = 0.02$, $\frac{wL}{2T_0} = 2$

So $S = \frac{100}{2}\sqrt{1+2^2} + \frac{100}{4}\ln\left(2+\sqrt{1+2^2}\right)$

$= 111.8 + 36.1 = \underline{147.9 \text{ ft}}$

5/206 Hole: $V = -9h$ (in.^{3}), $\bar{z} = h/2$

Cylinder: $V = \pi 6^2(10) = 1131$ in.^{3}, $\bar{z} = 5$ in.

$$\bar{z} = \frac{\Sigma \bar{z} V}{\Sigma V} = \frac{-9h\left(\frac{h}{2}\right) + 1131(5)}{-9h + 1131}$$

For max. $\bar{z}$, $\frac{d\bar{z}}{dh} = 0$

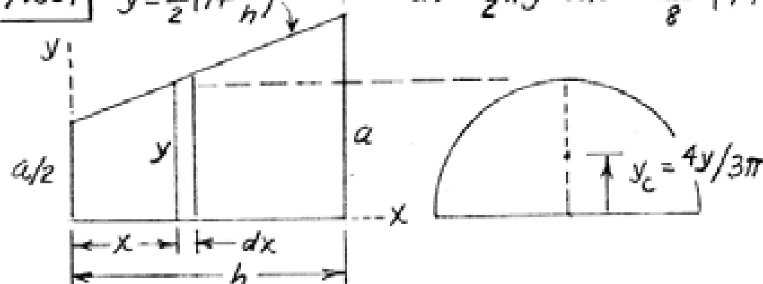
$$\frac{d\bar{z}}{dh} = \frac{(-9h + 1131)(-9h) - \left(-\frac{9h^2}{2} + 5655\right)(-9)}{(-9h + 1131)^2} = 0$$

$\Rightarrow 9h^2 - 1131h - 4.5h^2 + 5655 = 0$

$4.5h^2 - 1131h + 5655 = 0$

$$h = \frac{1131 \pm \sqrt{1131^2 - 4(4.5)(5655)}}{9} = \underline{5.10 \text{ in.}} \text{ or } 246 \text{ in.}$$

5/207



$dV = \frac{1}{2}\pi y^2 dx = \frac{\pi a^2}{8}(1+\frac{x}{h})^2 dx$

$$V = \frac{\pi a^2}{8}\int_0^h (1+\frac{x}{h})^2 dx = \frac{\pi a^2}{8}\frac{(1+\frac{x}{h})^3}{3}h\Big]_0^h = \frac{7\pi}{24}a^2 h$$

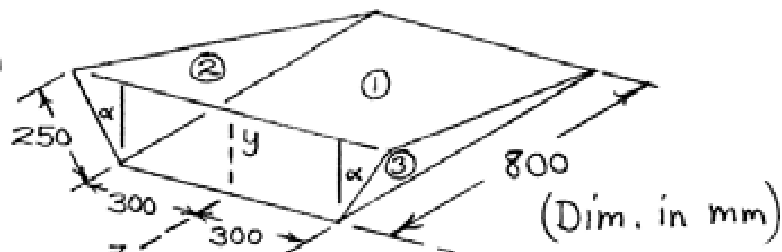
$$\int y_c\, dV = \int_0^h \frac{4}{3\pi}\frac{a}{2}(1+\frac{x}{h})\cdot\frac{\pi a^2}{8}(1+\frac{x}{h})^2 dx = \frac{a^3}{12}\int_0^h (1+\frac{x}{h})^3 dx$$

$$= \frac{a^3}{12}\frac{(1+\frac{x}{h})^4}{4}h\Big]_0^h = \frac{a^3}{48}15h = \frac{5a^3h}{16}$$

So $\bar{y} = \int y_c\, dV / V = \frac{5a^3h}{16}\frac{24}{7\pi a^2 h} = \underline{\frac{15a}{14\pi}}$

5/208

($\alpha = 30°$)



Comp.	A (mm^2)	$\bar{x}$ (mm)	$\bar{y}$ (mm)	$\bar{z}$ (mm)	$A\bar{x}$ (mm^3)	$A\bar{y}$ (mm^3)	$A\bar{z}$ (mm^3)
①	$600(800) = 4.8(10^5)$	0	0	−400	0	0	$-192(10^6)$
②	$\frac{250(800)}{2} = 10^5$	$-300 - \frac{250}{3}s30° = -342$	$\frac{250}{3}c30° = 72.2$	$\frac{-800}{3} = -267$	$-34.2(10^6)$	$7.22(10^6)$	$-26.7(10^6)$
③	10^5	342	72.2	−267	$+34.2(10^6)$	$7.22(10^6)$	$-26.7(10^6)$

$\Sigma A = 6.8(10^5)\ mm^2$

$$\begin{cases} \Sigma A\bar{x} = 0 \\ \Sigma A\bar{y} = 14.43(10^6)\ mm^3 \\ \Sigma A\bar{z} = -245(10^6)\ mm^3 \end{cases}$$

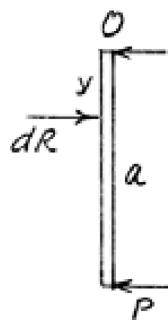
$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \underline{0}$$

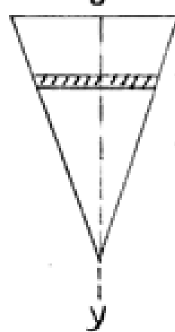
$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{14.43(10^6)}{6.8(10^5)} = \underline{21.2\ mm}$$

$$\bar{Z} = \frac{\Sigma A\bar{z}}{\Sigma A} = \frac{-245(10^6)}{6.8(10^5)} = \underline{-361\ mm}$$

5/209 $dA = 2x\,dy = \frac{b}{a}(a-y)\,dy$, $p = \rho g\,(h+y)$

$dR = \rho g\,(h+y)\frac{b}{a}(a-y)\,dy$





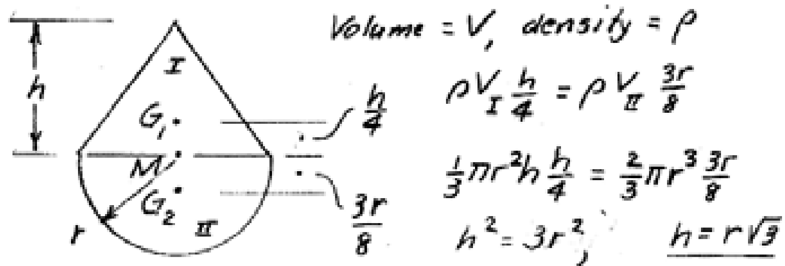
$\Sigma M_O = 0;\; Pa - \int y\,dR = 0$

$$Pa = \rho g \frac{b}{a}\int_0^a y(h+y)(a-y)\,dy$$

$$Pa = \rho g \frac{b}{a}\frac{a^3}{6}\left(h + \frac{a}{2}\right)$$

$$P = \frac{\rho g a b}{6}\left(h + \frac{a}{2}\right)$$

5/210 M = metacenter, so combined center of mass cannot be above M for stability



Volume = V, density = ρ

$$\rho V_I \frac{h}{4} = \rho V_{II} \frac{3r}{8}$$

$$\frac{1}{3}\pi r^2 h \frac{h}{4} = \frac{2}{3}\pi r^3 \frac{3r}{8}$$

$h^2 = 3r^2$, $\underline{h = r\sqrt{3}}$

5/211 Av. pressure $= \frac{1}{2}\rho g h = \frac{1}{2} 1.0(9.81)3$

$= 14.72$ kN/m^2

$R = pA = 14.72(3)(6)$

$= 264.9$ kN

$W = 8.5(9.81) = 83.4$ kN

C 3 m B

1 m

2 m

1 m

W

R

Length 6 m

A

P

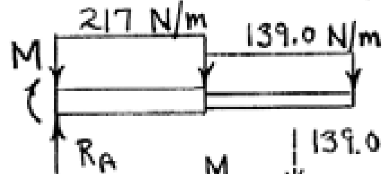
$\Sigma M_C = 0;\ 264.9(3) + 83.4(3) - P(3) = 0$

$\underline{P = 348 \text{ kN}}$

5/212 $\rho_{steel}\, g = 7830\,(9.81) = 76.8\,(10^3)\ \text{N/m}^3$

w = weight/meter = $\frac{\pi d^2}{4}(1)(76.8)10^3 = 60.3\,d^2(10^3)\ \frac{\text{N}}{\text{m}}$

For $d = 60(10^{-3})$ m, $w = 217$ N/m
For $d = 48(10^{-3})$ m, $w = 139.0$ N/m
(d in meters)



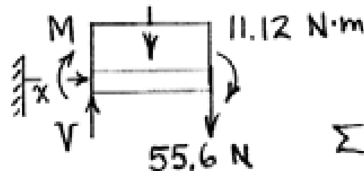
$0.4 < x < 0.8$ m

$\Sigma F = 0$: $V = 139.0(0.8 - x)$

$\Sigma M = 0$: $M + 139.0(0.8-x)\frac{0.8-x}{2} = 0$

$M = -69.5x^2 + 111.2x - 44.5$

At $x = 0.4$ m, $\begin{cases} M = -11.12\ \text{N·m} \\ V = 55.6\ \text{N} \end{cases}$



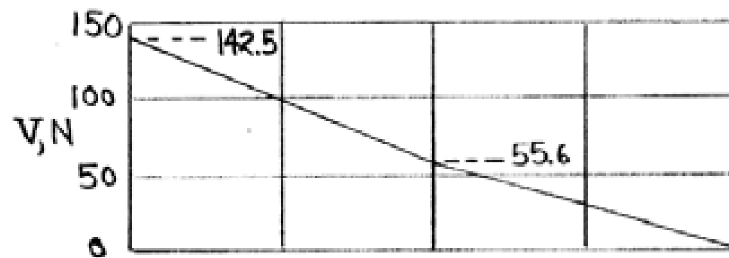
$0 < x < 0.4$ m

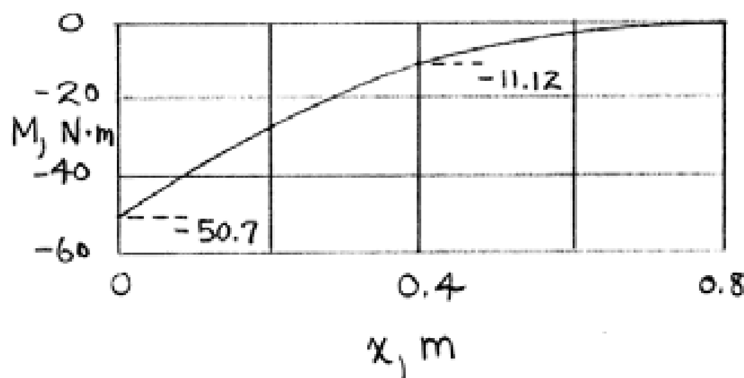
$\Sigma F = 0$: $V = 142.5 - 217x$

$\Sigma M = 0$: $M + 11.12 + 55.6(0.4-x) + 217\frac{(0.4-x)^2}{2} = 0$

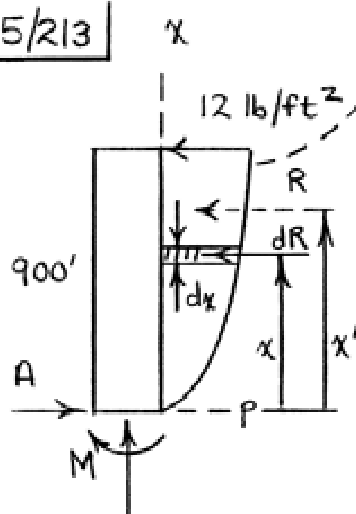
$M = -108.6x^2 + 142.5x - 50.7$

At $x = 0$, $\begin{cases} M = -50.7\ \text{N·m} \\ V = 142.5\ \text{N} \end{cases}$





5/213



$p = k\sqrt{x}$: $12 = k\sqrt{900}$

$k = 0.4 \dfrac{\text{lb}}{\text{ft}^{5/2}}$, $p = 0.4\sqrt{x}$

$dR = 200\,p\,dx = 200(0.4\sqrt{x})$

$= 80\sqrt{x}\,dx$

$R = \int dR = \int_0^{900'} 80\sqrt{x}\,dx$

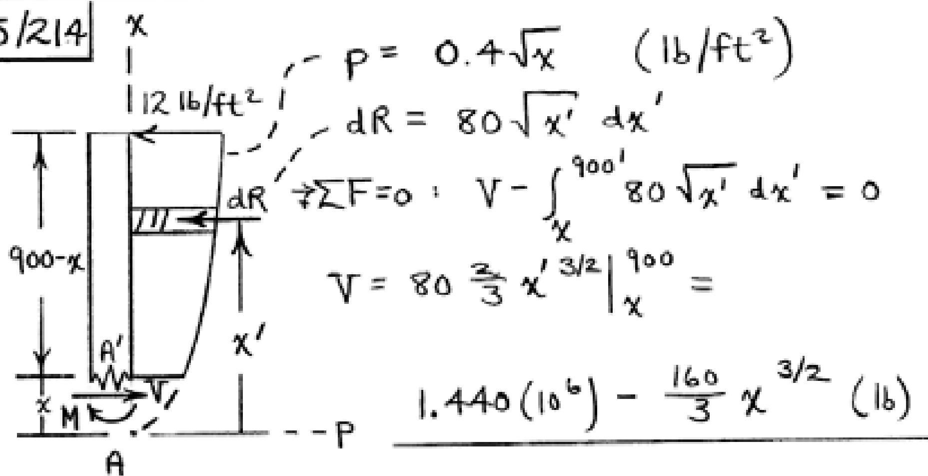
$= 80 \frac{2}{3} x^{3/2} \Big|_0^{900'} = 1.440(10^6)\text{ lb}$

$$\int x\,dR = \int_0^{900} 80\,x^{3/2}\,dx = 80\,\frac{2}{5}\,x^{5/2}\Big|_0^{900'}$$

$$= 7.78(10^8)\text{ lb-ft}$$

Thus, $\underline{A = 1.440(10^6)\text{ lb}}$

$\underline{M = 7.78(10^8)\text{ lb-ft}}$



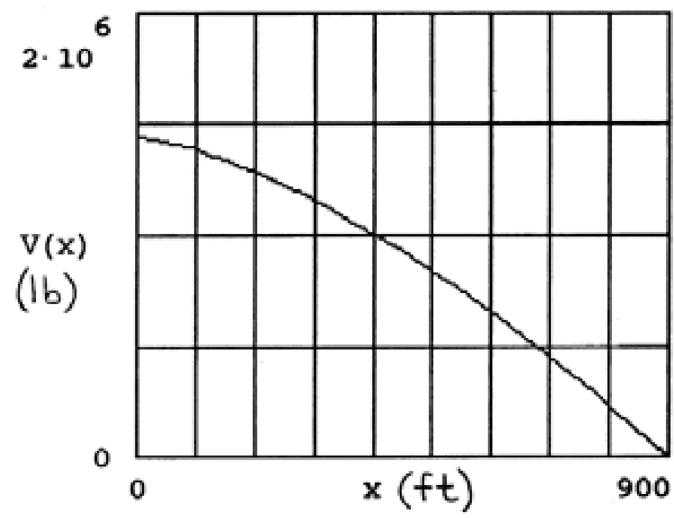
$$V\Big|_{x=450'} = 0.931(10^6)\ \text{lb}$$

$$\Sigma M_{A'} = 0: \quad M - \int_x^{900} 80\sqrt{x'}\,(x'-x)\,dx' = 0$$

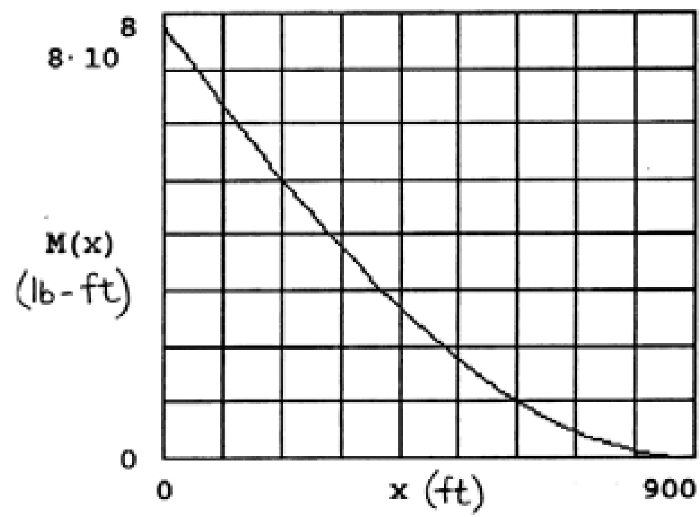
$$M = 80\left\{\frac{2}{5}x'^{5/2} - x\,\frac{2}{3}x'^{3/2}\right\}\Bigg|_x^{900}$$

$$= 7.78(10^8) - 1.440(10^6)x + \frac{64}{3}x^{5/2}$$

$$M\Big|_{x=450'} = 2.21(10^8)\ \text{lb-ft}$$

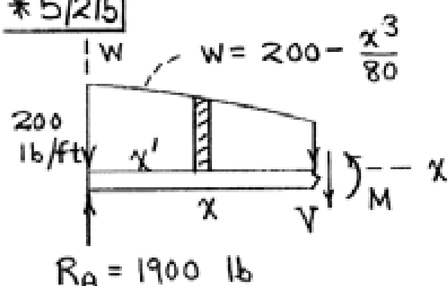


$2 \cdot 10^6$
V(x)
(lb)
0
0
x (ft)
900



$8 \cdot 10^8$
M(x)
(lb-ft)
0
0
x (ft)
900

*5/215 (w & R_A from Prob. 5/107)

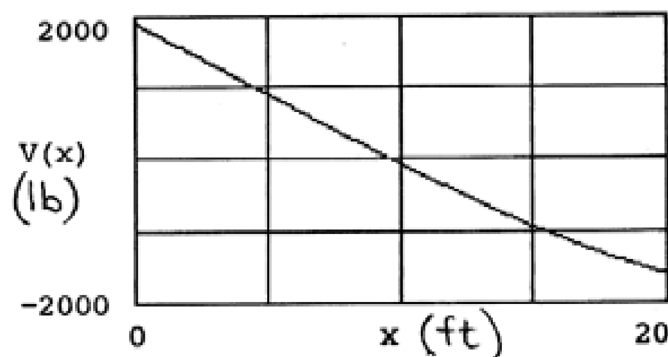


$+\uparrow\Sigma F = 0: \; 1900 - V - \int_0^x \left(200 - \frac{x'^3}{80}\right) dx'$

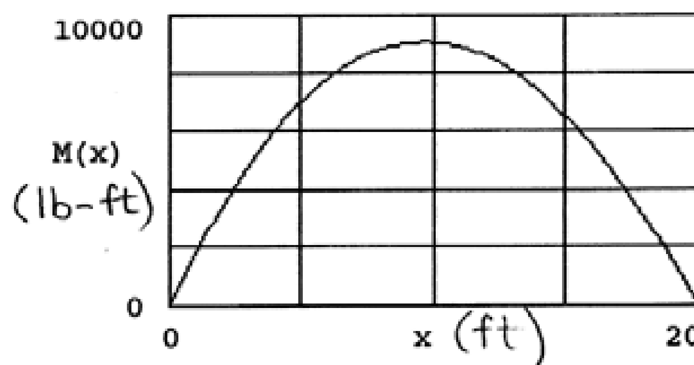
$$V = 1900 - 200x + \frac{x^4}{320} \quad \text{(lb)}$$

$\circlearrowleft \Sigma M = 0: \; M + \int_0^x \left(200 - \frac{x'^3}{80}\right)(x - x')\, dx' - 1900x = 0$

$$M = 1900x - 100x^2 + \frac{x^5}{1600} \quad \text{(lb-ft)}$$



$V_{max} = 1900$ lb @ $x = 0$

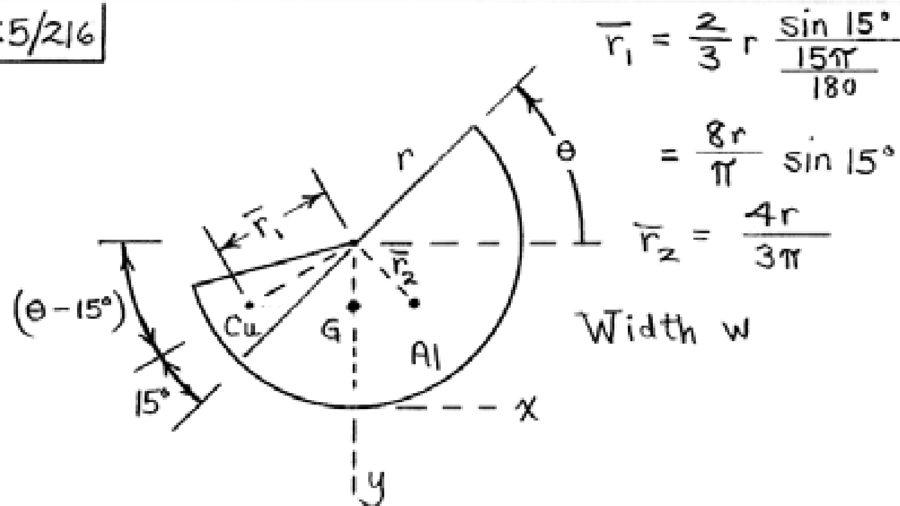


$M_{max} = 9080$ lb-ft @ $x = 9.63'$

***5/216**

$$\bar{r}_1 = \frac{2}{3} r \frac{\sin 15^\circ}{\frac{15\pi}{180}} = \frac{8r}{\pi} \sin 15^\circ$$

$$\bar{r}_2 = \frac{4r}{3\pi}$$



$$\bar{X} = \frac{\Sigma m\bar{x}}{\Sigma m} = 0 \quad \text{for equilibrium}$$

So $\quad m_{Cu}\left|\bar{x}_1\right| = m_{Al}\left|\bar{x}_2\right|$

$$\left(\frac{30}{360}\pi r^2 w \rho_{Cu}\right)\left(\frac{8r}{\pi}\sin 15^\circ\right)\cos(\theta - 15^\circ) = \left(\frac{\pi r^2}{2} w \rho_{Al}\right)\frac{4r}{3\pi}\sin\theta$$

Reduces to $\quad \dfrac{\rho_{Cu}}{\rho_{Al}} \sin 15^\circ \cos(\theta - 15^\circ) = \sin\theta$

With $\rho_{Cu} = 8910\ \text{kg/m}^3$ and $\rho_{Al} = 2690\ \text{kg/m}^3$,

a numerical solution yields $\underline{\theta = 46.8^\circ}$.

***5/217**

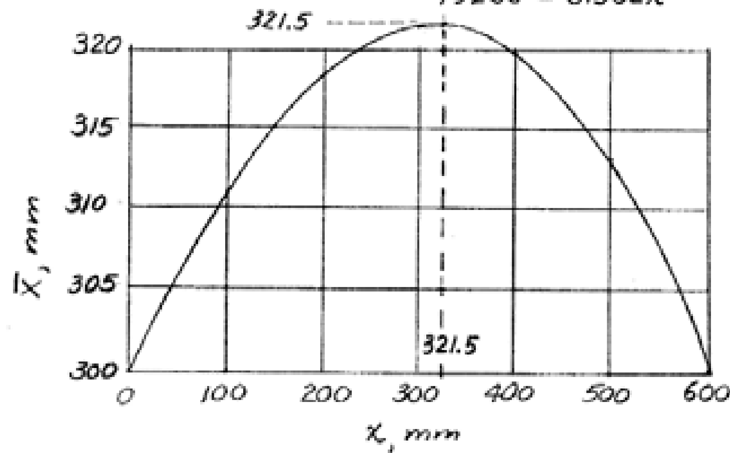
$$V_1 = \frac{\pi(180)^2}{4}600 = 15268\,(10^3)\ \text{mm}^3$$

$$\bar{x}_1 = 300\ \text{mm}$$

$$V_2 = -\frac{\pi(90)^2}{4}x = -6.362(10^3)x\,,\quad \bar{x}_2 = -x/2$$

$$\bar{X} = \frac{\Sigma V\bar{x}}{\Sigma V} = \frac{15268\,(300)10^3 - 3.181\,(10^3)x^2}{15268\,(10^3) - 6.362(10^3)x}$$

$$= \frac{4580(10^3) - 3.181x^2}{15268 - 6.362x}$$

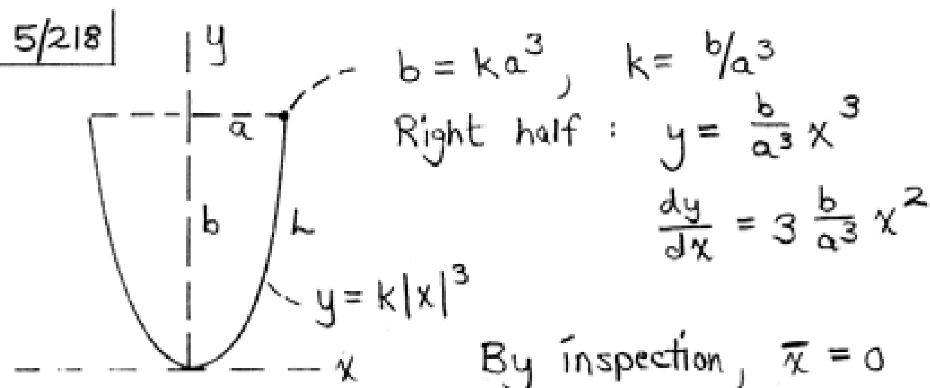


$\bar{X}_{max} = 322$ mm

at

$x = 322$ mm

5/218



$b = ka^3$, $k = b/a^3$

Right half: $y = \frac{b}{a^3} x^3$

$\frac{dy}{dx} = 3 \frac{b}{a^3} x^2$

By inspection, $\bar{x} = 0$

$$L = \int dL = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_0^a \sqrt{1 + 9 \frac{b^2}{a^6} x^4}$$

$$\int y\, dL = \int_0^a \frac{b}{a^3} x^3 \sqrt{1 + 9 \frac{b^2}{a^6} x^4}\, dx$$

$$= \frac{b}{a^3} \int_0^a x^3 \sqrt{1 + 9 \frac{b^2}{a^6} x^4}\, dx$$

$$\bar{y} = \frac{\int y\, dL}{\int dL} = \frac{\frac{b}{a^3} \int_0^a x^3 \sqrt{1 + 9 \frac{b^2}{a^6} x^4}\, dx}{\int_0^a \sqrt{1 + 9 \frac{b^2}{a^6} x^4}\, dx}$$

(Units of length ✓; integrals not in most tables)

For the given values,

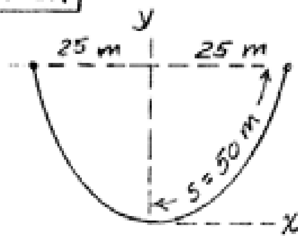
$k = b/a^3 = 8/2^3 = 1$, $y = x^3$, and

$$\bar{y} = \frac{\int_0^2 x^3 \sqrt{1 + 9x^4}\, dx}{\int_0^2 \sqrt{1 + 9x^4}\, dx} = \frac{N}{D}$$

Use a numerical approach such as the rectangular, trapezoidal, or Simpson technique for area determination and find

$N = 32.3$, $D = 8.63$, $\bar{y} = \frac{N}{D} = \underline{3.74}$

*5/219



Eq. 5/20 $s = \frac{T_0}{\mu} \sinh \frac{\mu x}{T_0}$

$50 = \frac{T_0}{\mu} \sinh \frac{25\mu}{T_0}$

$\frac{50\mu}{T_0} - \sinh \frac{25\mu}{T_0} = R = 0$

Write and run program for $R = f\left(\frac{\mu}{T_0}\right)$ & find μ/T_0 for $R=0$. Result is $\mu/T_0 = 0.0871$

From Eq. 5/19, $y = \frac{T_0}{\mu}\left(\cosh \frac{\mu x}{T_0} - 1\right)$

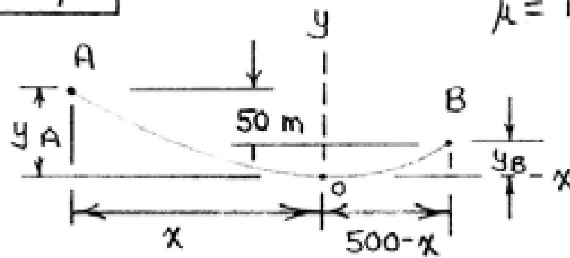
$= \frac{1}{0.0871}(\cosh 0.0871[25] \quad -1)$

$h = y = 3.468/0.0871 = \underline{39.8 \text{ m}}$

Result depends only on the geometry of the catenary.

*5/220

$\mu = 12(9.81) = 117.7 \ \text{N/m}$



From $s = \frac{T_0}{\mu} \sinh \frac{\mu}{T_0} x$:

$s_{OA} = \frac{T_0}{\mu} \sinh \frac{\mu}{T_0} x$ (1)

$s_{OB} = \frac{T_0}{\mu} \sinh\left[\frac{\mu}{T_0}(500 - x)\right]$ (2)

Also: $s_{OA} + s_{OB} = 505$ (3)

From $y = \frac{T_0}{\mu}\left[\cosh \frac{\mu}{T_0} x - 1\right]$:

$y_A = \frac{T_0}{\mu}\left[\cosh \frac{\mu}{T_0} x - 1\right]$ (4)

$y_B = \frac{T_0}{\mu}\left[\cosh \frac{\mu}{T_0}(500 - x) - 1\right]$ (5)

Also: $y_A - y_B = 50$ (6)

Numerical Sol. of (1) – (6):

$\frac{T_0}{\mu} = 1439 \ \text{m}$

$s_A = 398 \ \text{m}$

$s_B = 107.1 \ \text{m}$

$x = 393 \ \text{m}$

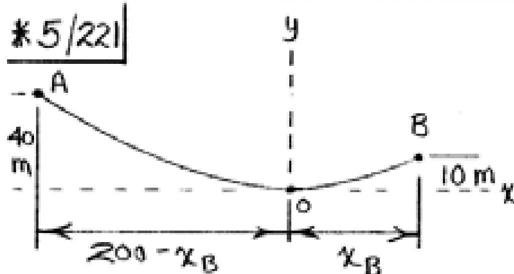
$y_A = 54.0 \ \text{m}$

$y_B = 3.98 \ \text{m}$

$\frac{T_0}{\mu} = \frac{T_0}{117.7} = 1439 \Rightarrow T_0 = 169\,400 \ \text{N}$

$T = T_0 + \mu y$: $\begin{cases} T_A = T_0 + 117.7(54) = \underline{175\,800 \ \text{N}} \\ T_B = T_0 + 117.7(3.98) = \underline{169\,900 \ \text{N}} \end{cases}$

***5/221**



$\mu_{cable} = 20(9.81) = 196.2\ \frac{N}{m}$

Eq. 5/19: $y = \frac{T_o}{\mu}\left[\cosh\frac{x}{T_o/\mu} - 1\right]$

At B: $10 = \frac{T_o}{\mu}\left[\cosh\frac{x_B}{T_o/\mu} - 1\right]$

At A: $40 = \frac{T_o}{\mu}\left[\cosh\frac{200 - x_B}{T_o/\mu} - 1\right]$

Simultaneous numerical solution: $\begin{cases} x_B = 67.1\text{ m} \\ T_o/\mu = 227\text{ m} \end{cases}$

$T_A = T_o + \mu y_A$: $75\,000 = 227\mu + \mu(40)$

$\mu = 281$ N/m

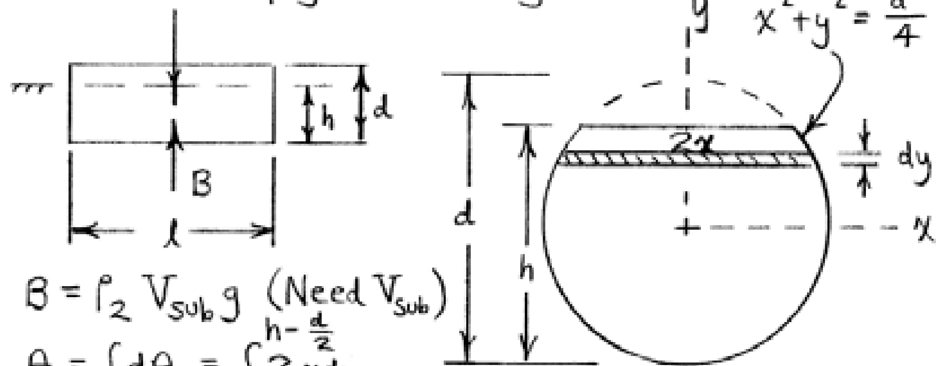
$\mu = \mu_{cable} + \mu_{ice}$: $281 = 196.2 + \mu_{ice}$

$\mu_{ice} = 84.7$ N/m

$\rho = \frac{\mu}{g} = \frac{84.7}{9.81} = \underline{8.63\text{ kg/m}}$

The configuration does not depend on μ.

*5/222 $W = \rho_1 V g = \rho_1 \pi \frac{d^2}{4} l g$



$B = \rho_2 V_{sub} g$ (Need V_{sub})

$$A = \int dA = \int_{-d/2}^{h-\frac{d}{2}} 2x\,dy$$

$$= 2\int_{-d/2}^{h-d/2} \sqrt{\frac{d^2}{4}-y^2}\,dy = 2\cdot\frac{1}{2}\left[y\sqrt{\frac{d^2}{4}-y^2} + \frac{d^2}{4}\sin^{-1}\frac{2y}{d}\right]_{-\frac{d}{2}}^{h-\frac{d}{2}}$$

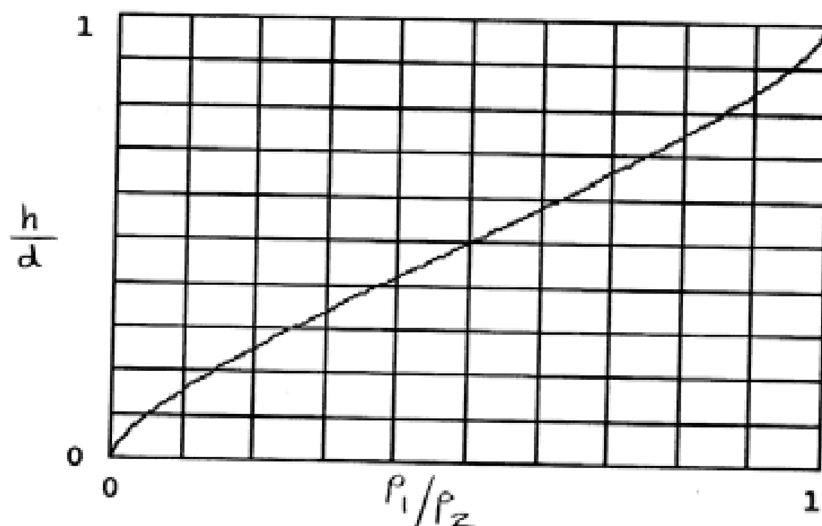
$$= \left(h-\frac{d}{2}\right)\sqrt{hd-h^2} + \frac{d^2}{4}\sin^{-1}\left(\frac{2h-d}{d}\right) + \frac{\pi d^2}{8}$$

So $B = \rho_2 g A l$, A given just above

$\uparrow \Sigma F = 0: \quad \rho_2 g A l - \rho_1 \pi \frac{d^2}{4} l g = 0$

or $\left(h-\frac{d}{2}\right)\sqrt{hd-h^2} + \frac{d^2}{4}\sin^{-1}\left(\frac{2h-d}{d}\right) + \frac{\pi d^2}{8} = \frac{\rho_1}{\rho_2}\pi\frac{d^2}{4}$

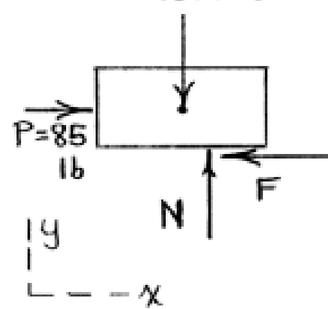
Strategy : Set $d=1$ and numerically solve the above equation for h for values of $\frac{\rho_1}{\rho_2}$ between 0 and 1:



For pine wood and salt water, $\rho_1 = 480\ \frac{kg}{m^3}$ and $\rho_2 = 1030\ \frac{kg}{m^3}$. So $\frac{\rho_1}{\rho_2} = \frac{480}{1030} = 0.466$

Numerical solution : $\frac{h}{d} = \underline{0.473} = r$

6/1



Assume equilibrium.

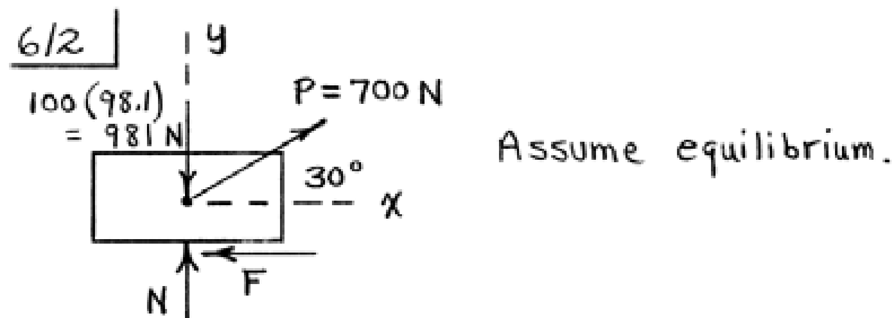
$\Sigma F_x = 0 \Rightarrow F = 85 \text{ lb}$

$\Sigma F_y = 0 \Rightarrow N = 200 \text{ lb}$

$F_{max} = \mu_s N = 0.5(200) = 100 \text{ lb}$

$F < F_{max}$, assumption valid, $\underline{F = 85 \text{ lb}}$

6/2



Assume equilibrium.

$\Sigma F_x = 0: \quad 700 \cos 30° - F = 0, \quad F = 606 \text{ N}$

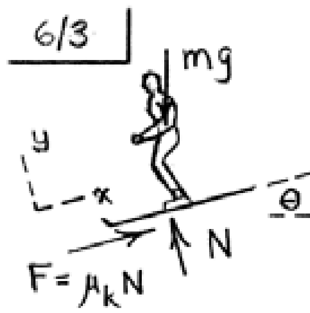
$\Sigma F_y = 0: \quad N - 981 + 700 \sin 30° = 0, \quad N = 631 \text{ N}$

$F_{max} = \mu_s N = 0.8 (631) = 505 \text{ N} < F = 606 \text{ N}$

Assumption invalid, motion occurs.

$F = \mu_k N = 0.6 (631) = \underline{379 \text{ N}}$

6/3



$\Sigma F_x = 0: \; 0.08N - mg\sin\theta = 0$

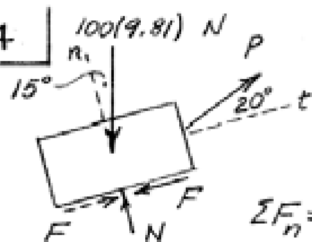
$N = \dfrac{mg\sin\theta}{0.08}$

$\Sigma F_y = 0: \; N - mg\cos\theta = 0$

$N = mg\cos\theta$

So $\dfrac{mg\sin\theta}{0.08} = mg\cos\theta$, $\tan\theta = 0.08$, $\underline{\theta = 4.57°}$

6/4



(a) Assume equilibrium

$\Sigma F_t = 0;\ 200 \cos 20° + F_{up} - 981 \sin 15° = 0$

$\underline{F = 66.0\ N}$

$\Sigma F_n = 0;\ N + 200 \sin 20° - 981 \cos 15° = 0$

$N = 879\ N$

& $F_{s_{max}} = 0.3(879) = 264\ N > 66.0\ N$ so equil. assumpt. OK

(b) $F_{dwn} = \mu_s N$; $\Sigma F_t = 0;\ P\cos 20° - 981 \sin 15° - 0.3N = 0$

$\Sigma F_n = 0;\ N + P\sin 20° - 981 \cos 15° = 0$

Solve & get $\underline{P = 516\ N}$

(c) Assume slipping up plane with $F_{dwn} = \mu_K N$

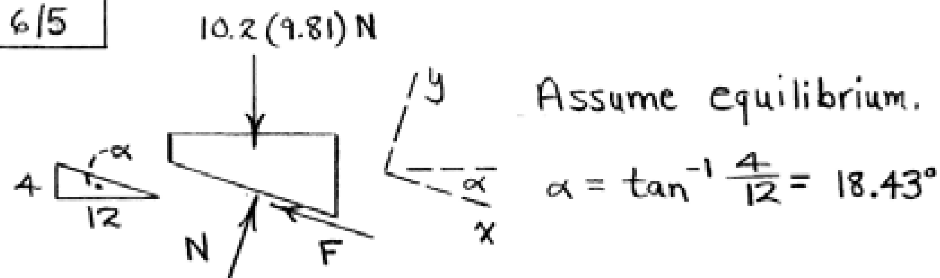
$\Sigma F_n = 0;\ 600 \sin 20° + N - 981 \cos 15° = 0,\ N = 742\ N$

$F = 0.2(742) = \underline{148.5\ N}$

$\Sigma F_t = 600 \cos 20° - 981 \sin 15° - 148.5 = 161.4\ N > 0$

so block moves up plane as assumed

6/5



Assume equilibrium.

$\alpha = \tan^{-1}\frac{4}{12} = 18.43°$

$\Sigma F_x = 0: \; -F + 10.2(9.81)\sin\alpha = 0, \quad F = 31.6 \text{ N}$

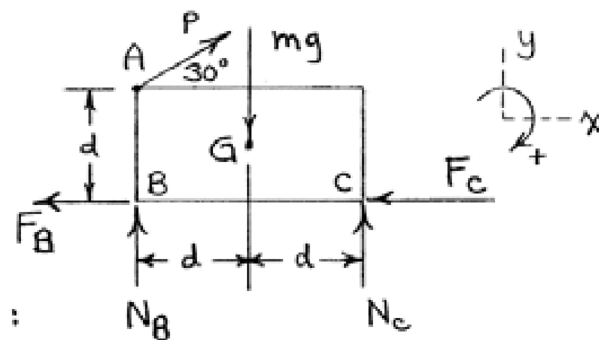
$\Sigma F_y = 0: \; N - 10.2(9.81)\cos\alpha = 0, \quad N = 94.9 \text{ N}$

$F_{max} = \mu_s N = 0.9(94.9) = 85.4 \text{ N} > F = 31.6 \text{ N}$

Assumption valid; $\underline{F = 31.6 \text{ N}}$ ↖

Total force $P = 10.2(9.81) = \underline{100.1 \text{ N} \; \uparrow}$

6/6



Slips:

$$\Sigma F_x = 0: \; -F_B - F_C + P\cos 30° = 0 \quad (1)$$

$$\Sigma F_y = 0: \; N_B + N_C + P\sin 30° - mg = 0 \quad (2)$$

With $F_B = \mu_s N_B$ & $F_C = \mu_s N_C$, combine (1) & (2) to obtain $P = \dfrac{\mu_s mg}{\mu_s \sin 30° + \cos 30°}$

With $\mu_s = 0.5$, $P = P_{slip} = \underline{0.448 mg}$

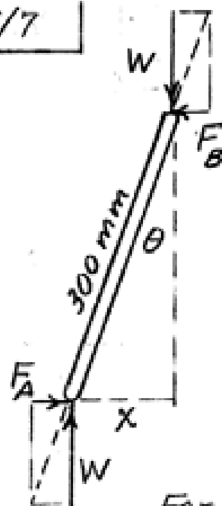
Tips ($N_B, F_B \rightarrow 0$):

$$\Sigma M_G = 0: \; (P\cos 30°)d + (P\sin 30°)(2d) - mg(d) = 0$$

$$\Rightarrow P = \frac{mg}{\cos 30° + 2\sin 30°} = \underline{0.536 mg = P_{tip}}$$

For these conditions, slipping would occur first.

6/7



For $x = 75$ mm, $\theta = \sin^{-1}\frac{75}{300} = 14.5°$

Friction angle $\phi = \tan^{-1}\mu_s$

for A is $\phi_A = \tan^{-1}0.40 = 21.8°$

" B " $\phi_B = \tan^{-1}0.30 = 16.7°$

Since $\theta < \phi_A$ & ϕ_B, bar does not slip & $F_A = F_B = W\tan\theta$

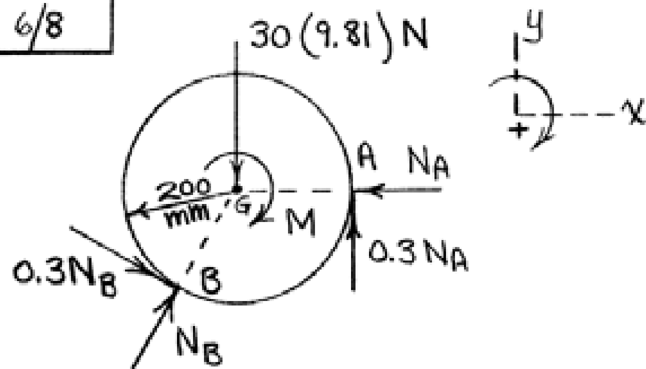
$= 50(9.81)\tan 14.5°$

$= \underline{126.6 \text{ N}}$

For increased x bar slips first at B with $\theta = \phi_B = 16.7°$. Thus $x_{max} = 300 \sin 16.7°$

$= \underline{86.2 \text{ mm}}$

6/8

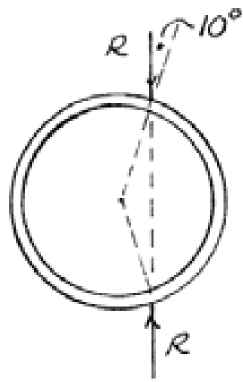


$$\begin{cases} \Sigma M_G = 0: & M - 0.3(N_A + N_B)\,0.2 = 0 & (1) \\ \Sigma F_x = 0: & N_B \sin 30° + 0.3 N_B \cos 30° - N_A = 0 & (2) \\ \Sigma F_y = 0: & N_B \cos 30° - 0.3 N_B \sin 30° - 30(9.81) + 0.3 N_A = 0 & (3) \end{cases}$$

Solution of Eqs. (1)–(3):

$$\begin{cases} N_B = 312 \text{ N} \\ N_A = 237 \text{ N} \\ \underline{M = 32.9 \text{ N}\cdot\text{m}} \end{cases}$$

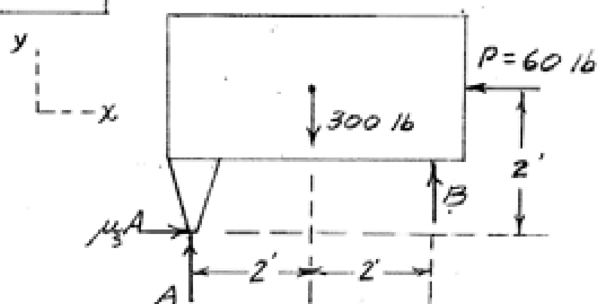
6/9



$\mu_{s\,min} = \tan\phi = \tan 10°$

$= \underline{0.176}$

6/10



$\Sigma M_A = 0;\ 60(2) + 4B - 300(2) = 0,\quad B = 120\text{ lb}$

$\Sigma F_y = 0;\ A + 120 - 300 = 0,\quad A = 180\text{ lb}$

$\Sigma F_x = 0;\ \mu_s(180) - 60 = 0,\quad \underline{\mu_s = 0.33}$

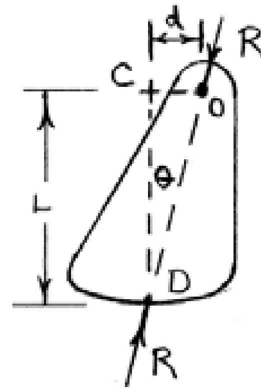
6/11

$\theta_{max} = \phi = \tan^{-1}\mu_s = \tan^{-1}\frac{d}{L}$

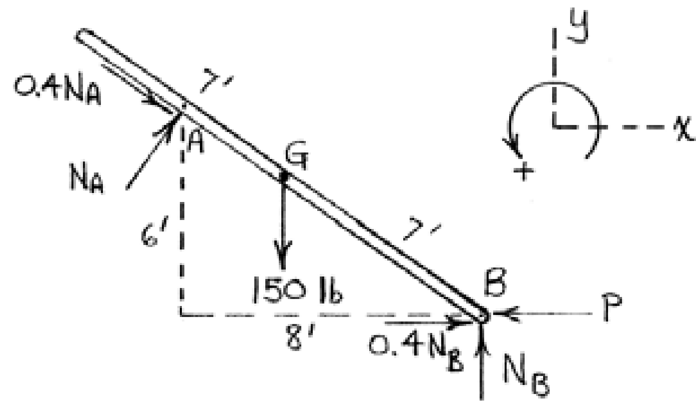
So $\mu_s = \frac{d}{L}$, $d = \mu_s L = 0.2L$

Device will work for all

$\underline{d \leq 0.2L}$



6/12



$$\Sigma M_B = 0: \; 150\left(\tfrac{4}{5}7\right) - 10\,N_A = 0 \qquad N_A = 84 \text{ lb}$$

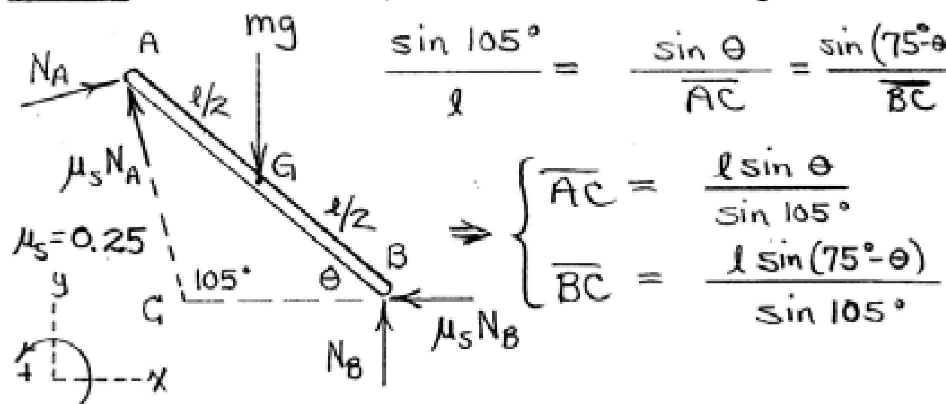
$$\Sigma F_y = 0: \; N_B - 150 + \tfrac{4}{5}(84) - 0.4(84)\tfrac{3}{5} = 0,$$

$$N_B = 103.0 \text{ lb}$$

$$\Sigma F_x = 0: \; -P + 0.4(103.0) + 84\left(\tfrac{3}{5}\right) + 0.4(84)\tfrac{4}{5} = 0$$

$$\underline{P = 118.5 \text{ lb}}$$

6/13



From law of sines:

$$\frac{\sin 105^\circ}{\ell} = \frac{\sin\theta}{\overline{AC}} = \frac{\sin(75^\circ-\theta)}{\overline{BC}}$$

$$\Rightarrow \begin{cases} \overline{AC} = \dfrac{\ell \sin\theta}{\sin 105^\circ} \\ \overline{BC} = \dfrac{\ell \sin(75^\circ-\theta)}{\sin 105^\circ} \end{cases}$$

$\Sigma F_x = 0$: $N_A \cos 15^\circ - 0.25 N_A \sin 15^\circ - 0.25 N_B = 0$

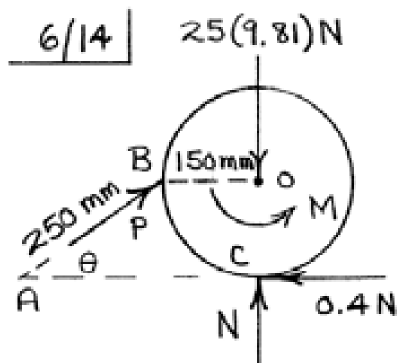
$\Sigma F_y = 0$: $N_A \sin 15^\circ + 0.25 N_A \cos 15^\circ + N_B - mg = 0$

$\Sigma M_C = 0$: $-N_A \left(\dfrac{\ell \sin\theta}{\sin 105^\circ}\right) + N_B \left(\dfrac{\ell \sin(75^\circ-\theta)}{\sin 105^\circ}\right)$

$$- mg\left[\frac{\ell \sin(75^\circ-\theta)}{\sin 105^\circ} - \frac{\ell}{2}\cos\theta\right] = 0$$

Solution: $\begin{cases} N_A = 0.244\,mg \\ N_B = 0.878\,mg \\ \theta = 59.9^\circ \end{cases}$

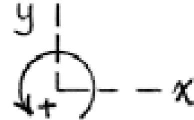
6/14



$\sin\theta = \frac{150}{250} = 0.6$

$\cos\theta = 0.8$

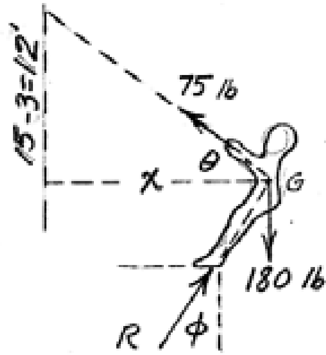
$\overline{AC} = 0.8(0.25) + 0.15 = 0.35\text{ m}$



$$\begin{cases} \Sigma F_x = 0: & 0.8P - 0.4N = 0 \\ \Sigma F_y = 0: & N + 0.6P - 25(9.81) = 0 \\ \Sigma M_A = 0: & M + (N - 25(9.81))(0.35) = 0 \end{cases}$$

Solution: $N = 188.7\text{ N}$, $P = 94.3\text{ N}$, $\underline{M = 19.81\text{ N}\cdot\text{m}}$

6/15



75 lb, 180 lb, θ, β, φ, R

$\phi = \tan^{-1} 0.40 = 21.8°$

$\beta = 90 - 21.8 = 68.2°$

Law of sines

$$\frac{180}{\sin(\theta+\beta)} = \frac{75}{\sin 21.8°}$$

$$\theta+\beta = \sin^{-1} \frac{180 \sin 21.8}{75}$$

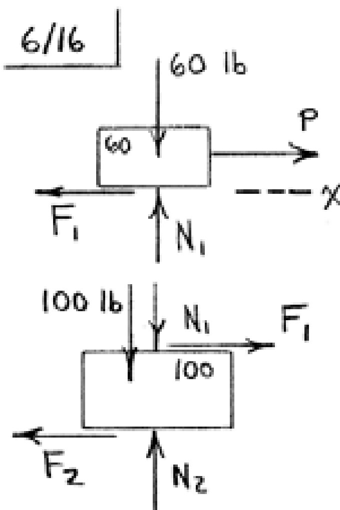
$$= \sin^{-1} 0.891 = 63.0° \quad \text{or } 117.0°$$

63.0° sol. not possible,

so $\theta = 117.0 - 68.2 = 48.8°$

$\frac{12}{x} = \tan 48.8°$, $x = 12/1.14 = \underline{10.52 \text{ ft}}$

6/16



(a) P applied to 60-lb block

(Note: $N_1 = 60$ lb & $N_2 = 160$ lb throughout)

Assume 60-lb block slips by itself ($F_1 = \mu_{s_1} N_1$)

$\Sigma F_x = 0$: $P - \mu_{s_1} N = 0$

$P = \mu_{s_1} N_1 = 0.4(60) = 24$ lb

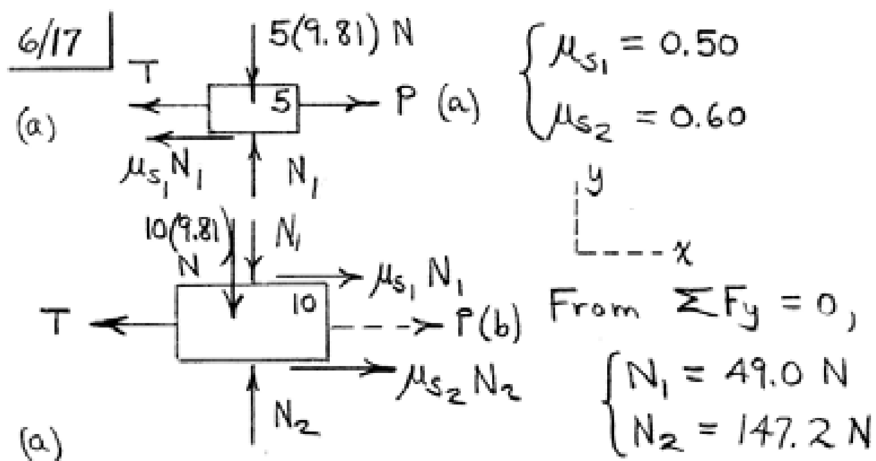
Check on 100-lb block:

$\Sigma F_x = 0$: $24 - F_2 = 0$, $F_2 = 24$ lb

But $F_{2max} = \mu_{s_2} N_2 = 0.12(160) = 19.2$ lb

So the 60-lb does not slip by itself; rather, the two blocks move as a unit. In both cases (a) & (b),

$$P = \mu_{s_2} N_2 = 0.12(160) = \underline{19.2 \text{ lb}}$$



$$\Sigma F_x = 0: \begin{cases} P - T - 0.50(49.0) = 0 \\ -T + 0.50(49.0) + 0.60(147.2) = 0 \end{cases}$$

$\underline{T = 112.8 \text{ N}}$, $\underline{P = 137.3 \text{ N}}$

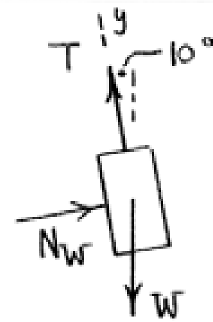
(b) Now P is applied to 10-kg block & we reverse all friction forces above:

$$\Sigma F_x = 0: \begin{cases} -T + 0.50(49.0) = 0 \\ -T - 0.50(49.0) - 0.60(147.2) + P = 0 \end{cases}$$

$\underline{T = 24.5 \text{ N}}$, $\underline{P = 137.3 \text{ N}}$

6/18 $\Sigma F_y = 0: T - W\cos 10° = 0$

$T = W\cos 10°$

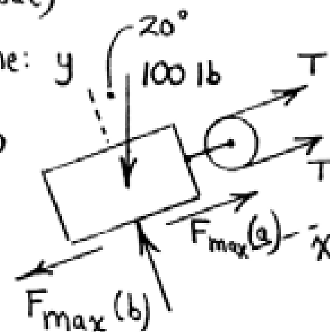


100-lb block:

$\Sigma F_y = 0: \quad N - 100\cos 20° = 0$

$N = 94.0$ lb (throughout)

(a) Motion impends down incline:



$\Sigma F_x = 0: \quad 2T - 100\sin 20° + F_{max} = 0$

With $F_{max} = \mu_s N = 0.3(94.0)$

$= 28.2$ lb and $T = W\cos 10°$,

$W = 3.05$ lb

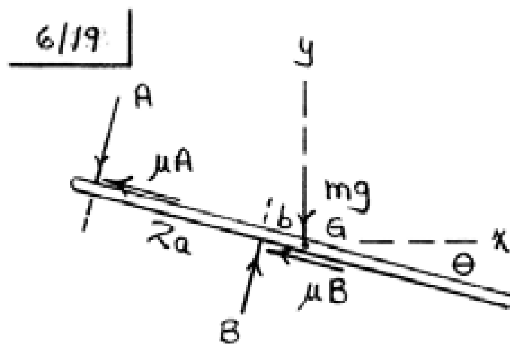
(b) Motion impends up incline:

$\Sigma F_x = 0: \quad 2T - 100\sin 20° - F_{max} = 0$

Similarly, $W = 31.7$ lb

Hence the allowable range is $\underline{3.05 \leq W \leq 31.7 \text{ lb}}$

6/19



$\circlearrowleft \Sigma M_G = 0: \quad A(2a+b) - Bb = 0 \; , \quad \dfrac{B}{A} = \dfrac{2a+b}{b}$

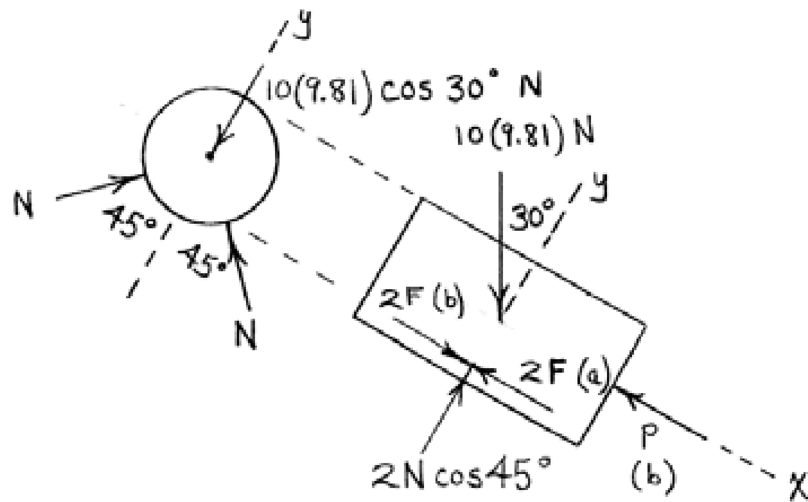
$\Sigma F_x = 0: \quad (B-A)\sin\theta - \mu(A+B)\cos\theta = 0$

$$\tan\theta = \mu\,\frac{A+B}{B-A} = \mu\,\frac{1+B/A}{\frac{B}{A}-1}$$

Substitute $\dfrac{B}{A}$: $\tan\theta = \mu\,\dfrac{1+(2a+b)/b}{(2a+b)/b - 1}$

or $\quad \underline{\theta = \tan^{-1}\left(\mu\,\dfrac{a+b}{a}\right)}$

6/20



$\Sigma F_y = 0$: $2N\cos 45° - 10(9.81)\cos 30° = 0$, $N = 60.1$ N

(a) $P = 0$

$\Sigma F_x = 0$: $-2F + 10(9.81)\sin 30° = 0$, $F = 24.5$ N

Check: $F_{max} = \mu_s N = 0.5(60.1) = 30.0\text{ N} > F = 24.5\text{ N}$

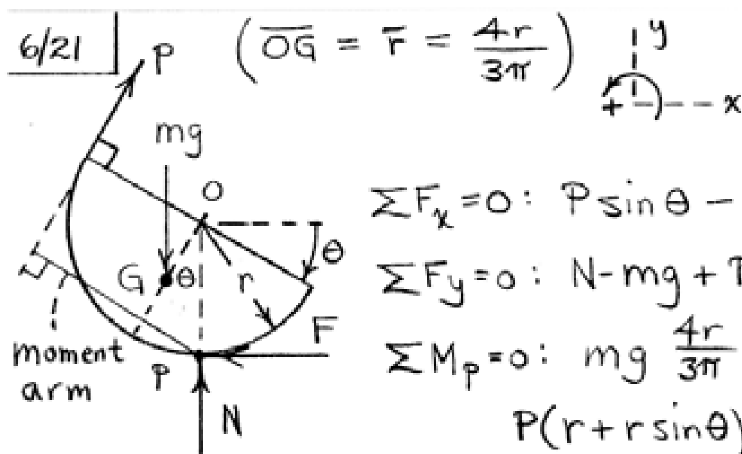
So we indeed have static equilibrium.

(b) $P \neq 0$

$\Sigma F_x = 0$: $-P + 10(9.81)\sin 30° + 2(0.5 \cdot 60.1) = 0$

$P = 109.1$ N

6/21 $\left(\overline{OG} = \bar{r} = \frac{4r}{3\pi}\right)$



$\Sigma F_x = 0: \quad P\sin\theta - F = 0 \quad (1)$

$\Sigma F_y = 0: \quad N - mg + P\cos\theta = 0 \quad (2)$

$\Sigma M_P = 0: \quad mg\,\frac{4r}{3\pi}\sin\theta - P(r + r\sin\theta) = 0 \quad (3)$

When slipping impends, $F = \mu_s N \quad (4)$

$(4) \rightarrow (1): \quad P\sin\theta - \mu_s N = 0 \quad (5)$

$(2): \quad N = mg - P\cos\theta \quad (6)$

$(6) \rightarrow (5): \quad P\sin\theta - \mu_s(mg - P\cos\theta) = 0$

$$\Rightarrow P = \frac{\mu_s mg}{\sin\theta + \mu_s\cos\theta} \quad (7)$$

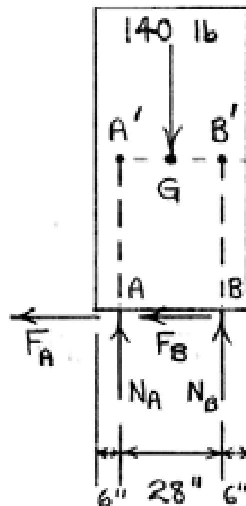
$(7) \rightarrow (3)$ & simplification yields

$$\mu_s = \frac{4\sin^2\theta}{3\pi(1+\sin\theta) - 4\sin\theta\cos\theta}$$

For $\theta = 40°$, $\underline{\mu_s = 0.1223}$

(7) then gives $\underline{P = 0.1661\,mg}$

6/22



(a) Set $F_B = 0$, $F_A = \mu N_A$.

$\circlearrowleft + \Sigma M_{B'} = 0 : 140(14) - N_A(28) - 0.3 N_A(40) = 0, \quad N_A = 49 \text{ lb}$

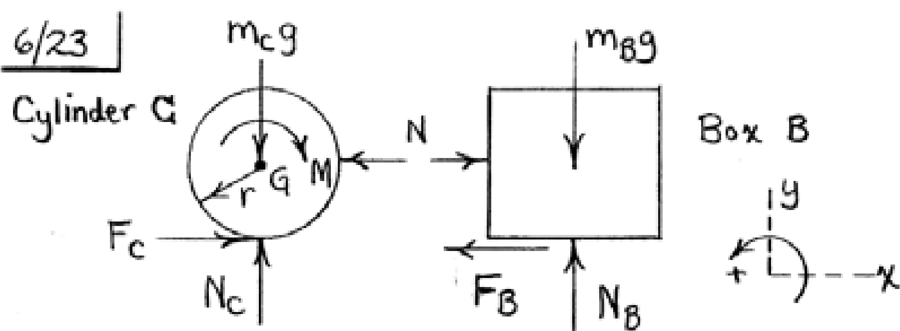
$\xrightarrow{+} \Sigma F = 0 : P - \mu N_A = 0, \quad \underline{P = 14.7 \text{ lb}}$

(b) Set $F_A = 0$, $F_B = \mu N_B$.

$\circlearrowleft + \Sigma M_{A'} = 0 : -140(14) + N_B(28) - 0.3 N_B(40) = 0, \quad N_B = 122.5 \text{ lb}$

$\xrightarrow{+} \Sigma F = 0 : P - \mu N_B = 0, \quad \underline{P = 36.8 \text{ lb}}$

(Note : Above $\mu = \mu_k = 0.3$)



Assume that box slips but cylinder does not.

$F_B = (\mu_s)_B N_B$.

B $\begin{cases} \Sigma F_x = 0: & N - F_B = 0 & (1) \\ \Sigma F_y = 0: & N_B - m_B g = 0 & (2) \end{cases}$

So $N_B = m_B g$, $N = F_B = (\mu_s)_B m_B g$

C $\begin{cases} \Sigma F_x = 0: & F_c - N = 0 & (3) \\ \Sigma F_y = 0: & N_c - m_c g = 0 & (4) \\ \Sigma M_G = 0: & F_c r - M = 0 & (5) \end{cases}$

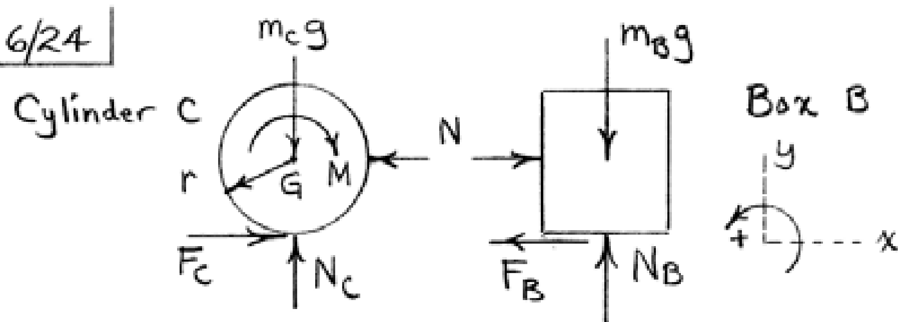
$M = F_c r = N r = (\mu_s)_B m_B g r$

$= 0.5(3)(9.81)(0.2) = \underline{2.94\ \text{N}\cdot\text{m}}$

$F_c = N = (\mu_s)_B m_B g = (0.5)(3)(9.81) = 14.72\ \text{N}$

$< (F_c)_{max} = (\mu_s)_c m_c g = (0.4)(6)(9.81) = 23.5\ \text{N}$

6/24



Assume that cylinder slips but box does not.

$F_C = (\mu_s)_C N_C$.

$$B \begin{cases} \Sigma F_x = 0: \quad N - F_B = 0 & (1) \\ \Sigma F_y = 0: \quad N_B - m_B g = 0 & (2) \end{cases}$$

So $N_B = m_B g$, $N = F_B$

$$C \begin{cases} \Sigma F_x = 0: \quad (\mu_s)_C N_C - N = 0 & (3) \\ \Sigma F_y = 0: \quad N_C - m_C g = 0 & (4) \\ \Sigma M_G = 0: \quad (\mu_s)_C N_C r - M = 0 & (5) \end{cases}$$

$M = (\mu_s)_C m_C g r = 0.2(6)(9.81)(0.2) = \underline{2.35 \text{ N}\cdot\text{m}}$

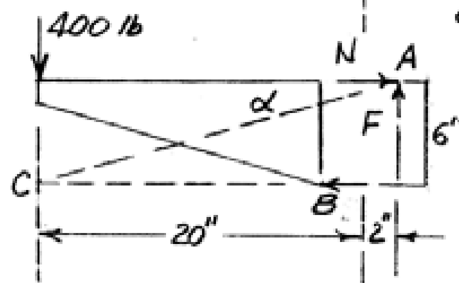
$N = (\mu_s)_C m_C g = 0.2(6)(9.81) = 11.77 \text{ N}$

$F_B = N = 11.77 \text{ N}$

$(F_B)_{max} = (\mu_s)_B N_B = 0.5(3)(9.81) = 14.72 \text{ N} > F_B$

Assumption OK

6/25



For equilibrium forces must be concurrent at C so that

$$\alpha = \tan^{-1}\frac{6}{22} = \tan^{-1} 0.27$$

Since 0.27 < 0.40, collar will not slip &

$\Sigma F = 0$; $\underline{F = 400 \text{ lb}}$

6/26 (a) P to the right.

y, x, $+$ (sign convention)

Labels on the free-body diagrams: B_y, mg, $l/2$, B_x, B, A, $l/2$, G, θ, $\mu_s N_A$, N_A, $\mu_s N_A$, $m_0 g$, P, $\mu_s N$, N

$\Sigma M_B = 0$: $mg\frac{l}{2}\cos\theta - N_A l\cos\theta + \mu_s N_A l\sin\theta = 0$ (1)

(Box) $\Sigma F_x = 0$: $P - \mu_s N_A - \mu_s N = 0$ (2)

$\Sigma F_y = 0$: $N - m_0 g - N_A = 0$ (3)

Solve for P as

$$P = \mu_s g\left[\frac{m\cos\theta}{\cos\theta - \mu_s\sin\theta} + m_0\right]$$

(b) P to the left. Reverse P and all friction forces in the above FBD's & obtain

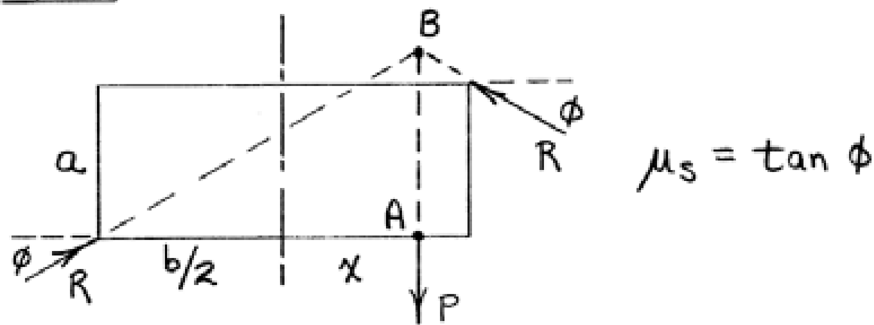
$$P = \mu_s g\left[\frac{m\cos\theta}{\cos\theta + \mu_s\sin\theta} + m_0\right]$$

With $\theta = 30°$, $m = m_0 = 3\text{kg}$, and $\mu_s = 0.60$, we obtain

(a) $P = 44.7$ N

(b) $P = 30.8$ N

6/27



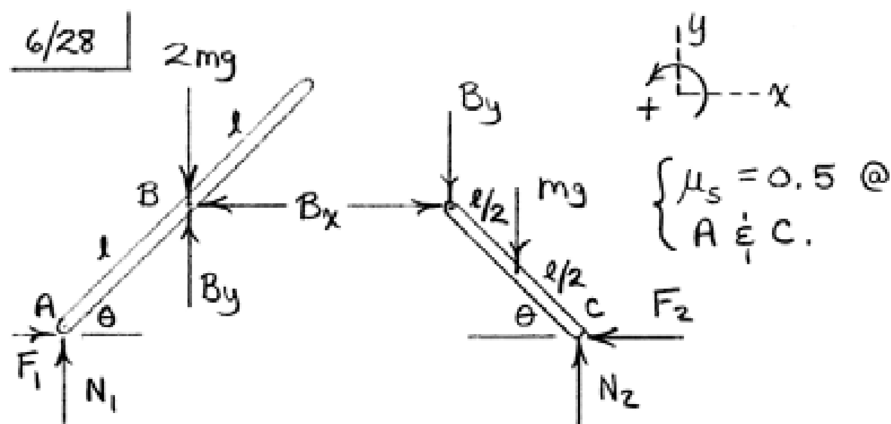
$\mu_s = \tan\phi$

Binding occurs for max. x when ϕ becomes the friction angle or less. For equilibrium, forces are concurrent at B. Thus

$$\overline{AB} = \left(\frac{b}{2}+x\right)\tan\phi = a + \left(\frac{b}{2}-x\right)\tan\phi$$

$$\Rightarrow \quad x = \frac{a}{2\mu_s}$$

6/28



AB
$$\begin{cases} \Sigma F_x = 0: & F_1 - B_x = 0 & (1) \\ \Sigma F_y = 0: & N_1 + B_y - 2mg = 0 & (2) \\ \Sigma M_A = 0: & B_x(l\sin\theta) + B_y(l\cos\theta) - 2mg(l\cos\theta) = 0 & (3) \end{cases}$$

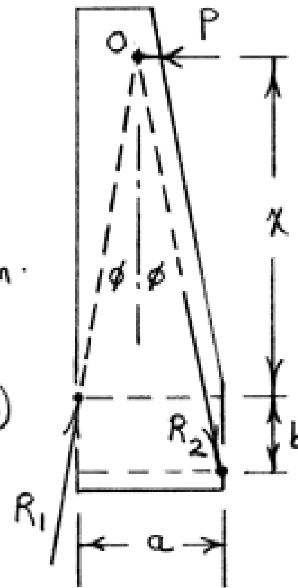
BC
$$\begin{cases} \Sigma F_x = 0: & B_x - F_2 = 0 & (4) \\ \Sigma F_y = 0: & -B_y - mg + N_2 = 0 & (5) \\ \Sigma M_C = 0: & mg\left(\frac{l}{2}\cos\theta\right) + B_y(l\cos\theta) - B_x(l\sin\theta) = 0 & (6) \end{cases}$$

Assume first slippage at A: $F_1 = 0.5N_1$. Solve seven equations to obtain $\underline{\theta = 63.4°}$, $F_2 = 0.625\,mg$, & $N_2 = 1.75\,mg$. Note $F_2 < F_{2max} = 0.875\,mg$.

Then assume first slippage at B: $F_2 = 0.5N_2$. Obtain $\theta = 55.0°$, $F_1 = 0.875\,mg$ & $N_1 = 1.25\,mg$. Note $F_1 > F_{1max} = 0.625\,mg$. So A slips first.

6/29

For equilibrium, the forces must be concurrent at O and $\phi = \tan^{-1}\mu_s$ for impending motion with $x = x_{min}$.

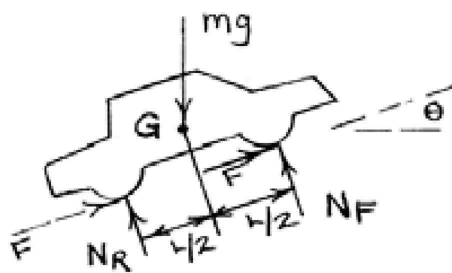


$$a = x\tan\phi + (x+b)\tan\phi$$

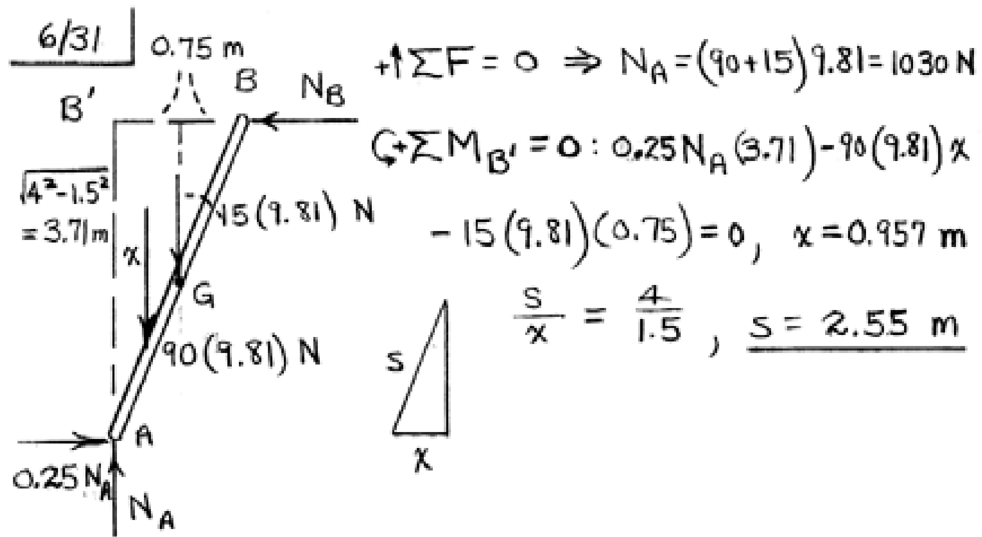
$$= x\mu_s + (x+b)\mu_s = \mu_s(2x+b)$$

$$x = \frac{a - b\mu_s}{2\mu_s}$$

6/30 Consider the FBD below and the equilibrium equation $\Sigma M_G = 0$. The presence of the propulsive friction forces F, whether applied at the front or at the rear, increases the rear normal forces and decreases the front ones. Increased normals mean increased available propulsive friction forces.

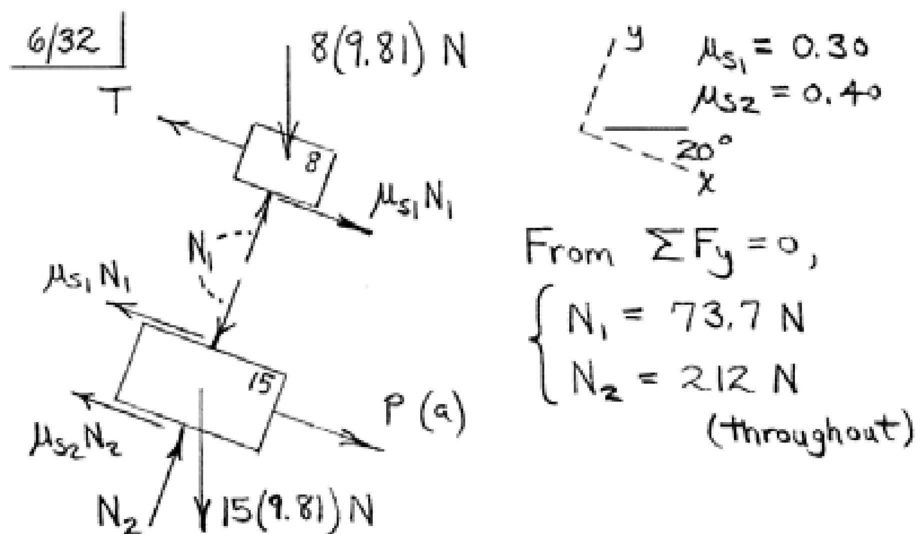


Thus the <u>rear-wheel drive</u> car would climb the steeper grade.



6/31
0.75 m
B
B′
N_B
√(4²−1.5²) = 3.71 m
x
15(9.81) N
G
90(9.81) N
A
0.25 N_A
N_A
+↑ΣF = 0 ⇒ N_A = (90+15)9.81 = 1030 N
ΣM_B′ = 0 : 0.25 N_A (3.71) − 90(9.81) x − 15(9.81)(0.75) = 0, x = 0.957 m
s/x = 4/1.5 , s = 2.55 m
s
x

6/32

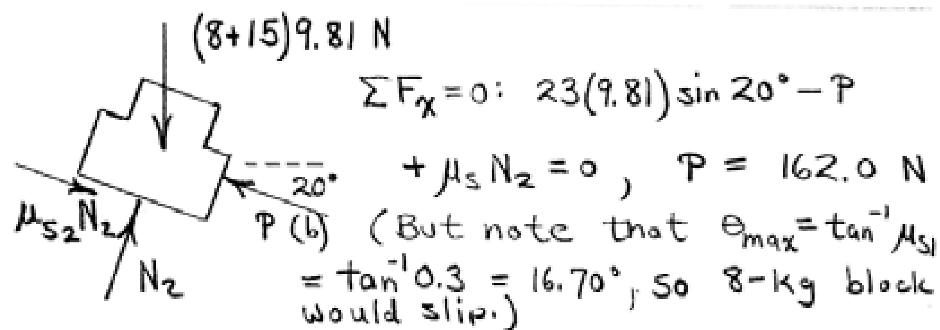


(a) $\Sigma F_x = 0$:

$$\left.\begin{aligned} -T + 8(9.81)\sin 20° + \mu_{s_1} N_1 &= 0 \\ -\mu_{s_1} N_1 - \mu_{s_2} N_2 + 15(9.81)\sin 20° + P &= 0 \end{aligned}\right\}$$

Solution: $\underline{P = 56.6 \text{ N}}$, $T = 49.0$ N

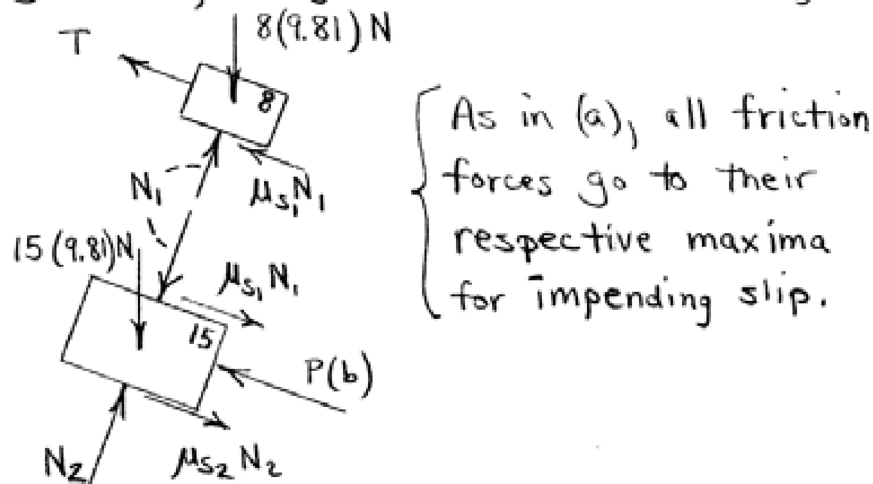
(b) Assume T goes slack and both masses move as one unit.



$\Sigma F_x = 0$: $23(9.81)\sin 20° - P + \mu_s N_2 = 0$, $P = 162.0$ N

(But note that $\theta_{max} = \tan^{-1}\mu_{s1} = \tan^{-1}0.3 = 16.70°$; so 8-kg block would slip.)

(b), continued. Assume that T does not go slack; 8-kg block remains stationary.



As in (a), all friction forces go to their respective maxima for impending slip.

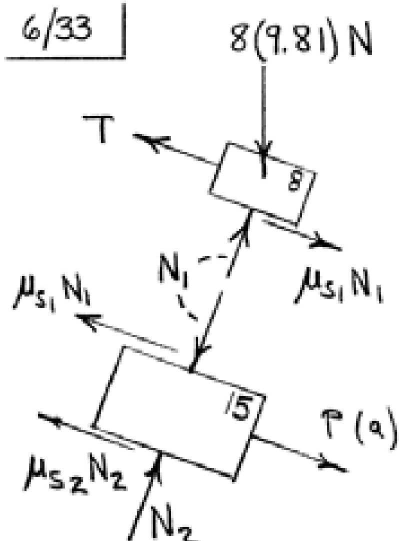
$\Sigma F_x = 0$:

$$\left.\begin{aligned} -T + 8(9.81)\sin 20° - \mu_{s1}N_1 &= 0 \\ \mu_{s1}N_1 + \mu_{s2}N_2 + 15(9.81)\sin 20° - P &= 0 \end{aligned}\right\}$$

Solution: $\underline{P = 157.3 \text{ N}}$, $T = 4.72$ N

The 15-kg block slips up the incline while the 8-kg block remains stationary.

6/33



$\mu_{s_1} = 0.50$, $\mu_{s_2} = 0.40$

From $\Sigma F_y = 0$,

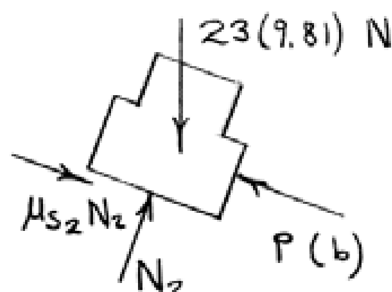
$N_1 = 73.7$ N
$N_2 = 212$ N
} throughout

(a) $\Sigma F_x = 0$:

$$\left.\begin{array}{l} -T + 8(9.81)\sin 20° + \mu_{s_1} N_1 = 0 \\ -\mu_{s_1} N_1 - \mu_{s_2} N_2 + 15(9.81)\sin 20° + P = 0 \end{array}\right\}$$

Solution, $\underline{P = 71.4 \text{ N}}$ $T = 63.7$ N

(b) Assume that T goes slack and motion impends for A and B as a unit.



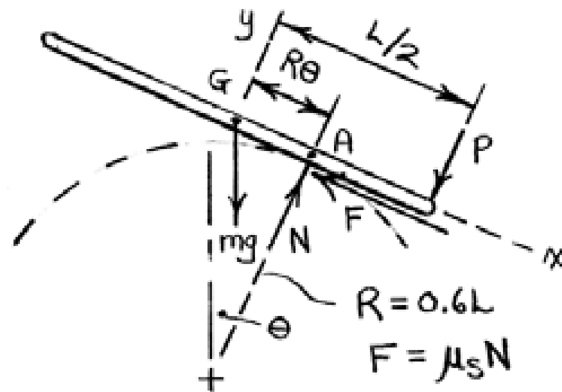
$\Sigma F_x = 0$: $23(9.81)\sin 20° - P + \mu_{s_2} N_2 = 0$

$\underline{P = 162.0 \text{ N}}$

Note that $\theta_{max} = \tan^{-1}\mu_{s_1} = \tan^{-1} 0.5$
$= 26.6° > 20°$

Hence, the possibility that the 8-kg block remains stationary as the 15-kg block slips beneath it is ruled out.

6/34



$\Sigma F_x = 0: \quad mg\sin\theta - \mu_s N = 0$ (1)

$\Sigma F_y = 0: \quad N - P - mg\cos\theta = 0$ (2)

$\circlearrowleft + \Sigma M_A = 0: \quad mg\,R\theta\cos\theta - P\left(\frac{L}{2} - R\theta\right) = 0$

With $R = 0.6L$: $\quad 0.6\,mg\,\theta\cos\theta - P\left(\frac{1}{2} - 0.6\theta\right) = 0$ (3)

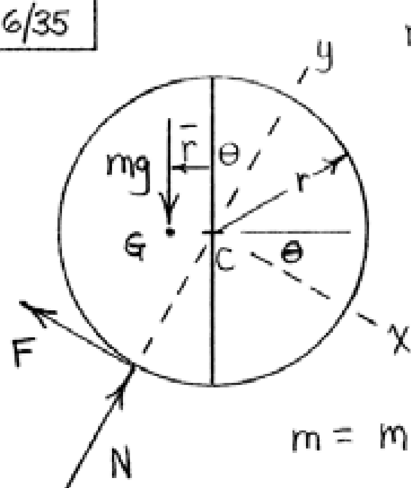
(1) & (2): $\quad mg\sin\theta = \mu_s\left(P + mg\cos\theta\right)$ (4)

(3) & (4): $\quad mg\sin\theta = \mu_s\left[\frac{0.6\,mg\,\theta\cos\theta}{0.5 - 0.6\theta} + mg\cos\theta\right]$

$= \mu_s\, mg\cos\theta\left(\frac{0.5}{0.5 - 0.6\theta}\right)$

$\mu_s = \left(1 - \frac{6}{5}\theta\right)\tan\theta = \left(1 - \frac{6}{5}\cdot 20^\circ\cdot\frac{\pi}{180^\circ}\right)\tan 20^\circ = \underline{0.212}$

6/35



$m_{Al} = \rho_{Al} \pi r^2 t_{Al}$ (t = depth)

$= 2690\pi(0.080)^2(0.040)$

$= 2.16$ kg

$m_{st} = \rho_{st} \pi r^2 t_{st} \cdot \frac{1}{2}$

$= 7830\pi(0.080)^2(0.016)\frac{1}{2}$

$= 1.259$ kg

$m = m_{Al} + m_{st} = 3.42$ kg

$\bar{r}_{st} = \frac{4r}{3\pi} = \frac{4(80)}{3\pi} = 34.0$ mm

$\bar{r} = \frac{\Sigma m\bar{r}}{\Sigma m} = \frac{2.16(0) + 1.259(34.0)}{3.42} = 12.49$ mm

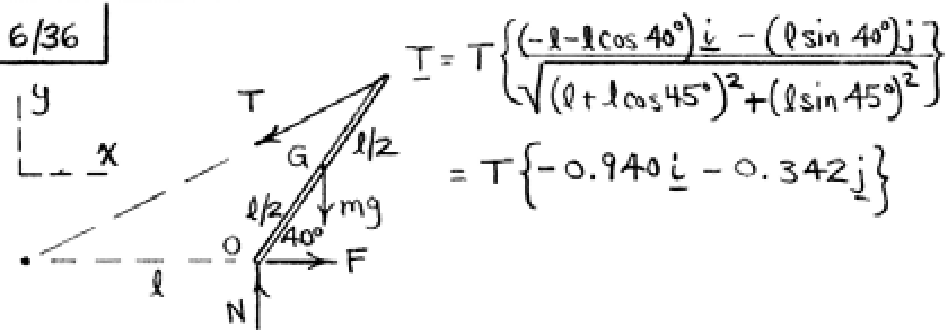
$\circlearrowleft \Sigma M_C = 0: \; mg\bar{r} - Fr = 0, \quad F = mg\frac{\bar{r}}{r}$

$F = 3.42(9.81)\frac{12.49}{80} = 5.24$ N

$\Sigma F_x = 0: \; -5.24 + 3.42(9.81)\sin\theta = 0, \quad \underline{\theta = 8.98°}$

$\Sigma F_y = 0: \; N - 3.42(9.81)\cos 8.98° = 0, \quad N = 33.2$ N

$\mu_s = \frac{F}{N} = \frac{5.24}{33.2} = \underline{0.158}$



$\circlearrowleft + \Sigma M_o = 0: \quad 0.342T(l) - mg\frac{l}{2}\cos 40° = 0$

$T = 1.120\ mg$

$\Sigma F_x = 0: \quad F - 1.120\ mg\ (0.940) = 0, \quad F = 1.052\ mg$

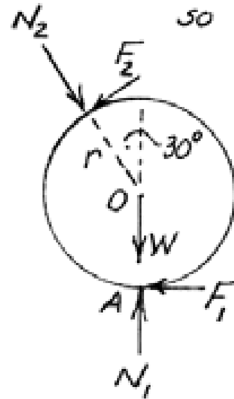
$\Sigma F_y = 0: \quad N - 1.120\ mg\ (0.342) - mg = 0$

$N = 1.383\ mg$

$\mu_s = \dfrac{F}{N} = \dfrac{1.052\ mg}{1.383\ mg} = \underline{0.761}$

6/37 Lower roller; $\Sigma M_O = 0$; $F_1 = F_2$ But $N_1 > N_2$

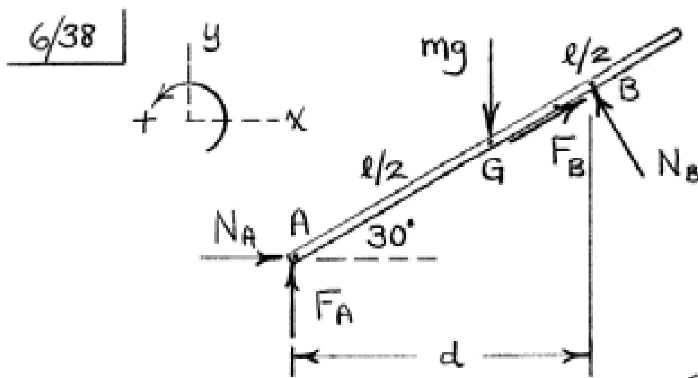
so F_2 reaches limiting value $\mu_s N_2$ before F_1.



$\Sigma M_A = 0$; $\mu_s N_2 \left(r + \frac{\sqrt{3}}{2} r\right) - N_2 \frac{r}{2} = 0$

$$\text{so } \mu_s = \frac{r/2}{r + \frac{\sqrt{3}r}{2}} = \frac{0.5}{1 + 0.866} = \underline{0.268}$$

6/38



FBD assumes slipping CCW { We expect $\frac{l}{2} > \frac{d}{\cos 30°}$!

$$\begin{cases} \Sigma F_x = 0: N_A + F_B \cos 30° - N_B \sin 30° = 0 \\ \Sigma F_y = 0: F_A + F_B \sin 30° + N_B \cos 30° - mg = 0 \\ \Sigma M_A = 0: -mg \frac{l}{2} \cos 30° + N_B \frac{d}{\cos 30°} = 0 \end{cases}$$

Limiting friction: $F_A = \mu_s N_A$, $F_B = \mu_s N_B$

Solve 5 equations to obtain

$$\begin{cases} N_A = 0.1362 mg & F_A = 0.0545 mg \\ N_B = 0.887 mg & F_B = 0.355 mg \end{cases} \quad l = 2.37 d$$

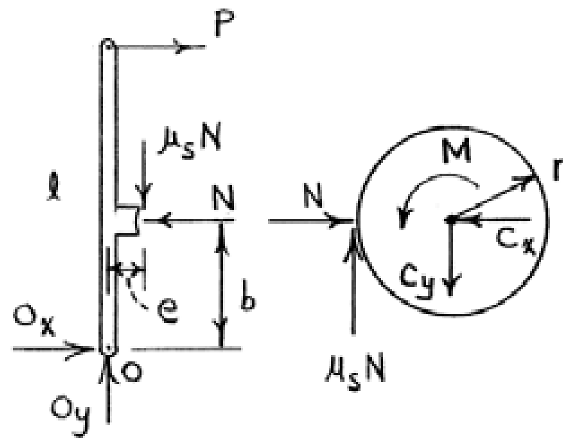
For slipping CW, reverse F_A & F_B on FBD and first two equations & obtain

$N_A = 2.58 mg \quad F_A = 1.034 mg$

$N_B = 3.05 mg \quad F_B = 1.222 mg \quad l = 8.14 d$

For equilibrium: $\underline{2.37 \leq \frac{l}{d} \leq 8.14}$

6/39



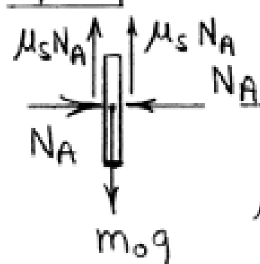
Wheel: $\curvearrowleft + \Sigma M_C = 0$: $M - \mu_s N r = 0$, $\mu_s N = M/r$

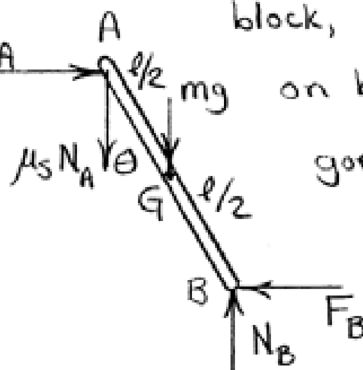
Lever: $\curvearrowleft + \Sigma M_O = 0$: $Nb - P\ell - \mu_s N e = 0$

$$\underline{P = \frac{M}{r\ell}\left(\frac{b}{\mu_s} - e\right)}$$

If $b = \mu_s e$, $P = 0$ & brake would be self-locking.

6/40





For impending slip of the block, the friction force on both of its faces goes to the maximum.

(Block) $+\uparrow \Sigma F = 0: \; 2\mu_s N_A - m_o g = 0, \quad N_A = \dfrac{m_o g}{2\mu_s}$ (1)

(Bar) $\curvearrowleft + \Sigma M_B = 0: \; mg\left(\frac{l}{2}\sin\theta\right) + \mu_s N_A (l \sin\theta) - N_A (l\cos\theta) = 0$ (2)

With Eq. (1), Eq. (2) becomes

$$mg\left(\frac{l}{2}\sin\theta\right) + \mu_s \frac{m_o g}{2\mu_s} l\sin\theta - \frac{m_o g}{2\mu_s} l\cos\theta = 0$$

Solving for θ: $\theta = \tan^{-1}\left[\dfrac{1}{\mu_s\left(1 + \frac{m}{m_o}\right)}\right] = \theta_{min}$

(Bar) $\overset{+}{\rightarrow} \Sigma F = 0: \; N_A - F_B = 0$

$$F_B = N_A = \frac{m_o g}{2\mu_s}$$

$+\uparrow \Sigma F = 0: \; N_B - mg - \mu_s N_A = 0$

$$N_B = mg + \mu_s \frac{m_o g}{2\mu_s} = \left(m + \frac{m_o}{2}\right)g$$

$$(\mu_s)_B = \frac{F_B}{N_B} = \frac{m_o g/(2\mu_s)}{\left(m + \frac{m_o}{2}\right)g} = \frac{1}{\mu_s\left(1 + 2\frac{m}{m_o}\right)}$$

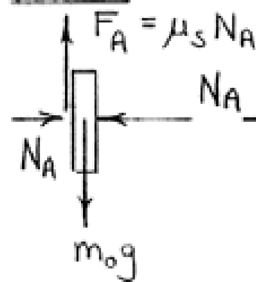
Numbers ($\mu_s = 0.5$ throughout)

(a) $\frac{m}{m_o} = 0.1$: $\theta_{min} = 61.2°$, $(\mu_s)_B = 1.667$ (not possible)

(b) $\frac{m}{m_o} = 1$: $\theta_{min} = 45°$, $(\mu_s)_B = 0.667$

(c) $\frac{m}{m_o} = 10$: $\theta_{min} = 10.30°$, $(\mu_s)_B = 0.0952$

6/41



A

$\ell/2$ mg

θ

G

$\ell/2$

F_B

B

N_B

The friction force F_A between the block m_o and the vertical wall is set to the maximum.

Block: $+\uparrow \Sigma F = 0$:

$$\mu_s N_A = m_o g$$

$$N_A = \frac{m_o g}{\mu_s} \quad (1)$$

Bar: $\circlearrowleft + \Sigma M_B = 0$: $mg\left(\frac{\ell}{2}\sin\theta\right) - N_A(\ell\cos\theta) = 0$

$$N_A = \frac{mg}{2}\tan\theta \quad (2)$$

Combine (1) & (2): $\frac{m_o g}{\mu_s} = \frac{mg}{2}\tan\theta$

$$\Rightarrow \underline{\theta = \tan^{-1}\left(\frac{2}{\mu_s}\frac{m_o}{m}\right) = \theta_{min}}$$

Numbers: $\theta_{min} = \tan^{-1}\left(\frac{2}{0.5}\cdot\frac{1}{10}\right) = \underline{21.8°}$

Check to see that no slippage occurs at B.

(Bar) $\xrightarrow{+} \Sigma F = 0$: $N_A - F_B = 0$

So $F_B = N_A = \frac{mg}{2}\tan\theta$ (from Eq. (2))

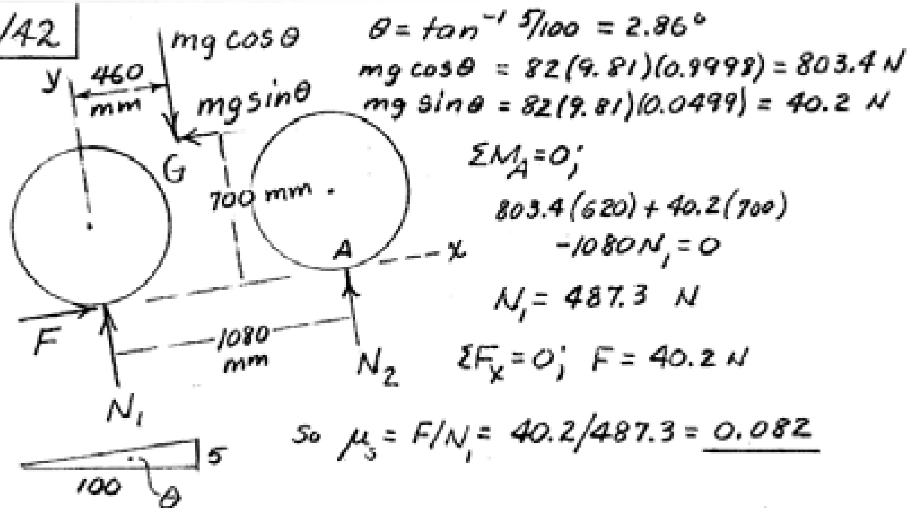
$+\uparrow \Sigma F = 0$: $N_B - mg = 0$, $N_B = mg$

$(F_B)_{max} = \mu_s N_B = \mu_s mg$

Numbers: $\begin{cases} F_B = \frac{mg}{2}\tan 21.8° = 0.2mg \\ (F_B)_{max} = 0.5mg \end{cases}$

$F_B < (F_B)_{max}$, so no slippage at B.

6/42



$\theta = \tan^{-1} 5/100 = 2.86°$

$mg \cos\theta = 82(9.81)(0.9998) = 803.4 \text{ N}$

$mg \sin\theta = 82(9.81)(0.0499) = 40.2 \text{ N}$

$\Sigma M_A = 0$; $803.4(620) + 40.2(700) - 1080 N_1 = 0$

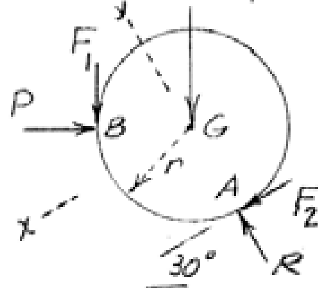
$N_1 = 487.3 \text{ N}$

$\Sigma F_x = 0$; $F = 40.2 \text{ N}$

So $\mu_s = F/N_1 = 40.2/487.3 = \underline{0.082}$

If μ_s were doubled, friction force remains $\underline{40.2 \text{ N}}$

6/43



Assume roll slips at B but not at A. Then

$F_1 = 0.4P$

$\Sigma M_A = 0;$

$mg\, r \sin 30° + 0.4P(r + r\sin 30°) - P(r\cos 30°) = 0$

$P(0.866 - 0.4[1 + 0.5]) = \frac{1200(9.81)}{2}$

$P = 22\ 126$ N or $\underline{P = 22.1 \text{ kN}}$

Check on assumption:

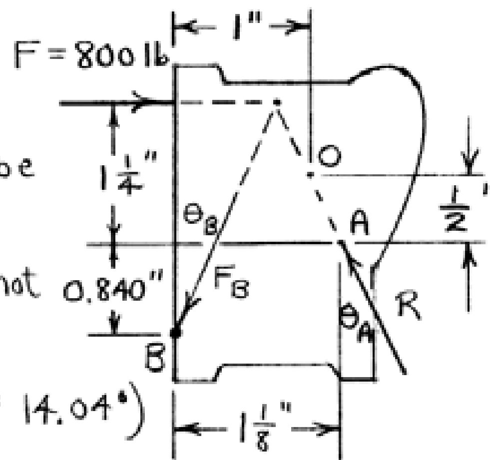
$\Sigma F_y = 0;$ $R - 1200(9.81)\cos 30° - 0.4(22\ 126)\cos 30° - 22\ 126 \sin 30° = 0$

$R = 28\ 922$ N

$\Sigma M_G = 0;$ $F_2 r - F_1 r = 0,$ $F_2 = F_1 = 0.4(22\ 126) = 8850$ N

But $(F_2 = 8850 \text{ N}) < (0.4R = 11\ 569 \text{ N})$ so assumption OK

6/44



For equilibrium, the three forces must be concurrent.

(a) Slipping at A is not possible as long as

$\phi_A > (\theta_A = \tan^{-1}\frac{1/8}{1/2} = 14.04°)$

So $(\mu_s)_{min} = \tan\theta_A = \underline{0.25 \text{ @ } A}$

Slipping at B is prevented as long as $\phi_B > \theta_B$.

$$\tan\theta_B = \frac{1.125 - 1.25(0.25)}{1.25 + 0.840}$$

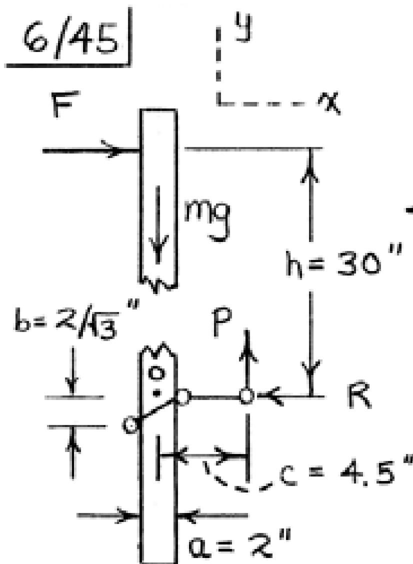
$$= 0.389$$

$(\mu_s)_{min} = \underline{0.389 \text{ @ } B}$

(b) $\Sigma M_B = 0$: $800(1.25 + 0.840) - R\cos 14.04°(1.125)$

$- R\sin 14.04°(0.840) = 0$

$\underline{R = 1291 \text{ lb}}$

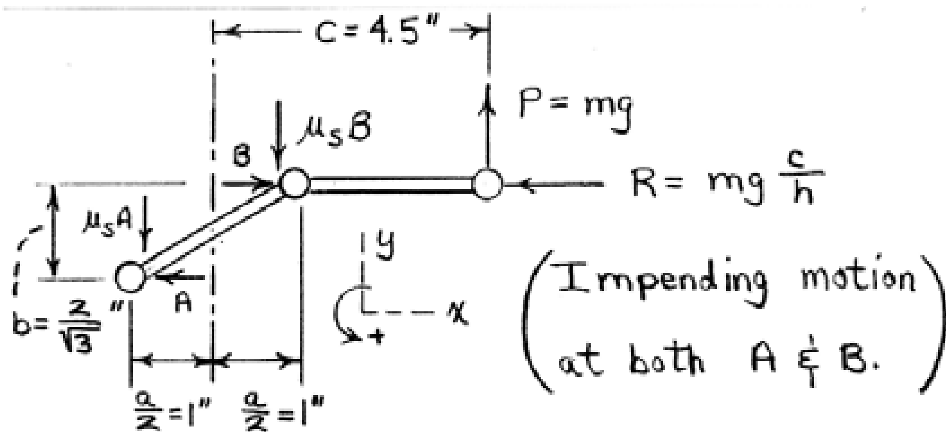


System of panel & carrier:

$$\begin{cases} \Sigma F_x = 0: & F = R \\ \Sigma F_y = 0: & P = mg \\ \Sigma M_o = 0: & Fh = Pc \end{cases}$$

$$\Rightarrow R = mg\frac{c}{h}$$

(m is $\frac{1}{2}$ panel mass)



$$\begin{cases} \Sigma F_x = 0: & B - A - mg\frac{c}{h} = 0 & (1) \\ \Sigma F_y = 0: & mg - \mu_s A - \mu_s B = 0 & (2) \\ \Sigma M_B = 0: & mg\left(c - \frac{a}{2}\right) + \mu_s A a - Ab = 0 & (3) \end{cases}$$

Eliminate B from (1) & (2):

$$mg - \mu_s A - \mu_s\left(A + mg\frac{c}{h}\right) = 0, \quad A = \frac{mg}{2}\left(\frac{1}{\mu_s} - \frac{c}{h}\right)$$

$$(3): \quad mg\left(c - \frac{a}{2}\right) + \frac{mg}{2}\left(\frac{1}{\mu_s} - \frac{c}{h}\right)\left(\mu_s a - b\right) = 0$$

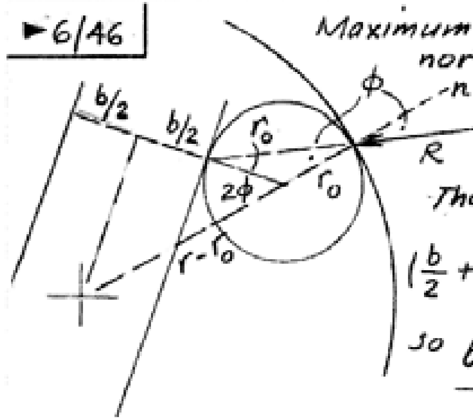
Simplify to get $\mu_s^2 - \mu_s\frac{b+2h}{a} + \frac{bh}{ac} = 0$

$$\mu_s = \frac{b+2h}{2a} \pm \frac{1}{2}\sqrt{\frac{b^2 + 4bh + 4h^2}{a^2} - \frac{4bh}{ac}}$$

With $a = 2"$, $b = \frac{2}{\sqrt{3}}"$, $c = 4.5"$ and $h = 30"$,

$$\underline{\mu_s = 0.1264}$$

►6/46



Maximum angle between R and normal is the friction angle

$\phi = \tan^{-1}\mu$

so ϕ increases as b decreases

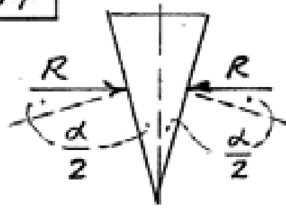
Thus min. value of b is given by

$$\left(\frac{b}{2} + r_0\right)/\cos 2\phi + r_0 = r$$

so $b = 2\left[(r - r_0)\cos 2\phi - r_0\right]$

where $\phi = \tan^{-1}\mu$

6/47



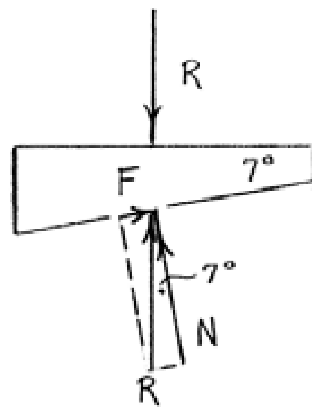
Critical α occurs when

$$\frac{\alpha}{2} = \phi = \tan^{-1}\mu$$

so $\alpha = 2\tan^{-1} 0.20$

$= 2(11.31°) = \underline{22.6°}$

6/48



$$\frac{F}{N} = \mu_s = \tan 7°$$

$\underline{\mu_s = 0.1228}$

6/49 Helix angle $\alpha = \tan^{-1}\frac{24}{40\pi} = 10.81°$

Friction angle $\phi = \tan^{-1}\mu = \tan^{-1} 0.15 = 8.53°$

$\alpha > \phi$ so screw is not self-locking.

$\alpha + \phi = 19.34°$; $\alpha - \phi = 2.28°$

(a) $M = Pr \tan(\alpha - \phi)$: $60 = P(0.020) \tan 2.28°$

$P = 75\,300$ N or 75.3 kN

(b) $M = P r \tan(\alpha + \phi)$: $60 = P(0.020) \tan 19.34°$

$P = 8550$ N or 8.55 kN

6/50 Friction angle $\phi = \tan^{-1}\mu = \tan^{-1} 0.15 = 8.53°$

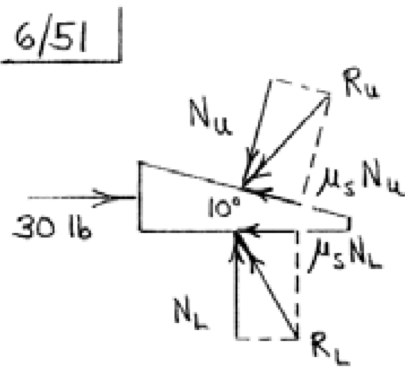
$\tan\alpha = \frac{L}{2\pi r}$; Critical when $\alpha = \phi$

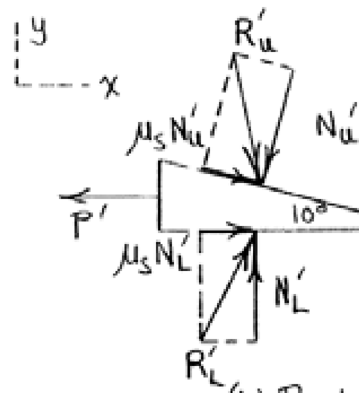
$\therefore$ Lead $L = 2\pi r \tan\phi = 2\pi \frac{3/8}{2} \tan 8.53°$

$= 0.1767$ in. per revolution

$N = 1/L = 1/0.1767 =$ <u>5.66 Threads per inch</u>

6/51





(a) Insertion (b) Retraction

(a) $\Sigma F_x = 0 : \; 30 - N_u \sin 10° - \mu_s N_u \cos 10° - \mu_s N_L = 0$

$\Sigma F_y = 0 : \; -N_u \cos 10° + \mu_s N_u \sin 10° + N_L = 0$

Solution: $N_u = 53.5$ lb, $N_L = 50.8$ lb

(b) Primes denote new values of N_u and N_L. There are now 3 unknowns (P', N'_u, and N'_L), so we cannot solve for P' without more information.

6/52

Helix angle $\alpha = \tan^{-1} \frac{L}{\pi d} = \tan^{-1} \frac{1/12}{\pi(3/8)} = 4.05°$

Friction angle $\phi = \tan^{-1} \mu_s = \tan^{-1}(0.20) = 11.31°$

Tighten : $M = Pr \tan(\alpha + \phi) = 80 \frac{3/8}{2} \tan(4.05° + 11.31°)$

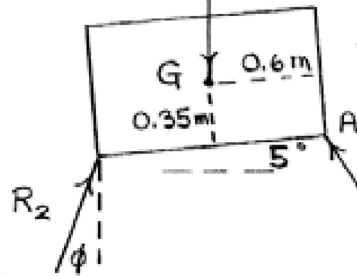
$= \underline{4.12 \text{ lb-in.}}$

Loosen : $M' = Pr \tan(\phi - \alpha) = 80 \frac{3/8}{2} \tan(11.31° - 4.05°)$

$= \underline{1.912 \text{ lb-in.}}$ (in direction opposite to that of M)

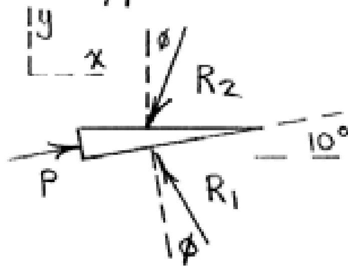
Note : $\alpha < \phi$, so screw is self-locking (a good feature for a clamp!)

6/53



$\phi = \tan^{-1}\mu = \tan^{-1}0.4 = 21.8°$

$\circlearrowleft + \Sigma M_A = 0: \; 90(9.81)\left[\cos 5°(0.6) + \sin 5°(0.35)\right] - R_2\cos(\phi + 5°) \times 1.2 = 0, \quad R_2 = 518 \text{ N}$



$\Sigma F_x = 0: \; P\cos 10° - 518\sin\phi - R_1\sin(\phi + 10°) = 0$

$\Sigma F_y = 0: \; P\sin 10° - 518\cos\phi + R_1\cos(\phi + 10°) = 0$

Solve simultaneously to obtain

$R_1 = 471$ N $\qquad$ $P = 449$ N

6/54 Friction angle $\phi = \tan^{-1}(0.2) = 11.31°$

(a) Rollers under wedge

$\Sigma F_y = 0: \; -100 + R_2 \cos(15° + 11.31°) = 0$

$R_2 = 111.6 \text{ lb}$

$\Sigma F_x = 0: \; R_2 \sin(15° + 11.31°) - P = 0$

$\underline{P = 49.4 \text{ lb}}$

(b) Rollers removed

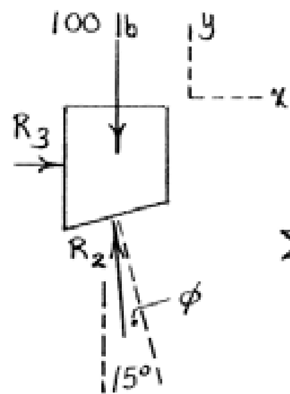
Value of R_2 from 100-lb body is unchanged.

$\Sigma F_x = 0: \; R_2 \sin(15° + 11.31°) - P + R_1 \sin(11.31°) = 0$

With R_1 determined from overall equilibrium as $R_1 = 100/\cos 11.31° = 102.0 \text{ lb}$, we solve for P as

$\underline{P = 69.4 \text{ lb}}$

6/55 Friction angle $\phi = \tan^{-1}(0.2) = 11.31°$



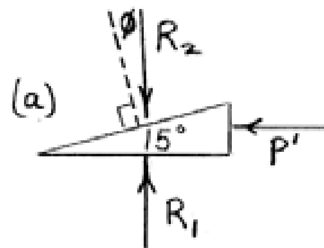
(a) Rollers under wedge

$\Sigma F_y = 0: \; -100 + R_2 \cos(15° - 11.31°) = 0$

$R_2 = 100.2$ lb

$\Sigma F_x = 0: \; R_2 \sin(15° - 11.31°) - P' = 0$

$P' = 6.45$ lb (to the left)

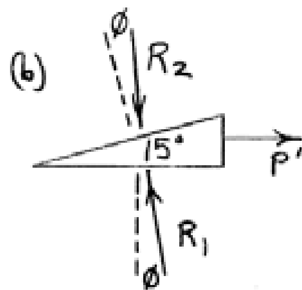


(b) Rollers removed

$R_2 = 100.2$ lb, from (a).

From overall equilibrium,

$R_1 = \frac{100}{\cos 11.31°} = 102.0$ lb



$\Sigma F_x = 0: \; R_2 \sin(15° - 11.31°) + P'$

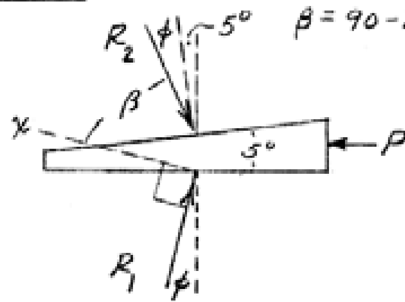
$- R_1 \sin(11.31°) = 0$

$P' = 13.55$ lb (to the right)

6/56

$\phi = \tan^{-1} 0.40 = 21.80°$

$\beta = 90 - 2\phi - 5° = 41.40°$



Column & wedge

$\Sigma F_{vert.} = 0$

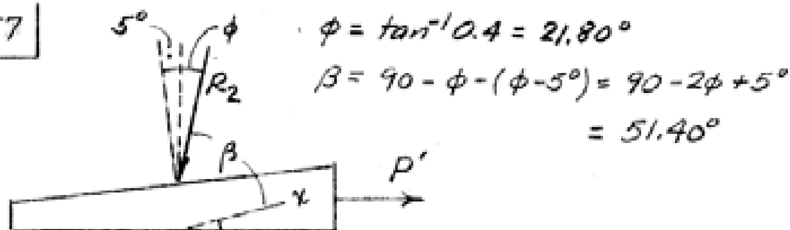
$R_2 \cos(21.80 + 5)° = 5$ kN

$R_2 = \dfrac{5}{\cos 26.80°} = 5.60$ kN

Wedge: $\Sigma F_x = 0$; $P \cos 21.80° - 5.60 \cos 41.40 = 0$

$P = 5.60(0.7501)/0.9285 = \underline{4.53 \text{ kN}}$

6/57



$\phi = \tan^{-1} 0.4 = 21.80°$

$\beta = 90 - \phi - (\phi - 5°) = 90 - 2\phi + 5°$

$= 51.40°$

Column & wedge

$\Sigma F_{vert.} = 0;$

$R_2 \cos(21.80 - 5)° = 5 \text{ kN}$

$R_2 = \frac{5}{\cos 16.80°} = 5.22 \text{ kN}$

Wedge; $\Sigma F_x = 0;$ $P' \cos 21.80 - 5.22 \cos 51.40 = 0$

$P' = 5.22(0.6239)/0.9285 = \underline{3.51 \text{ kN}}$

6/58 | Helix angle: $\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{1/5}{2\pi\left(\frac{1.150}{2}\right)} = 3.17°$

Friction angle $\phi = \tan^{-1}\mu = \tan^{-1}(0.25) = 14.04°$

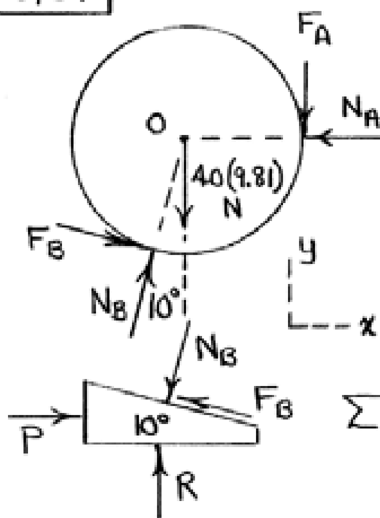
(a) To tighten, $M_a = 2Tr\tan(\alpha + \phi)$

$M_a = 2(10{,}000)\frac{1.150}{2}\tan(3.17° + 14.04°) = \underline{3560 \text{ lb-in.}}$

(b) To loosen, $M_b = 2Tr\tan(\phi - \alpha)$

$M_b = 2(10{,}000)\frac{1.150}{2}\tan(14.04° - 3.17°) = \underline{2210 \text{ lb-in.}}$

6/59



By inspection $N_A < N_B$, so slipping occurs first at A. Thus $F_A = 0.25 N_A$

Cylinder: $\Sigma M_O = 0$:

$F_B r - F_A r = 0$, $F_B = F_A = 0.25 N_A$

$\Sigma F_y = 0$: $N_B \cos 10° - F_A \sin 10° - 40(9.81) - F_A = 0$

or $N_B \cos 10° - (0.25 N_A) \sin 10° - 40(9.81) - 0.25 N_A = 0$ (a)

$\Sigma F_x = 0$: $N_B \sin 10° + F_B \cos 10° - N_A = 0$

or $N_B \sin 10° + (0.25 N_A) \cos 10° - N_A = 0$ (b)

Solve (a) & (b): $N_A = 98.6$ N, $N_B = 428$ N; $\underline{F_B = F_A = 24.6 \text{ N}}$

Wedge: $\Sigma F_x = 0$: $P - 24.6 \cos 10° - 428 \sin 10° = 0$, $\underline{P = 98.6 \text{ N}}$

6/60 $M = Tr \tan(\alpha + \phi) + Tr \tan(\alpha - \phi)$

where $\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{4}{2\pi(16/2)} = 4.55°$

$\phi = \tan^{-1}\mu = \tan^{-1} 0.24 = 13.50°$

$\alpha + \phi = 18.05°$, $\phi - \alpha = 13.50 - 4.55 = 8.95°$

So $M = 8000\left(\frac{16}{2}\right)\left[\tan 18.05° + \tan 8.95°\right]$

$= 8000(8)(0.3258 + 0.1574) = 30\,926$ N·mm

or $\underline{M = 30.9 \text{ N·m}}$ (Same for both directions)

6/61 For the screw,

$$\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{6}{20\pi} = 5.45°$$

$$\phi = \tan^{-1}(0.25) = 14.04°$$

Eq. 6/3: $M = Wr\tan(\alpha + \phi)$

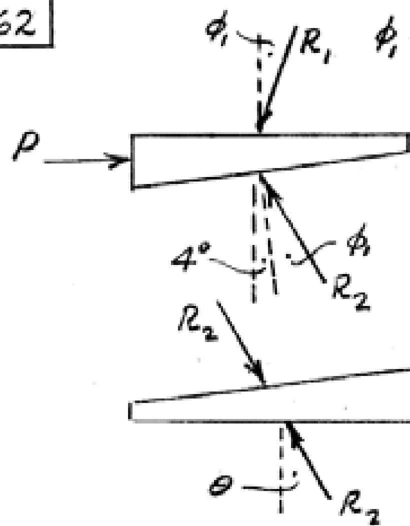
$$24 = P\,\frac{20/2}{1000}\tan(5.45° + 14.04°)$$

$$P = 6780\text{ N} \quad \text{(to remove collar)}$$

Collar: $\mu pA = P$: $0.30p\,(0.050\pi \cdot 0.060) = 6780$

$$p = 2.40(10^6)\text{ Pa or } \underline{2400\text{ kPa}}$$

6/62



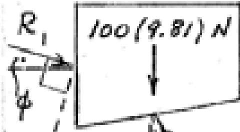
$\phi_1 = \tan^{-1} 0.3 = 16.70°$

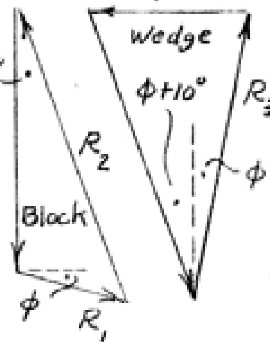
In order for column to be raised, bottom wedge must not move. Therefore μ_2 must be greater than $\tan\theta$ or

$(\mu_2)_{min} = \tan(4 + 16.70)$

$= 0.378$

6/63 $\phi = \tan^{-1} 0.40 = 21.80°$





$2\phi + 10° = 53.60°$

$90° - (2\phi + 10°) = 36.40°$

Block: $\Sigma F_y = 0$; $981 \cos 21.80° = R_2 \cos 53.60°$

$R_2 = 1535$ N

Wedge: $\Sigma F_x = 0$; $1535 \cos 36.40° = P \cos 21.80°$

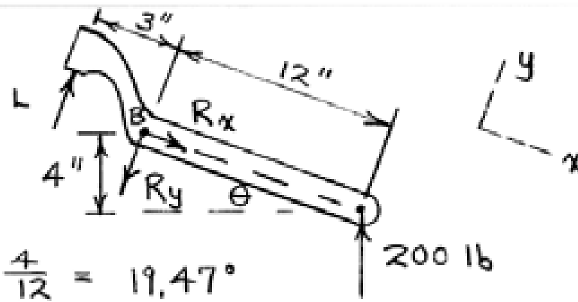
$P = 1331$ N

Screw: $\alpha = \tan^{-1} \frac{L}{2\pi r} = \tan^{-1} \frac{10}{2\pi(15)} = 6.06°$; $\phi = \tan^{-1} 0.25 = 14.04°$

$\phi + \alpha = 20.09°$

$M = Pr \tan(\phi + \alpha) = 1331(0.015) \tan 20.09° = \underline{7.30 \text{ N}\cdot\text{m}}$

6/64



$\theta = \sin^{-1}\frac{4}{12} = 19.47°$

$\Sigma M_B = 0$: $200 \cos 19.47° (12) - 3L = 0$, $L = 754$ lb

Screw: helix angle $\alpha = \tan^{-1}\frac{1/12}{\pi(1/2)} = 3.04°$

friction angle $\phi = \tan^{-1}(0.20) = 11.31°$

Tighten screw: $M = Lr\tan(\phi + \alpha)$

$= 754(0.25)\tan(11.31° + 3.04°) = \underline{48.2 \text{ lb-in.}}$

Loosen screw: $M' = Lr\tan(\phi - \alpha)$

$= 754(0.25)\tan(11.31° - 3.04°) = \underline{27.4 \text{ lb-in.}}$

▸6/65 For equil. of screw (refer to prob. illust.)

$\Sigma F = 0$; $W = \Sigma R_1 \cos(\alpha+\gamma) = \cos(\alpha+\gamma)\,\Sigma R_1$

$\Sigma M = 0$; $M = \Sigma R_1 r \sin(\alpha+\gamma) = r\sin(\alpha+\gamma)\,\Sigma R_1$

Combine & get $M = Wr\tan(\alpha+\gamma) = Wr\,\dfrac{\tan\alpha + \tan\gamma}{1-\tan\alpha\tan\gamma}$

But $\tan\gamma = \dfrac{R\sin\phi}{R\cos\phi\cos\beta/2} = \mu/\cos\beta/2$

& $\tan\beta/2 = \dfrac{L}{2h}\cos\alpha$, $\tan\dfrac{\theta}{2} = \dfrac{L}{2h}$, so $\tan\dfrac{\beta}{2} = \tan\dfrac{\theta}{2}\cos\alpha$

& $\cos\dfrac{\beta}{2} = 1\Big/\sqrt{1+\tan^2\dfrac{\theta}{2}\cos^2\alpha}$. Thus $\tan\gamma = \mu\sqrt{1+\tan^2\dfrac{\theta}{2}\cos^2\alpha}$

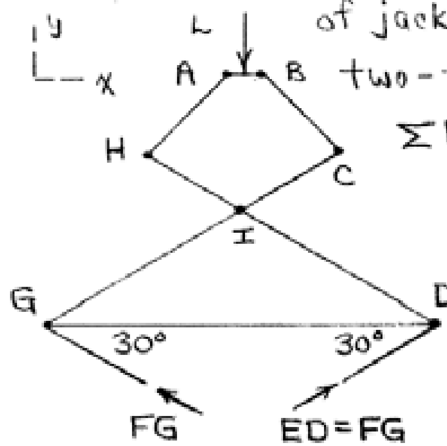
Hence

$$M = Wr\,\frac{\tan\alpha + \mu\sqrt{1+\tan^2\frac{\theta}{2}\cos^2\alpha}}{1-\mu\tan\alpha\sqrt{1+\tan^2\frac{\theta}{2}\cos^2\alpha}}$$

---

where $\tan\alpha = \dfrac{L}{2\pi r}$

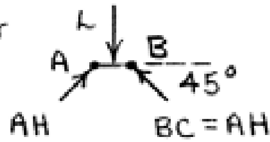
▸6/66 Calculate force in member FG. FBD of jack with FG & DE (both two-force members) cut:



$\Sigma F_y = 0: \; 2FG \sin 30° = L$

$FG = L$

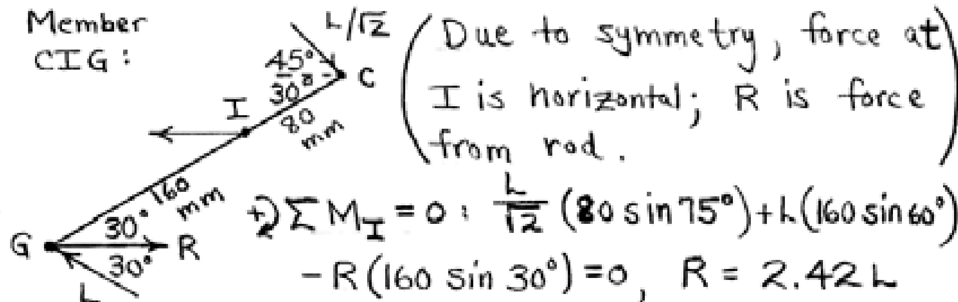
Member AB:



$\Sigma F_y = 0: \; 2AH \sin 45° = L$

$AH = L/\sqrt{2}$

Member CIG:



(Due to symmetry, force at I is horizontal; R is force from rod.)

$\circlearrowleft + \Sigma M_I = 0: \; \frac{L}{\sqrt{2}}(80 \sin 75°) + L(160 \sin 60°) - R(160 \sin 30°) = 0, \quad R = 2.42L$

Friction angle $\phi = \tan^{-1}\mu = \tan^{-1}(0.20) = 11.31°$

$\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{0.003}{2\pi(0.010/2)} = 5.45°$

To raise load: $M = Rr\tan(\alpha + \phi)$

$= [2.42(1000)(9.81)]\left[\frac{0.010}{2}\right]\tan(11.31° + 5.45°)$

$=$ <u>35.7 N·m</u>

To lower load: $M' = Rr\tan(\phi - \alpha)$

$= [2.42(1000)(9.81)]\left[\frac{0.010}{2}\right]\tan(11.31° - 5.45°)$

$=$ <u>12.15 N·m</u>

6/67

$M = \frac{2}{3}\mu P R$

A on B: $M = \frac{2}{3}(0.40)(80)\left(\frac{9}{2}\right) = \underline{96.0 \text{ lb-in.}}$

B on C: $96.0 = \frac{2}{3}\mu(80)\left(\frac{12}{2}\right)$, $\underline{\mu = 0.300}$

6/68

$M = Rr\sin\phi$, $\sin\phi = \dfrac{M}{Rr} = \dfrac{3}{2(40)(9.81)(0.040/2)}$

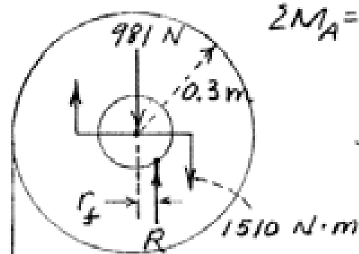
$\phi = 11.02°$

$\mu = \tan\phi = \underline{0.1947}$

$r_f = r\sin\phi = \dfrac{0.040}{2}\sin 11.02° = 0.00382$ m

or $\underline{r_f = 3.82 \text{ mm}}$

6/69 $r_f = r \sin\phi = 0.025 \sin\phi$



$\Sigma M_A = 0$; $1510 - 981(0.025 \sin\phi)$

$-500(9.81)(0.3 + 0.025 \sin\phi) = 0$

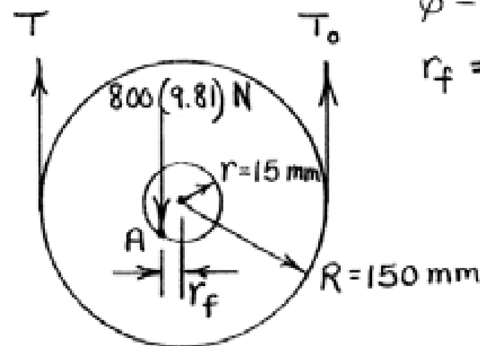
$\sin\phi = 0.2616$, $\phi = 15.17°$

$\mu = \tan\phi = \underline{0.271}$

(using $r_f \approx r\phi$, $\phi = 15.0°$

$\mu = \tan\phi = \underline{0.268}$)

6/70



$\phi = \tan^{-1} 0.25 = 14.04°$

$r_f = r \sin\phi = 15 \sin 14.04$

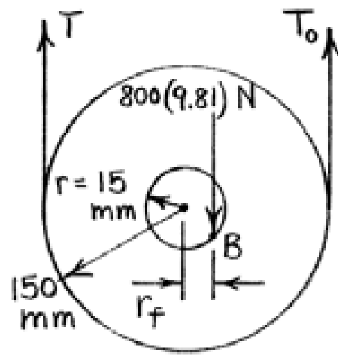
$= 3.64$ mm

$+\uparrow \Sigma F = 0: \quad T + T_o - 800(9.81) = 0 \qquad (1)$

$\circlearrowleft+ \Sigma M_A = 0: \quad T(150 - 3.64) - T_o\,(150 + 3.64) = 0 \qquad (2)$

Solve (1) & (2) : $\begin{cases} T = 4020 \text{ N} \\ T_o = 3830 \text{ N} \end{cases}$

6/71



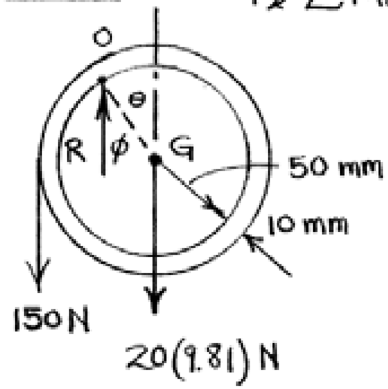
From the solution to Prob. 6/70, $r_f = 3.64$ mm

$+\uparrow \Sigma F = 0: \quad T + T_o - 800(9.81) = 0 \qquad (1)$

$\Sigma M_B = 0: \quad T(150 + 3.64) - T_o(150 - 3.64) = 0 \qquad (2)$

Solve (1) & (2): $\begin{cases} T = 3830 \text{ N} \\ T_o = 4020 \text{ N} \end{cases}$

6/72



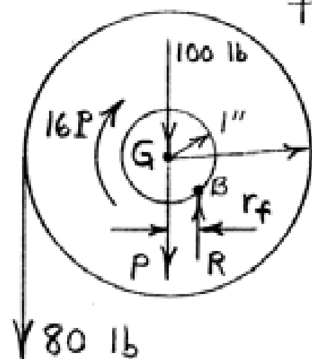
$\circlearrowleft + \Sigma M_O = 0: \; 20(9.81)(50 \sin\phi)$

$- 150(60 - 50 \sin\phi) = 0$

$\phi = \underline{\theta = 31.3°}$

$\mu = \tan\phi = \underline{0.609}$

6/73 FBD of shaft and attached drum, with force P replaced by a force-couple system at G:



(a) $r_f = 0$

$\Sigma M_G = 0: \quad 80(10) - 16P = 0$

$P = 50$ lb

(b) $\phi = \tan^{-1}(0.2) = 11.31°$

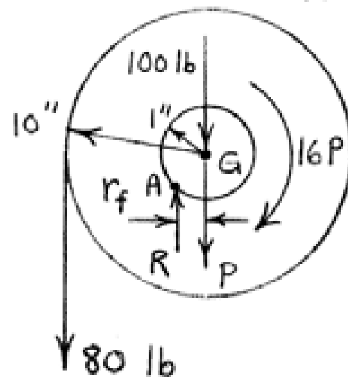
$r_f = r \sin\phi = 1 \sin 11.31° = 0.1961$ in.

$\Sigma M_B = 0: \quad 80(10 + 0.1961) + 100(0.1961) + P(0.1961) - 16P = 0$

$P = 52.9$ lb

(This solution assumes that the bearing reaction can be represented by a single force R as shown above.)

6/74 $r_f = 0.1961$ in. (from Prob. 6/73)



$\circlearrowleft + \Sigma M_A = 0$: $80(10 - 0.1961) - 16P - (100 + P)(0.1961) = 0$

$\underline{P = 47.2 \text{ lb}}$

The values are NOT symmetric:

$$\begin{cases} P_{down} = 47.215 \text{ lb} \\ P_{n.f.} = 50 \text{ lb} \\ P_{up} = 52.854 \text{ lb} \end{cases}$$

6/75 $dM = (\mu p \, dA) r$ where $p = k/r^2$

$$M = \int_0^{2\pi}\int_{r_i}^{r_o} \mu p r (r \, dr \, d\theta) = 2\pi\mu k \int_{r_i}^{r_o} dr = 2\pi\mu k (r_o - r_i)$$

Also $L = \int p \, dA = \int_0^{2\pi}\int_{r_i}^{r_o} \frac{k}{r^2} r \, dr \, d\theta = 2\pi k \ln r \Big]_{r_i}^{r_o}$

or $L = 2\pi k \ln \frac{r_o}{r_i}$, $2\pi k = \frac{L}{\ln r_o/r_i}$

Thus $M = \mu L \frac{r_o - r_i}{\ln (r_o/r_i)}$

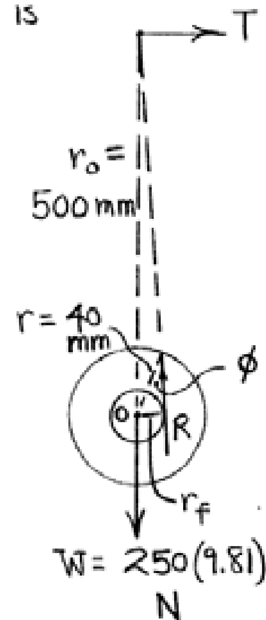
6/76 For constant pressure Eq. 6/5a gives

$$M = \frac{2}{3}\mu P \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2} = \frac{2}{3}(0.35)(1)\frac{(150)^3 - (75)^3}{(150)^2 - (75)^2} = 40.8 \text{ N}\cdot\text{m}$$

For wheel $\Sigma M = 0$; $F(0.3) - 40.8 = 0$, $\underline{F = 136.1 \text{ N}}$

6/77 Force of shaft on bearing is

$$R = \sqrt{W^2 + T^2}$$



$\Sigma M_O = 0$: $\quad T r_o - R r_f = 0$

$$T = \frac{r_f}{r_o}\sqrt{T^2 + W^2}$$

where $r_f = r\sin\phi$ and $\phi = \tan^{-1}\mu$

Thus $T^2 = \dfrac{r^2 \sin^2\phi}{r_o^2}\,(T^2 + W^2)$

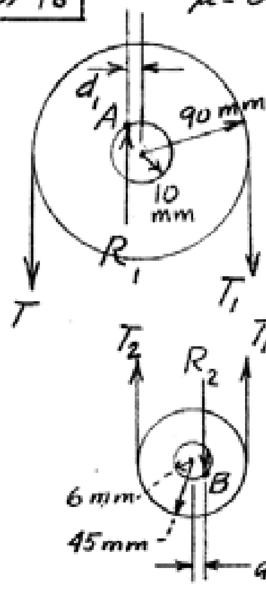
$$T = \frac{Wr\sin\phi}{\sqrt{r_o^2 - r^2\sin^2\phi}}$$

$\phi = \tan^{-1}(0.30) = 16.70°$

$\sin\phi = 0.287$

So $T = \dfrac{250(9.81)(0.040)(0.287)}{\sqrt{0.5^2 - 0.040^2\,(0.287)^2}} = \underline{56.4\text{ N}}$

6/78



$\mu = 0.25$; $\phi = \tan^{-1} 0.25 = 14.04°$

$d_1 = r_o \sin\phi = 10 \sin 14.04° = 2.43$ mm

$d_2 = 6 \sin 14.04° = 1.455$ mm

$\Sigma M_A = 0$; $T(90 - 2.43) - T_1(90 + 2.43) = 0$

$$T = \frac{92.43}{87.57} T_1 = 1.055 T_1$$

$\Sigma M_B = 0$; $T_1(45 - 1.456) - T_2(45 + 1.456) = 0$

$$T_1 = \frac{46.455}{43.54} T_2 = 1.067 T_2$$

$\Sigma F = 0$; $T_1 + T_2 - R_2 = 0$, $T_1 + T_2 = 200(9.81)$

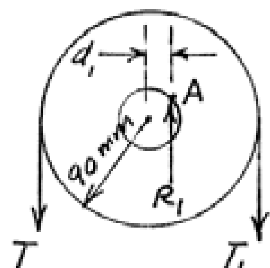
Combine & get $2.067 T_2 = 1962$

so $\underline{T_2 = 949 \text{ N}}$, $\underline{T_1 = 1013 \text{ N}}$

$T = 1.055(1013) = \underline{1069 \text{ N}}$

6/79 From Prob. 6/78, $d_1 = 2.43$ mm, $d_2 = 1.455$ mm

$R_2 = 200(9.81) = 1962$ N

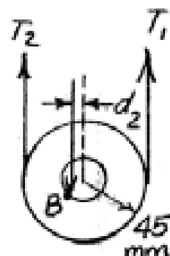


$\Sigma M_A = 0$; $T_1(90 - 2.43) - T(90 + 2.43) = 0$

$$T = \frac{87.57}{92.43} T_1 = 0.948\, T_1$$

$\Sigma M_B = 0$; $T_2(45 - 1.455) - T_1(45 + 1.455) = 0$

$$T_1 = \frac{43.54}{46.455} T_2 = 0.937\, T_2$$



$\Sigma F = 0$; $T_1 + T_2 - 1962 = 0$

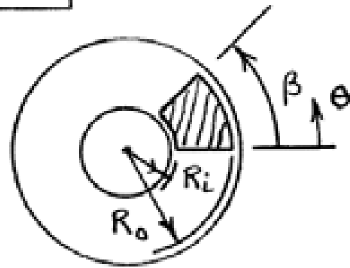
combine & get

$1.937 T_2 = 1962$, $\underline{T_2 = 1013 \text{ N}}$

$\underline{T_1 = 949 \text{ N}}$

$T = 0.948(949) \quad = \underline{899 \text{ N}}$

6/80



$$P = pA = p\int_0^\beta \int_{R_i}^{R_o} r\,dr\,d\theta$$

$$= \frac{p}{2}\int_0^\beta (R_o^2 - R_i^2)\,d\theta$$

$$= \frac{p}{2}(R_o^2 - R_i^2)\beta$$

$$M = 2\int \mu p r\,dA = 2\mu p \int_0^\beta \int_{R_i}^{R_o} r^2\,dr\,d\theta$$

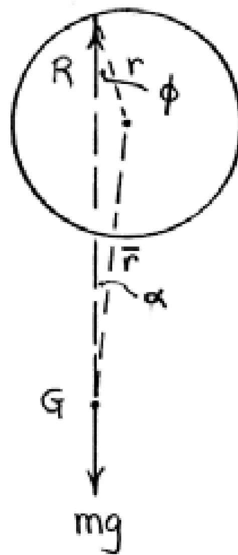
$$= \frac{2\mu p}{3}(R_o^3 - R_i^3)\beta$$

$$= \frac{2\mu}{3}\,\frac{2P}{(R_o^2 - R_i^2)\beta}(R_o^3 - R_i^3)\beta$$

$$= \frac{4\mu P}{3}\,\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}$$

Same form as Eq. 6/5a except for factor of 2 for 2 pads. No β dependence. Pressure variation with θ would not change the moment M.

6/81



$$r \sin\phi = \bar{r} \sin\alpha$$

$$\mu = \tan\phi = \frac{\bar{r}\sin\alpha}{\sqrt{r^2 - \bar{r}^2\sin^2\alpha}}$$

or

$$\mu = \frac{1}{\sqrt{\left(\frac{d/2}{\bar{r}\sin\alpha}\right)^2 - 1}}$$

6/87 $p = p_0\left(1 - \frac{r}{2a}\right)$; $dA = 2\pi r\,dr$

$$L = \int p\,dA = \int_0^a p_0\left(1 - \frac{r}{2a}\right) 2\pi r\,dr = 2\pi p_0\left[\frac{r^2}{2} - \frac{r^3}{6a}\right]_0^a$$

$$= \frac{2}{3}\pi p_0 a^2 \quad \text{so} \quad p_0 = \frac{3L}{2\pi a^2}$$

$$M = \int \mu p r\,dA = \int_0^a \mu p_0\left(r - \frac{r^2}{2a}\right) 2\pi r\,dr = 2\pi\mu p_0\left[\frac{r^3}{3} - \frac{r^4}{8a}\right]_0^a$$

$$= \frac{5}{12}\pi\mu p_0 a^3 = \underline{\frac{5}{8}\mu L a}$$

6/83 $L = \frac{1}{4}(960 - 4 \times 40) = 200$ lb

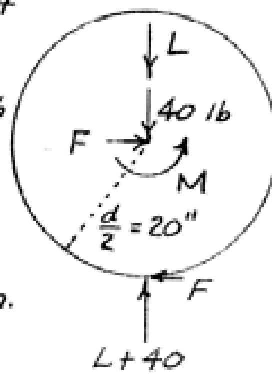
$P = 4F$

$F = \frac{1}{4}16 = 4$ lb

$M = F\frac{d}{2}$

$= 4(20)$

$= 80$ lb-in.



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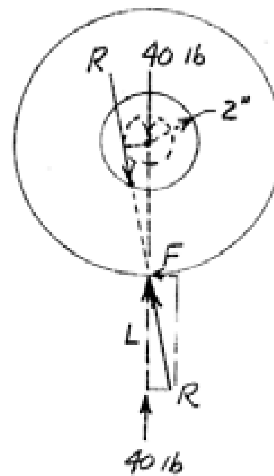


$M = Rr\sin\phi \approx Lr\sin\phi$

$80 \approx 200(2)\sin\phi$

$\phi = \sin^{-1}0.2 = 11.54°$, $\mu = \tan\phi = \underline{0.204}$

6/84

$$\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{11}{2\pi\,\frac{120}{2}} = 1.671°$$

$$\phi = \tan^{-1} 0.15 = 8.53°$$

Screw: (a) Raise : $M_s = Wr\tan(\alpha + \phi)$

$$= \frac{1}{2}(10+3)(9.81)\frac{120}{2}\tan(1.671° + 8.53°) = 689 \text{ N·m}$$

(b) Lower : $M_s = Wr\tan(\phi - \alpha)$

$$= \frac{1}{2}(10+3)(9.81)\frac{120}{2}\tan(8.53° - 1.671°) = 460 \text{ N·m}$$

Collar bearing : $M_c = \frac{2}{3}\mu P\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}$

$$= \frac{2}{3}(0.15)\left(\frac{10+3}{2} + 0.9\right)(9.81)\frac{(250/2)^3 - (125/2)^3}{(250/2)^2 - (125/2)^2}$$

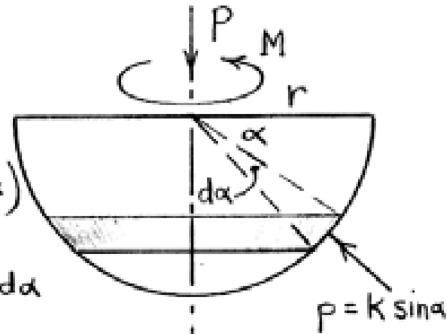
$$= 1059 \text{ N·m}$$

Total moment per screw:

(a) $M = 689 + 1059 = \underline{1747 \text{ N·m}}$

(b) $M = 460 + 1059 = \underline{1519 \text{ N·m}}$

6/85



$$dM = \mu p\, dA \times r\cos\alpha$$

$$= \mu(k\sin\alpha)(r\,d\alpha \cdot 2\pi r\cos\alpha)(r\cos\alpha)$$

$$M = 2\pi\mu k r^3 \int_0^{\pi/2} \sin\alpha\cos^2\alpha\, d\alpha$$

$$= 2\pi\mu k r^3 \left[-\frac{\cos^3\alpha}{3}\right]_0^{\pi/2} = \frac{2}{3}\pi\mu k r^3$$

$$\Sigma F = 0 \Rightarrow P = \int p\sin\alpha\, dA$$

$$P = \int_0^{\pi/2} (k\sin^2\alpha)(r\,d\alpha \cdot 2\pi r\cos\alpha)$$

$$= 2\pi k r^2 \int_0^{\pi/2} \sin^2\alpha\cos\alpha\, d\alpha = 2\pi k r^2 \left[\frac{\sin^3\alpha}{3}\right]_0^{\pi/2}$$

$$= \frac{2}{3}\pi k r^2$$

So $\underline{M = \mu P r}$

6/86

$dL = p(2\pi r)\, ds \sin\frac{\alpha}{2}$ where $ds = dz/\cos\frac{\alpha}{2}$

$$dL = 2\pi p \sin\frac{\alpha}{2}\left(z \tan\frac{\alpha}{2}\right)\frac{dz}{\cos\alpha/2}$$

$$= 2\pi p \tan^2\frac{\alpha}{2}\, z\, dz$$

$$L = 2\pi p \tan^2\frac{\alpha}{2}\int_{z_1}^{z_2} z\, dz \quad \text{where} \quad z_1 = \frac{d_1/2}{\tan\alpha/2}, \quad z_2 = \frac{d_2/2}{\tan\alpha/2}$$

$$L = \frac{\pi p}{4}\left(d_2^2 - d_1^2\right)$$

$$M = \int r \mu p\, dA = \mu p \int_{z_1}^{z_2}\left(z \tan\frac{\alpha}{2}\right)\left(2\pi z \tan\frac{\alpha}{2}\right)\frac{dz}{\cos\alpha/2}$$

$$= 2\pi\mu p \frac{\tan^2\frac{\alpha}{2}}{\cos\alpha/2}\int_{z_1}^{z_2} z^2\, dz = 2\pi\mu p \frac{1}{24\sin\frac{\alpha}{2}}\left(d_2^3 - d_1^3\right)$$

$$M = \frac{\mu L}{3\sin\frac{\alpha}{2}}\,\frac{d_2^3 - d_1^3}{d_2^2 - d_1^2}$$

6/87

$\frac{T_2}{T_1} = e^{\mu\beta}$, $\frac{100}{50} = e^{\pi\mu}$, $\pi\mu = \ln 2$

$\mu = \frac{0.6931}{\pi} = \underline{0.221}$

6/88 Use $T_2 = T_1 e^{\mu\beta}$

(a) $P = 40(9.81)e^{0.3(\pi)} = \underline{1007 \text{ N}}$

(b) $40(9.81) = P e^{0.3(\pi)}$, $\underline{P = 152.9 \text{ N}}$

6/89

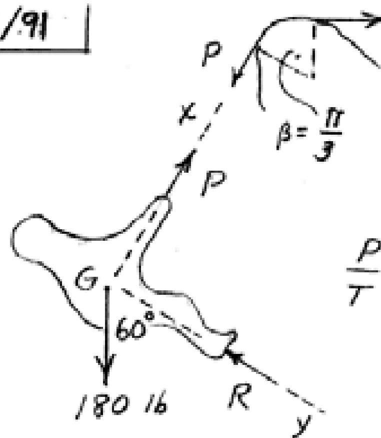
$$\frac{T_2}{T_1} = e^{\mu\beta}: \quad \frac{mg}{mg/10} = e^{\mu(3\pi)}, \quad \underline{\mu = 0.244}$$

6/90

$$\frac{mg}{P} = e^{\mu\beta}, \quad \frac{mg}{mg/6} = e^{\mu\left(\frac{5}{4}2\pi\right)}$$

$$\frac{5}{2}\pi\mu = \ln 6 = 1.792, \quad \mu = \frac{2(1.792)}{5\pi} = \underline{0.228}$$

6/91



Equil. of climber, $\Sigma F_x = 0$ gives $P = 180 \sin 60° = 155.9$ lb

$\frac{P}{T} = e^{\mu\beta}$; $\frac{155.9}{75} = e^{\pi\mu/3}$

$\frac{\pi}{3}\mu = \ln 2.078 = 0.732$

$\mu = 3(0.732)/\pi = \underline{0.699}$

6/92

$T_2 = T_1 e^{\mu\beta}$, $\beta = 2$ turns $+ 60°$

$= 2(360°) + 60° = 780°$

or 13.61 rad

$T = \left(\frac{2}{16}\right) e^{0.7(13.61)} = \underline{1720 \text{ lb}}$

6/93 $\frac{4}{mg} = e^{\mu\beta}$, $\frac{mg}{1.6} = e^{\mu\beta}$

Thus $\frac{4}{mg} = \frac{mg}{1.6}$, $m^2 g^2 = 4(1.6)$

$m = \frac{\sqrt{4(1.6)}}{9.81} = 0.258$ Mg or $m = 258$ kg

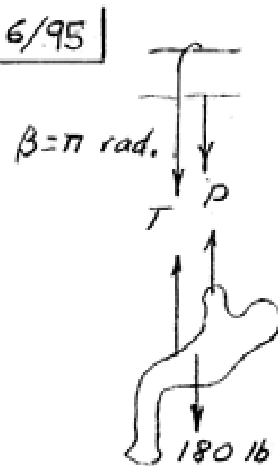
6/94 Use $T_2 = T_1 e^{\mu\beta}$

(a) Lower: $50(9.81) = 70\, e^{\mu\pi}$, $\mu = 0.620$

(b) Raise: $P' = 50(9.81)e^{0.620\pi}$

$P' = 3440$ N or $P' = 3.44$ kN

6/95



$T = Pe^{\mu\beta}$

$= Pe^{0.6\pi} = 6.59\,P$

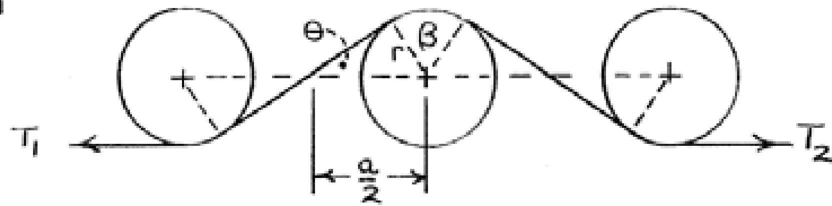
Man; $\Sigma F = 0$; $T + P = 180$

Combine & get

$P(1 + 6.59) = 180$ lb

$P = \frac{180}{7.59} = \underline{23.7 \text{ lb}}$

6/96



$\theta = \sin^{-1} \frac{r}{a/2}$, $\beta = 2\theta = 2\sin^{-1} \frac{2r}{a}$

$\frac{T_2}{T_1} = e^{\mu\beta}$: $1.15 = e^{0.1\beta}$, $\beta = 1.398$ rad or $80.1°$

Thus $80.1° = 2\sin^{-1} \frac{2(20)}{a}$, $\underline{a = 62.2 \text{ mm}}$

6/97 | Normal force under section BC is

$N = 5(1.2)(9.81) = 58.9$ N and friction

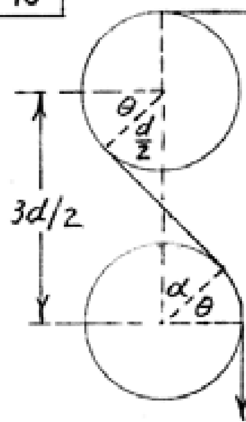
there is $F = T_1 = \mu N = 0.5(58.9) = 29.4$ N

$T_2 = T_1 e^{\mu\beta} = 29.4 e^{0.4(\pi/2)} = 55.2$ N

Neglecting effects of the hose mass from

A to B, $P_x = 55.2$ N

6/98



100(9.81) N

$\frac{P}{981} = e^{0.4\beta}$

where $\beta = \pi/2 + 2\theta$

From geometry

$\theta = \frac{\pi}{2} - \alpha = \frac{\pi}{2} - \cos^{-1}\frac{d/2}{3d/4}$

$= 90^\circ - 48.19^\circ = 41.81^\circ$

so $\beta = 90 + 2(41.81) = 173.6^\circ$

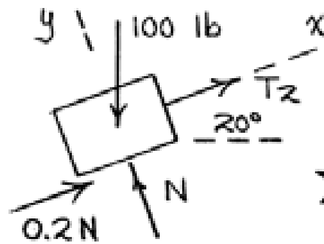
or $\beta = \frac{173.6}{180}\pi = 3.03$ rad

so $P = 981 e^{0.4(3.03)}$

$= 981(3.36) = 3297$ N

or $\underline{P = 3.30 \text{ kN}}$

6/99 Motion impending down plane:



$\Sigma F_y = 0 \Rightarrow N = 100 \cos 20°$

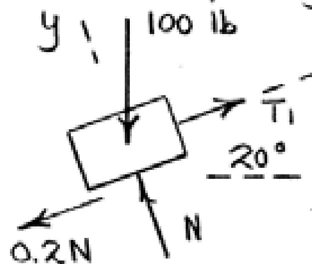
$= 94.0$ N (throughout)

$\Sigma F_x = 0: \; T_2 - 100 \sin 20° + 0.2(94.0) = 0$

$T_2 = 15.41$ lb

$T_2 = T_1 e^{\mu\beta}: \; 15.41 = W e^{0.3(1.920)}, \; W = 8.66$ lb

Motion impending up plane:



$\Sigma F_x = 0: \; T_1 - 100 \sin 20° - 0.2(94.0) = 0$

$T_1 = 53.0$ lb

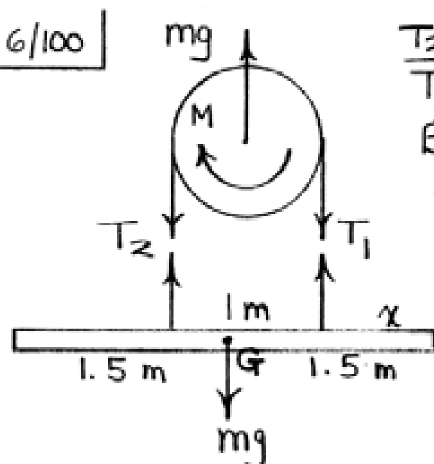
$T_2 = T_1 e^{\mu\beta}: \; W = 53.0 e^{0.3(1.920)}$

$W = 94.3$ lb

So range is $8.66 \leq W \leq 94.3$ lb

(Note that $\beta = 90° + 20° = 110°$ or 1.920 rad)

6/100



$$\frac{T_2}{T_1} = e^{\mu\beta} = e^{0.25\pi} = 2.19$$

Beam : $\Sigma M_G = 0$:

$$T_2\,(1 + x - 1.5) = T_1\,(1.5 - x)$$

Combine to get

$$\frac{1.5 - x}{x - 0.5} = 2.19$$

$$\underline{x = 0.813\ \text{m}}$$

6/101

$L = 75(9.81)$ N, $T_2 = T_1 e^{\mu\beta}$,

75(9.81) N

10 N

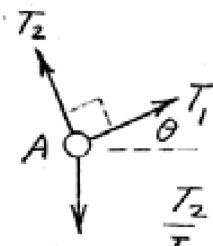
10 N

$$75(9.81) - 10 = 10e^{\mu(3+\frac{1}{2})2\pi}$$

$$72.6 = e^{21.99\mu}$$

$$21.99\mu = 4.285, \quad \underline{\mu = 0.195}$$

6/102



Equilibrium of A gives

$T_1 = mg\sin\theta$

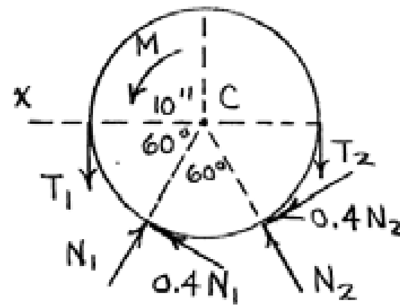
$T_2 = mg\cos\theta$

$\frac{T_2}{T_1} = e^{\mu\beta}, \quad \frac{mg\cos\theta}{mg\sin\theta} = e^{3\pi\mu/2}$

For $\theta = 20°$, $\text{ctn}\,20° = e^{3\pi\mu/2} = 2.747$

or $\mu = \frac{2}{3\pi}\ln 2.747 = \underline{0.214}$

6/103

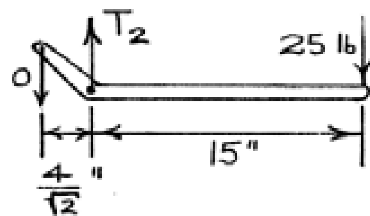


Lever: $\Sigma M_O = 0: \quad \frac{4T_2}{\sqrt{2}} = 25\left(15 + \frac{4}{\sqrt{2}}\right)$

$T_2 = 157.6$ lb

Band: $\frac{T_2}{T_1} = e^{\mu\beta}$

$\frac{157.6}{T_1} = e^{0.3\pi}, \quad T_1 = 61.4$ lb



Pipe: $\Sigma F_y = 0$:

$N_1\left(\frac{\sqrt{3}}{2} + 0.4\left(\frac{1}{2}\right)\right) + N_2\left(\frac{\sqrt{3}}{2} - 0.4\left(\frac{1}{2}\right)\right)$

$-157.6 - 61.4 = 0 \qquad$ (a)

$\Sigma F_x = 0: \quad N_1\left(\frac{1}{2} - 0.4\frac{\sqrt{3}}{2}\right)$

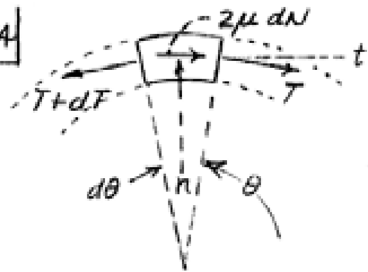
$-N_2\left(\frac{1}{2} + 0.4\frac{\sqrt{3}}{2}\right) = 0 \qquad$ (b)

Solve (a) & (b): $N_1 = 184.5$ lb, $N_2 = 33.5$ lb

$\circlearrowleft + \Sigma M_C = 0: \quad M - 157.6(10) + 61.4(10)$

$-0.4(184.5 + 33.5)(10) = 0, \qquad \underline{M = 1834 \text{ lb-in.}}$

6/104



Cross section of belt

$\Sigma F_n = 0;\quad T \sin\frac{d\theta}{2} + (T+dT)\sin\frac{d\theta}{2} = 2\,dN \sin\frac{\alpha}{2}$

or $\quad T\,d\theta = 2\,dN \sin\frac{\alpha}{2}$

$\Sigma F_t = 0;\quad T\cos\frac{d\theta}{2} + 2\mu\,dN = (T+dT)\cos\frac{d\theta}{2}$

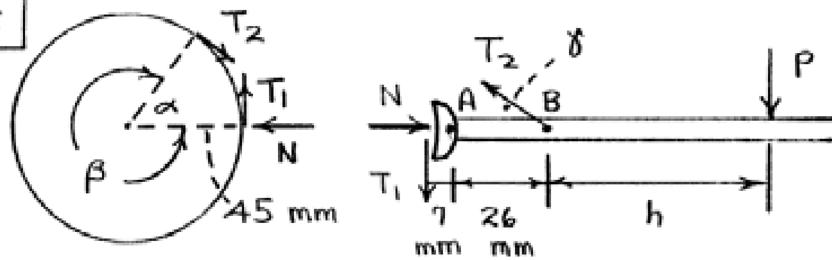
or $\quad 2\mu\,dN = dT$

combine & get $\quad \frac{dT}{T} = \frac{\mu}{\sin\frac{\alpha}{2}}\,d\theta$

$\int_{T_1}^{T_2}\frac{dT}{T} = \frac{\mu}{\sin\frac{\alpha}{2}}\int_0^{\beta} d\theta,\quad \ln\frac{T_2}{T_1} = \frac{\mu\beta}{\sin\frac{\alpha}{2}},\quad \underline{T_2 = T_1 e^{\mu\beta/\sin\frac{\alpha}{2}}}$

$n = 1/\sin 17.5° = \underline{3.33}$

6/105



$\cos\alpha = \frac{45}{45+7+26}$, $\alpha = 54.8°$, $\gamma = 180-90-\alpha = 35.2°$

$\beta = 360-\alpha = 305°$ or 5.33 rad

$$\frac{T_2}{T_1} = e^{\mu\beta} = e^{0.25(5.33)} = 3.79 \qquad (a)$$

Bar: $\Sigma M_P = 0$: $T_1(h+7+26) - T_2 \sin\gamma\,(h) = 0$

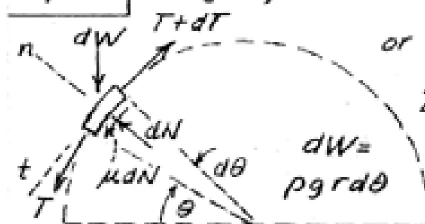
or $$\frac{T_2}{T_1} = \frac{h+33}{0.577\,h} \qquad (b)$$

From (a) & (b), $\underline{h = 27.8 \text{ mm}}$

(For actual wrench, $h \cong 100$ mm)

▸6/106

$\Sigma F_t = 0;\ (T+dT)\cos\frac{d\theta}{2} - T\cos\frac{d\theta}{2} - \mu\, dN = \rho g r\, d\theta \cos\theta$



or $dT - \mu\, dN = \rho g r \cos\theta\, d\theta$

$\Sigma F_n = 0;\ (T+dT)\sin\frac{d\theta}{2} + T\sin\frac{d\theta}{2}$
$+ \rho g r\, d\theta \sin\theta - dN = 0$

or $T\,d\theta + \rho g r \sin\theta\, d\theta = dN$

Eliminate dN & get $\frac{dT}{d\theta} - \mu T = \rho g r(\mu\sin\theta + \cos\theta)$

which is standard form for linear, nonhomogeneous Eq.

Sol. is $T = Ce^{\mu\theta} + e^{\mu\theta}\int e^{-\mu\theta}\rho g r(\mu\sin\theta + \cos\theta)\,d\theta$

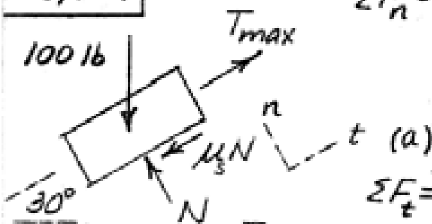
$= Ce^{\mu\theta} + \frac{\rho g r}{1+\mu^2}\left[0 - 2\mu(1)\right],\ C = \frac{2\rho g \mu r}{1+\mu^2}$

so $T = \frac{\rho g r}{1+\mu^2}\left[2\mu e^{\mu\theta} + (1-\mu^2)\sin\theta - 2\mu\cos\theta\right]$

$T_{\theta=\pi} = \rho g h = \frac{\rho g r}{1+\mu^2}\left[2\mu e^{\mu\pi} + 0 - 2\mu(-1)\right]$

so $\underline{h = \frac{2\mu r}{1+\mu^2}(1+e^{\mu\pi})}$

6/107



$\Sigma F_n = 0;\quad N = 100 \cos 30° = 86.6 \text{ lb}$

$\mu_s N = 0.3(86.6) = 26.0 \text{ lb}$

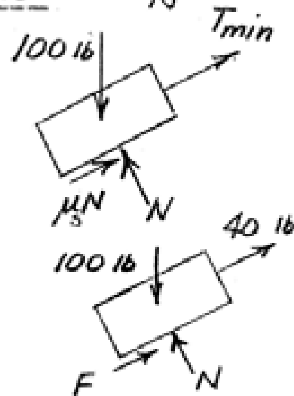
(a)

$\Sigma F_t = 0;\quad T_{max} - 100 \sin 30° - 26.0 = 0$

$T_{max} = \underline{76.0 \text{ lb}}$

$T_{min} - 100 \sin 30° + 26.0 = 0$

$T_{min} = \underline{24.0 \text{ lb}}$



(b) For $T = 40$ lb, $\Sigma F_t = 0$ gives

$F - 100 \sin 30° + 40 = 0$

$\underline{F = 10 \text{ lb}} < \mu_s N$

6/108 From $\theta_{max} = \tan^{-1} \mu_s$, we have

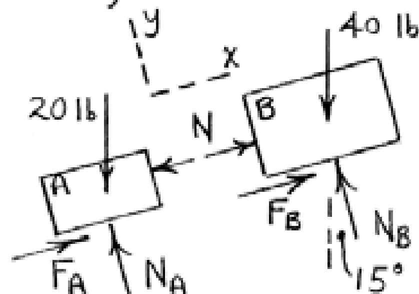
$(\theta_{max})_A = \tan^{-1} 0.30 = 16.70°$

$(\theta_{max})_B = \tan^{-1} 0.20 = 11.31°$

$(\theta_{max})_C = \tan^{-1} 0.35 = 19.29°$

So C remains stationary

By themselves, B would slide, A would not.



From $\Sigma F_y = 0$:

$N_A = 19.32$ lb

$N_B = 38.6$ lb

Assume that slipping impends for B:

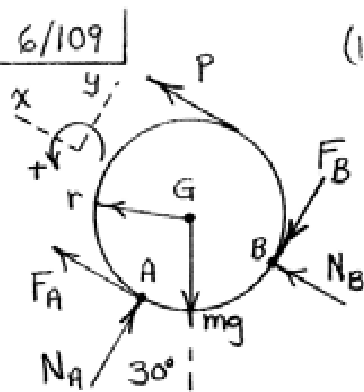
$\Sigma F_x = 0$: $N + 0.2(38.6) - 40 \sin 15° = 0$, $N = 2.63$ lb

$\Sigma F_x = 0$ for A: $F_A - 20 \sin 15° - 2.63 = 0$

$F_A = 7.80$ lb; $(F_A)_{max} = 0.30(19.32) = 5.80$ lb

Because $(F_A)_{max} < F$, both A and B slip.

6/109



(1) Assume no slippage until contact at B is lost :

$F_B = N_B = 0$

$$\begin{cases} \Sigma F_x = 0: \ P + F_A - mg \sin 30° = 0 \\ \Sigma F_y = 0: \ N_A - mg \cos 30° = 0 \\ \Sigma M_G = 0: \ Pr - F_A r = 0 \end{cases}$$

Solution : $P = F_A = \frac{mg}{4}$, $N_A = 0.866 mg$

$(F_A)_{max} = \mu_s N_A = 0.25(0.866 mg) = 0.217 mg < F_A$;

Assumption invalid

(2) Assume rotational slippage impends :

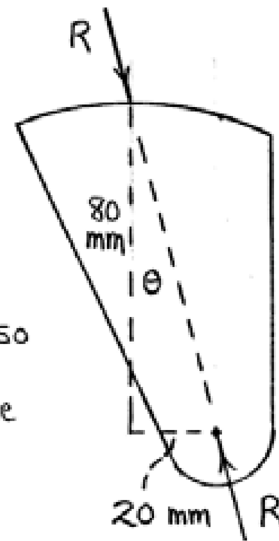
$F_A = \mu_s N_A = 0.25 N_A$, $F_B = \mu_s N_B = 0.25 N_B$

$$\left.\begin{aligned} \Sigma F_x = 0: &\ P + 0.25 N_A - mg \sin 30° + N_B = 0 \\ \Sigma F_y = 0: &\ N_A - mg \cos 30° - 0.25 N_B = 0 \\ \Sigma M_G = 0: &\ Pr - 0.25 N_A r - 0.25 N_B r = 0 \end{aligned}\right\}$$

Solution: $P = 0.232 mg$, $N_A = 0.878 mg$, $N_B = 0.0487 mg$

So rotational slippage occurs first, at $P = 0.232 mg$

6/110 $\theta = \tan^{-1}\frac{20}{80} = 14.04°$



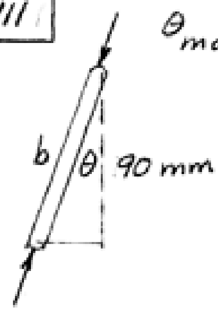
For the rope,

$$2R\sin 14.04° = 900$$

$$\underline{R = 1855 \text{ N}}$$

$\theta < (\phi = \tan^{-1} 0.8 = 38.7°)$, so there is no slipping and the friction force is less than $F_{max} = \mu_s N$.

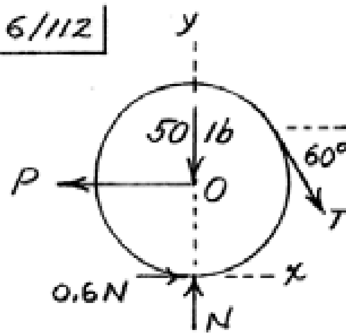
6/111



$\theta_{max} = \phi = \tan^{-1}\mu = \tan^{-1} 0.40 = 21.80°$

$b = 90/\cos 21.80° = \underline{96.9 \text{ mm}}$

6/112



$\Sigma M_O = 0;\ T = 0.6N$ (a)

$\Sigma F_x = 0;\ T\cos 60° + 0.6N - P = 0$ (b)

$\Sigma F_y = 0;\ T\sin 60° + 50 - N = 0$ (c)

(a) & (c) $0.6N(0.866) + 50 - N = 0$

$0.4804N = 50,\ N = 104.1\text{ lb}$

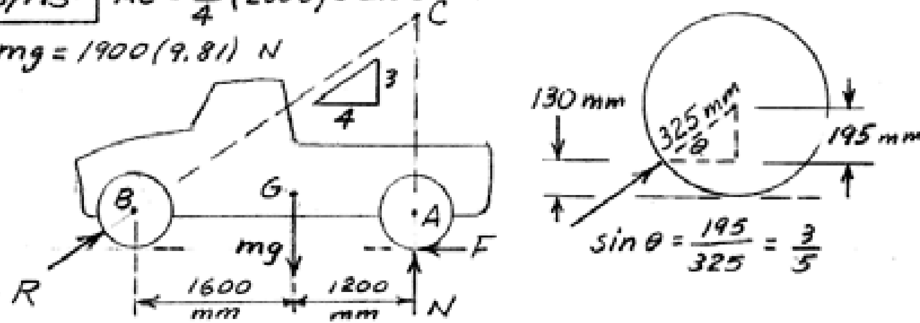
(c) $0.866T + 50 - 104.1 = 0$

$T = 62.4\text{ lb}$

(b) $P = 62.4(0.5) + 0.6(104.1),\ \underline{P = 93.7\text{ lb}}$

6/113 $\overline{AC} = \frac{3}{4}(2800) = 2100$ mm

$mg = 1900(9.81)$ N



$\sin\theta = \frac{195}{325} = \frac{3}{5}$

$\Sigma M_C = 0$; $F(2100 + 325) - 1200\,mg = 0$, $F = 0.495\,mg$

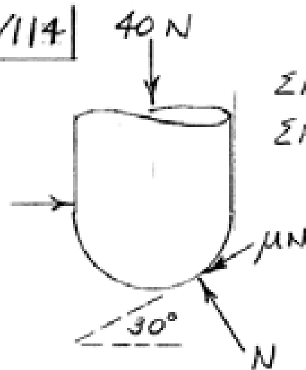
$\Sigma M_B = 0$; $1600\,mg + 0.495\,mg\,(325) - 2800\,N = 0$, $N = 0.629\,mg$

Thus $\mu_{min} = F/N = 0.495/0.629 = \underline{0.787}$

$M = Fr = 0.495(1900)(9.81)(0.325) = 2998$ N·m

or $\underline{M = 3.00 \text{ kN·m}}$

6/114



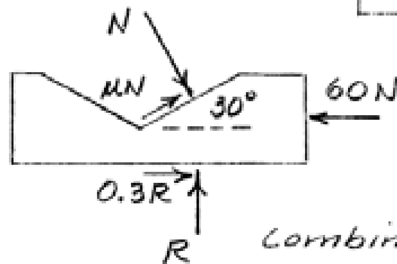
Detent:

$\Sigma F_x = 0$; $0.3R + 0.866\mu N + 0.5N - 60 = 0$

$\Sigma F_y = 0$; $R + 0.5\mu N - 0.866N = 0$

Eliminate R & get

$$N = \frac{60}{0.7598 + 0.716\mu} \qquad (a)$$



Plunger:

$\Sigma F_y = 0$;

$0.866N - 0.5\mu N - 40 = 0$

$N(0.866 - 0.5\mu) = 40$

$$N = \frac{40}{0.866 - 0.5\mu} \qquad (b)$$

Combine (a) & (b) & get

$60(0.866 - 0.5\mu) = 40(0.7598 + 0.716\mu)$

Solve for μ & get $\underline{\mu = 0.368}$ $\quad (N = 58.6 \text{ N})$

6/115

$$\phi = \tan^{-1}\mu_s = \tan^{-1}(0.2) = 11.31°$$

$$\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{2.5}{2\pi(5)} = 4.55°$$

(a) Tighten: $M = Pr\tan(\phi + \alpha)$

$$F(100) = 600\left(\frac{10}{2}\right)\tan(11.31° + 4.55°)$$

$$\underline{F = 8.52 \text{ N}}$$

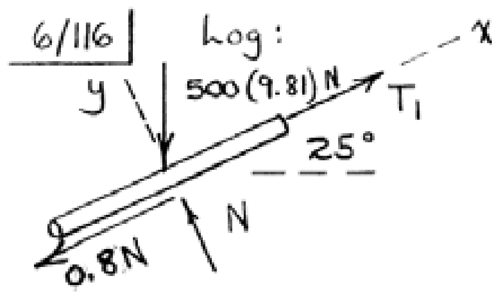
(b) Loosen: $M = Pr\tan(\phi - \alpha)$

$$F(100) = 600\left(\frac{10}{2}\right)\tan(11.31° - 4.55°)$$

$$\underline{F = 3.56 \text{ N}}$$

6/116

Log:



$\Sigma F_y = 0$: $N - 500(9.81)\cos 25° = 0$, $N = 4450$ N

$\Sigma F_x = 0$: $T_1 - 500(9.81)\sin 25° - 0.8(4450) = 0$

$T_1 = 5630$ N

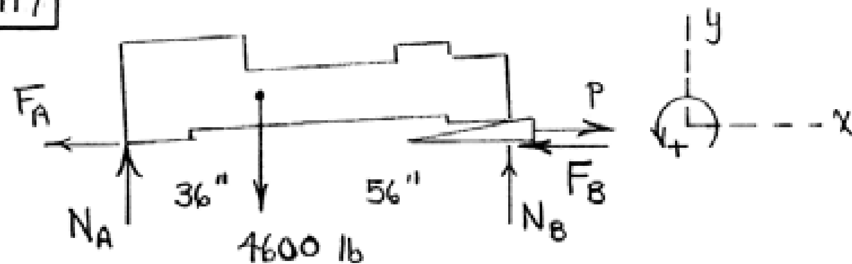
Rock:

$$T = T_1 e^{\mu\beta}$$

$$= 5630\, e^{0.5\left[40 \frac{\pi}{180}\right]}$$

$$= \underline{7980 \text{ N}}$$

6/117

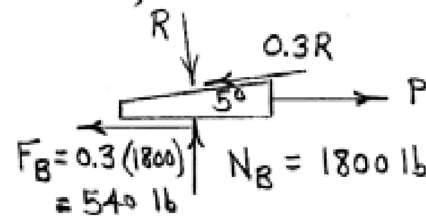


Lathe and wedge as a unit:

$\Sigma M_A = 0: \quad 92N_B - 4600(36) = 0, \quad N_B = 1800 \text{ lb}$

$\Sigma F_y = 0: \quad N_A - 4600 + 1800 = 0, \quad N_A = 2800 \text{ lb}$

Wedge:



$\Sigma F_y = 0: \quad 1800 - R\cos 5° - 0.3R\sin 5° = 0, \quad R = 1761 \text{ lb}$

$\Sigma F_x = 0: \quad P - 540 - 0.3(1761)\cos 5° + 1761\sin 5° = 0$

$\underline{P = 913 \text{ lb}}$

Lathe & wedge: $\Sigma F_x = 0: \quad 913 - 540 - F_A = 0, \quad F_A = 373 \text{ lb}$

But $373 < (0.3)(2800) = 840$ lb; A cannot slip.

6/118

$$\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{13}{2\pi(78)} = 3.04°$$

$$\phi = \tan^{-1}\mu = \tan^{-1}0.25 = 14.04°$$

(a) To raise, $M = Wr\tan(\alpha + \phi)$

$$= \frac{2.2(10^3)9.81}{2}\frac{(0.078)}{2}\tan(3.04° + 14.04°)$$

$$= \underline{129.3\ N\cdot m}$$

(b) To lower, $M = Wr\tan(\phi - \alpha)$

$$= \frac{2.2(10^3)9.81}{2}\frac{(0.078)}{2}\tan(14.04 - 3.04)$$

$$= \underline{81.8\ N\cdot m}$$

6/119

$\ell\sqrt{5-\cos^2\theta} - \ell\sin\theta = 1.622\ell$ @ $\theta = 25°$

N_A, A, mg, $\ell\sqrt{5}$, 115°, θ, O, B, ℓ, $\mu_s N_B$, N_B, y, x, +

$\Sigma F_x = 0: N_A + N_B \sin 25° - \mu_s N_B \cos 25° = 0 \quad (1)$

$\Sigma F_y = 0: N_B \cos 25° + \mu_s N_B \sin 25° - mg = 0 \quad (2)$

$\Sigma M_O = 0: N_B \ell - N_A(1.622\ell) - mg\frac{\ell}{2}\cos 25° = 0 \quad (3)$

(2): $N_B = \dfrac{mg}{\cos 25° + \mu_s \sin 25°} = \dfrac{mg}{0.906 + \mu_s(0.423)}$

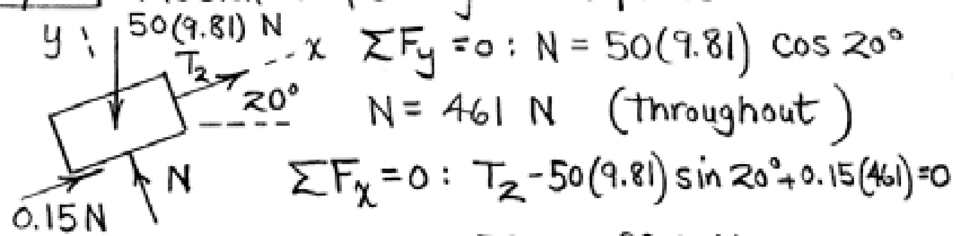
Rewrite (1) & (3):
$$\left.\begin{aligned} N_A + [0.423 - \mu_s(0.906)]N_B &= 0 \\ -1.622N_A + N_B &= 0.453\,mg \end{aligned}\right\}$$

Multiply first eq. by 1.622 & add: $N_B = \dfrac{0.453\,mg}{1.685 - 1.470\mu_s}$

Equate two expressions for N_B & solve for μ_s:

$\underline{\mu_s = 0.767}$

6/120 Motion impending down plane:



$\Sigma F_y = 0$: $N = 50(9.81)\cos 20°$

$N = 461$ N (throughout)

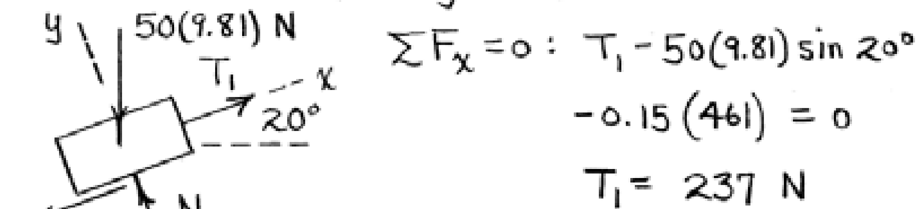
$\Sigma F_x = 0$: $T_2 - 50(9.81)\sin 20° + 0.15(461) = 0$

$T_2 = 98.6$ N

$T_2 = T_1 e^{\mu\beta}$: $98.6 = \frac{mg}{2} e^{0.25(110 \cdot \frac{\pi}{180})}$

$m = 12.44$ kg

Motion impending up plane:



$\Sigma F_x = 0$: $T_1 - 50(9.81)\sin 20°$

$-0.15(461) = 0$

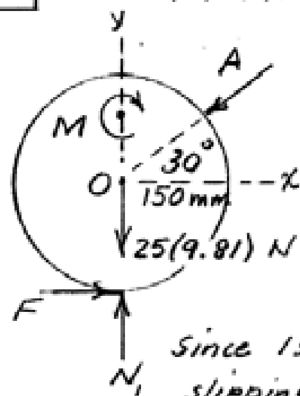
$T_1 = 237$ N

$T_2 = T_1 e^{\mu\beta}$: $\frac{mg}{2} = 237 e^{0.25(110 \cdot \frac{\pi}{180})}$

$m = 78.0$ kg

So range is $\underline{12.44 \leq m \leq 78.0 \text{ kg}}$

6/121



(a) Assume complete equilibrium

$\Sigma M_O = 0$; $M - Fr = 0$

$F = \frac{20}{0.150} = 133.3$ N

$\Sigma F_x = 0$; $133.3 - A\cos 30° = 0$

$A = 154.0$ N

$\Sigma F_y = 0$; $N_1 - 154\sin 30° - 25(9.81) = 0$

$N_1 = 322.2$ N

Since $133.3 < (\mu_s N_1 = 0.5[322.2] = 161\text{ N})$, slipping does not occur & assumption is valid. Thus $\underline{F = 133.3\text{ N}}$

(b) Assume wheel slips & $\Sigma M_O \neq 0$ so $F = \mu_k N_1 = 0.4 N_1$

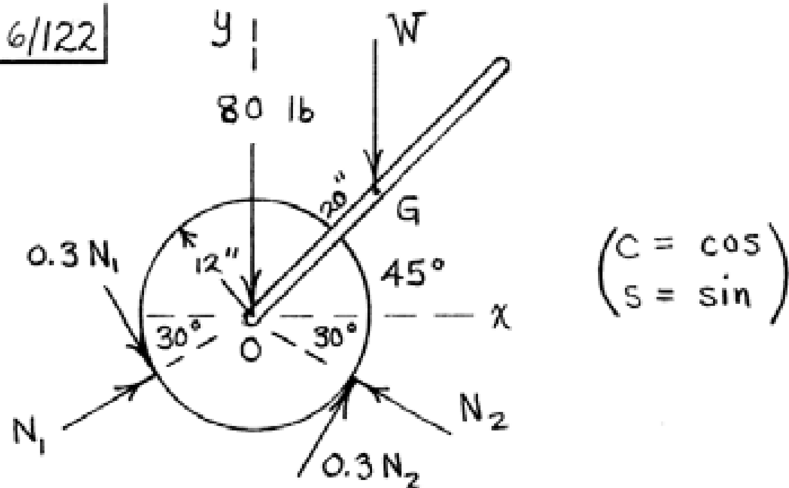
$\Sigma F_x = 0$; $0.4N_1 - A\cos 30° = 0$

$\Sigma F_y = 0$; $N_1 - 25(9.81) - A\sin 30° = 0$

Solve & get $A = 147.3$ N, $N_1 = 318.9$ N, $F = 0.4(318.9) = \underline{127.6\text{ N}}$

Assumption valid since $(M = 40\text{ N}\cdot\text{m}) > (127.6[0.150] = 19.1\text{ N}\cdot\text{m})$

6/122



(C = cos
S = sin)

$\Sigma F_x = 0: \; N_1 c30° + 0.3N_1 s30° - N_2 c30° + 0.3N_2 s30° = 0$

$\Sigma F_y = 0: \; N_1 s30° - 0.3N_1 c30° + N_2 s30° + 0.3N_2 c30° - 80 - W = 0$

$\Sigma M_O = 0: \; (0.3N_1 + 0.3N_2)(12) - W(20 c45°) = 0$

Solve to obtain
$N_1 = 113.9$ lb
$N_2 = 161.6$ lb
$W = 70.1$ lb

6/123

$\circlearrowleft + \Sigma M_C = 0: \; 500(9.81)(127.3) - R(100\cos 45°) = 0, \; R = 8830 \text{ N}$

Friction angle $\phi = \tan^{-1}\mu = \tan^{-1}(0.20) = 11.31°$

$\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{0.002}{2\pi(0.005)} = 3.64°$

Raise load: $M = Rr\tan(\phi + \alpha)$

$P(0.150) = 8830(0.005)\tan(11.31° + 3.64°)$

$\underline{P = 78.6 \text{ N}}$

Lower load: $M = Rr\tan(\phi - \alpha)$

$P(0.150) = 8830(0.005)\tan(11.31° - 3.64°)$

$\underline{P = 39.6 \text{ N}}$

6/124 For the slab

$\Sigma F_x = 0: \mu_k N \cos\alpha - N \sin\alpha = 0$

$\mu_k = \tan\alpha$

$\frac{d}{2} + \frac{a}{2} = \frac{b}{2} + \frac{d}{2}\cos\alpha$

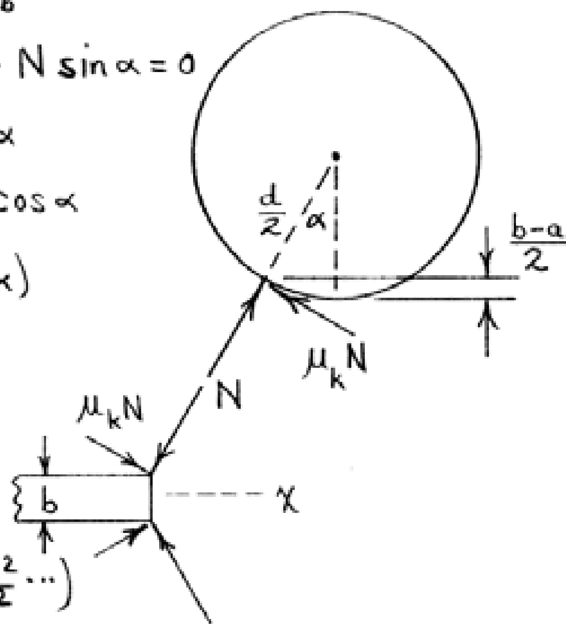
$b = a + d(1 - \cos\alpha)$

For small α,

$\begin{cases} \cos\alpha \cong 1 - \frac{\alpha^2}{2} + \cdots \\ \tan\alpha \cong \alpha \end{cases}$

So $b = a + d\left(1 - 1 + \frac{\alpha^2}{2} \cdots\right)$

$= a + \frac{\mu_k^2 d}{2}$



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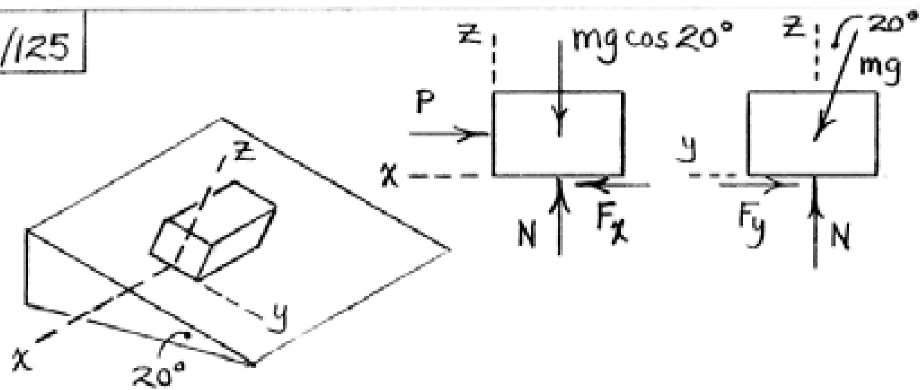


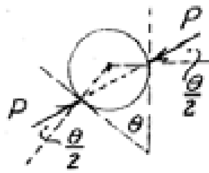
$$(x-z)\begin{cases} \Sigma F_z = 0: \; N - 8(9.81)\cos 20° = 0, \quad N = 73.7 \text{ N} \\ \Sigma F_x = 0: \; F_x - P = 0, \quad F_x = P \end{cases}$$

$$(y-z)\left\{ \Sigma F_y = 0: \; -F_y + 8(9.81)\sin 20° = 0, \; F_y = 26.8 \text{ N} \right.$$

$$F = \sqrt{F_x^2 + F_y^2} = \mu_s N: \sqrt{P^2 + 26.8^2} = 0.5(73.7)$$

$$\underline{P = 25.3 \text{ N}}$$

▶ 6/126

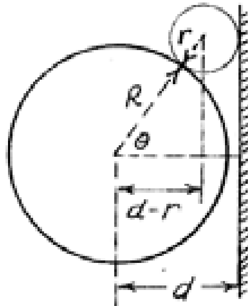


Rollers can support equal & opposite forces P provided $\frac{\theta}{2} < \tan^{-1}\mu$

Thus minimum d occurs when

$\tan\frac{\theta}{2} = \mu$

But $\tan^2 x = \frac{1-\cos 2x}{1+\cos 2x}$ so



$\tan^2\frac{\theta}{2} = \frac{1-\cos\theta}{1+\cos\theta}$ & $\mu^2 = \frac{1-\cos\theta}{1+\cos\theta}$

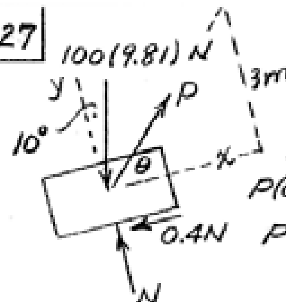
or $\cos\theta = \frac{1-\mu^2}{1+\mu^2}$ but $\cos\theta = \frac{d-r}{R+r}$

so $\frac{d-r}{R+r} = \frac{1-\mu^2}{1+\mu^2}$ which gives

$$d_{min} = \frac{2r + (1-\mu^2)R}{1+\mu^2}$$

& clearly $d_{max} = R + 2r$

*6/127



$\Sigma F_x = 0$; $P\cos\theta - 0.4N - 981\sin 10° = 0$

$\Sigma F_y = 0$; $N + P\sin\theta - 981\cos 10° = 0$

Eliminate N & get

$P(\cos\theta + 0.4\sin\theta) = 981(0.4\cos 10° + \sin 10°)$

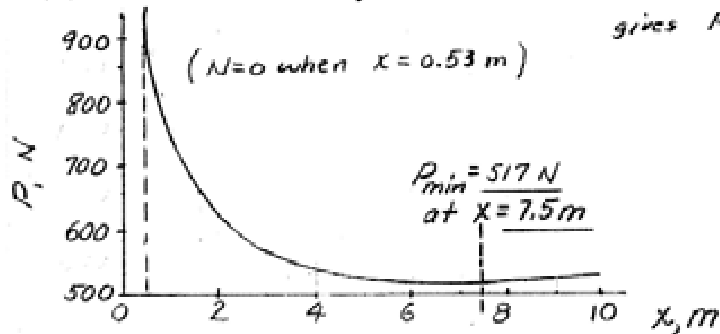
$P = \dfrac{556.8}{\cos\theta + 0.4\sin\theta}$

$\dfrac{dP}{d\theta}(\cos\theta + 0.4\sin\theta) + P(-\sin\theta + 0.4\cos\theta) = 0$

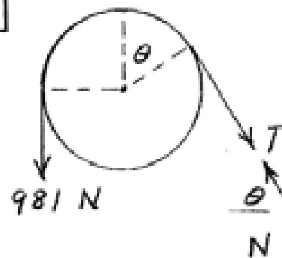
$\dfrac{dP}{d\theta} = P\,\dfrac{-\sin\theta + 0.4\cos\theta}{\cos\theta + 0.4\sin\theta} = 0$ for stationary value

Thus $\sin\theta = 0.4\cos\theta$, $\tan\theta = 0.4$, $x = \dfrac{3}{\tan\theta} = \dfrac{3}{0.4} = 7.5$ m

gives $P = 517$ N



*6/128



$T = 981 e^{0.5(\pi/2 + \theta)}$

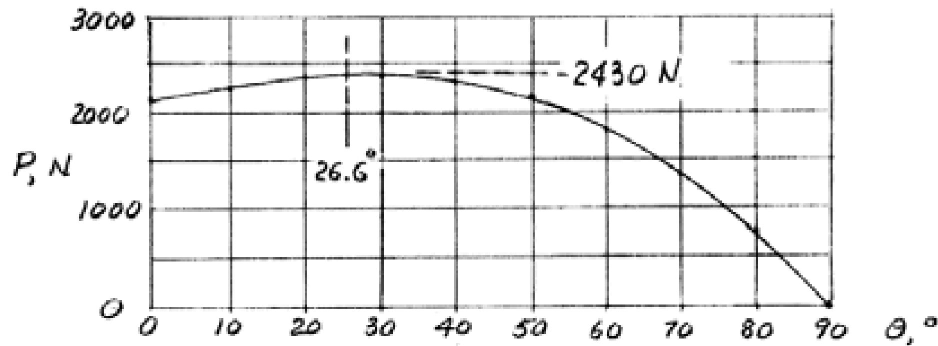
$P = T\cos\theta$

so $P = 981 e^{(\frac{\pi}{4} + \frac{\theta}{2})} \cos\theta$

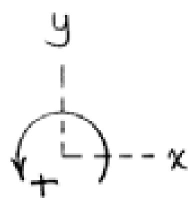
Carry out numerical solution & plot

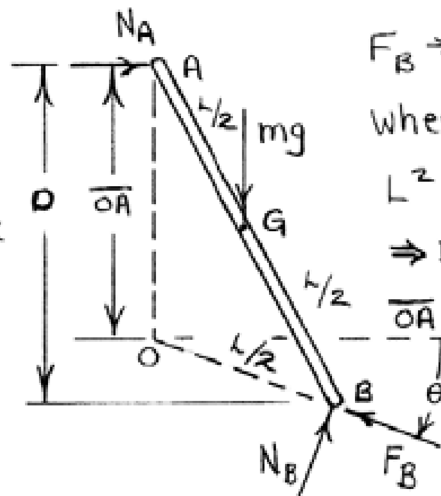
$\frac{dP}{d\theta} = 981 \left\{ -e^{\frac{\pi}{4}+\frac{\theta}{2}} \sin\theta + \frac{1}{2} e^{\frac{\pi}{4}+\frac{\theta}{2}} \cos\theta \right\} = 0$ for max. P

$\theta = \tan^{-1} 1/2 = \underline{26.6^\circ}$ & $P_{max} = 981 e^{1.017}(0.894) = \underline{2430\text{ N}}$



*6/129





$F_B \rightarrow \mu_s N_B = 0.4 N_B$ when slipping impends.

$$L^2 = D^2 + \left(\frac{L}{2}\cos\theta\right)^2$$

$$\Rightarrow D = L\sqrt{1 - \frac{\cos^2\theta}{4}}$$

$$\overline{OA} = L\sqrt{1 - \frac{\cos^2\theta}{4}} - \frac{L}{2}\sin\theta = \frac{L}{2}\left[\sqrt{4 - \cos^2\theta} - \sin\theta\right]$$

$$\Sigma F_x = 0: \; N_A + N_B \sin\theta - 0.4 N_B \cos\theta = 0 \qquad (1)$$

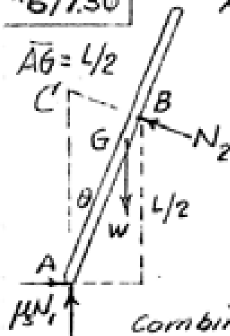
$$\Sigma F_y = 0: \; N_B \cos\theta + 0.4 N_B \sin\theta - mg = 0 \qquad (2)$$

$$\Sigma M_O = 0: \; N_B\left(\frac{L}{2}\right) - N_A \frac{L}{2}\left[\sqrt{4 - \cos^2\theta} - \sin\theta\right] - mg\frac{L}{4}\cos\theta = 0 \qquad (3)$$

Numerical solution :

$$\begin{cases} N_A = 0.287\,mg \\ N_B = 0.966\,mg \\ \underline{\theta = 5.80^\circ} \end{cases}$$

*6/130



$\overline{AB} = \frac{L/2}{\cos\theta}$, $\overline{GB} = \frac{L}{2}\left(\frac{1}{\cos\theta} - 1\right)$

$\Sigma M_B = 0$; $W \frac{L}{2}\left(\frac{1}{\cos\theta} - 1\right)\sin\theta + \mu_s N_1 \frac{L}{2} - N_1 \frac{L}{2}\tan\theta = 0$

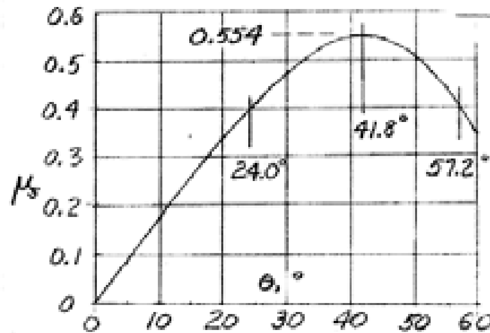
$$W(\tan\theta - \sin\theta) = N_1(\tan\theta - \mu_s)$$

$\Sigma M_C = 0$; $W \frac{L}{2}\sin\theta - \mu_s N_1 \frac{L/2}{\cos\theta}\frac{1}{\cos\theta} = 0$

$$W\sin\theta = \mu_s N_1 / \cos^2\theta$$

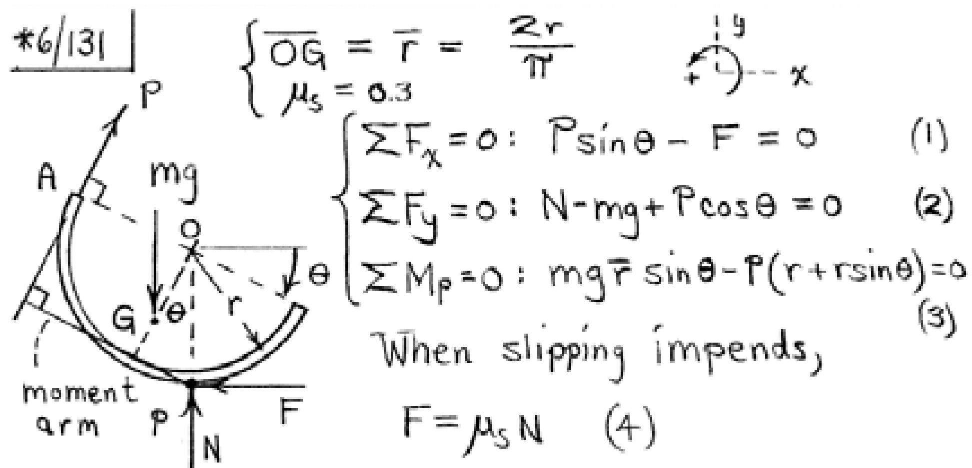
Combine & get $\frac{\tan\theta - \sin\theta}{\sin\theta} = \frac{\tan\theta - \mu_s}{\mu}\cos^2\theta$

Solve for μ_s & get $\mu_s = \frac{\sin\theta}{1 + \tan\theta(\tan\theta - \sin\theta)}$



(a) For $\mu_s = 0.4$, pole is unstable for $24.0° < \theta < 57.2°$

(b) $\theta = 41.8°$ requires $\mu_s = 0.554$



$\overline{OG} = \bar{r} = \dfrac{2r}{\pi}$

$\mu_s = 0.3$

$\Sigma F_x = 0: \; P\sin\theta - F = 0 \qquad (1)$

$\Sigma F_y = 0: \; N - mg + P\cos\theta = 0 \qquad (2)$

$\Sigma M_P = 0: \; mg\bar{r}\sin\theta - P(r + r\sin\theta) = 0 \qquad (3)$

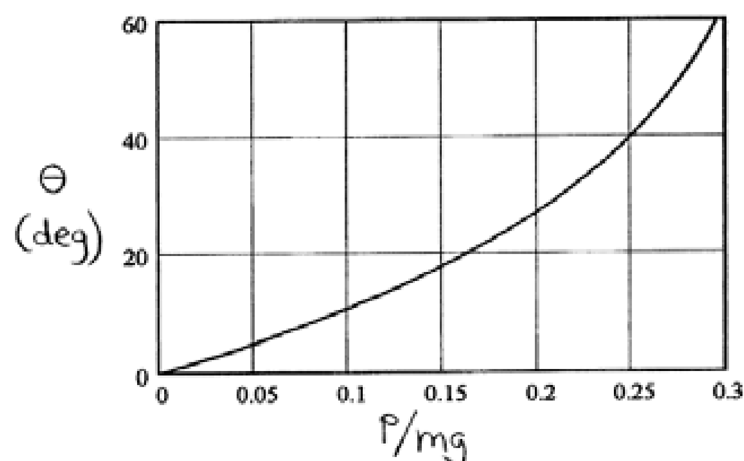
When slipping impends,

$F = \mu_s N \qquad (4)$

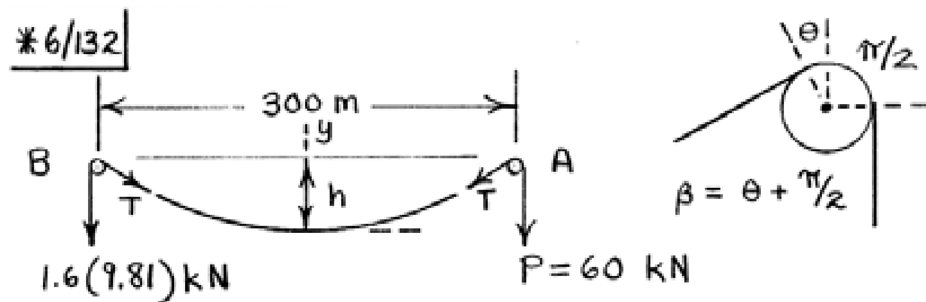
Numerical solution of (1) - (4): $\underline{\theta_{max} = 59.9°}$, $\underline{P_{max} = 0.295mg}$

Eqs. (1) - (3) may be solved for P as

$P = \dfrac{2\sin\theta}{\pi(1+\sin\theta)}$ (plotted below)



*6/132



During slipping, Eq. 6/7 @ A: $60 = Te^{\mu_k \beta}$

Eq. 6/7 @ B: $T = 1.6(9.81)e^{\mu_k \beta}$

Eliminate $\mu_k \beta$ & get $\underline{T = 30.7 \text{ kN}}$

Then $e^{\mu_k \beta} = 60/30.7$, $\mu_k \beta = 0.670$

Eq. 5/21: $T = T_o \cosh \frac{\rho g x}{T_o}$: $30.7 = T_o \cosh \frac{12(9.81)10^{-3} \cdot 150}{T_o}$

or $30.7 = T_o \cosh \frac{17.66}{T_o}$. Solve numerically: $T_o = 23.8$ kN

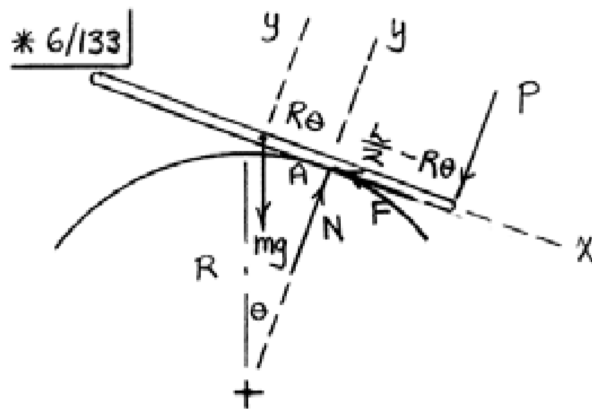
$\frac{dy}{dx} = \sinh \frac{\rho g x}{T_o}$: $\tan\theta = \sinh \frac{17.66}{23.8} = 0.810$

$\theta = 0.681$ rad, $\beta = \theta + \pi/2 = 2.25$ rad

$\mu_k = \frac{0.670}{2.25} = \underline{0.298}$

Eq. 5/22: $T = T_o + \rho g h$, $h = \frac{T - T_o}{\rho g}$

$= \frac{30.7 - 23.8}{12(9.81)(10^{-3})} = \underline{58.1 \text{ m}}$



$$\Sigma F_x = 0: \quad mg\sin\theta - \mu_s N = 0 \qquad (1)$$

$$\Sigma F_y = 0: \quad N - mg\cos\theta - P = 0 \qquad (2)$$

$$\Sigma M_A = 0: \quad P\left(\frac{L}{2} - R\theta\right) - mgR\theta\cos\theta = 0 \qquad (3)$$

(1): $N = \dfrac{mg\sin\theta}{\mu_s}$

(2): $\dfrac{mg\sin\theta}{\mu_s} - mg\cos\theta = P$

(3): $mg\left(\dfrac{\sin\theta}{\mu_s} - \cos\theta\right)\left(\dfrac{L}{2} - R\theta\right) - mgR\theta\cos\theta = 0$

Simplify: $\tan\theta = \mu_s \dfrac{1}{1 - \frac{2R\theta}{L}} = 0.15\dfrac{1}{1 - 2(0.6)\theta}$

or $\tan\theta - \dfrac{0.15}{1 - 1.2\theta} = 0$. Numerical solution: $\underline{\theta = 11.04°}$

*6/134

$\mu = 0.4$

From geometry,

upper wheel:

$$\theta + \frac{\pi}{4} + \frac{\pi}{2} + \beta_2 = 2\pi$$

$$\beta_2 = \frac{5\pi}{4} - \theta$$

lower wheel:

$$\theta + \frac{\pi}{4} + \beta_1 + \pi = 2\pi$$

$$\beta_1 = \frac{3\pi}{4} - \theta$$

Forces: $\begin{cases} mg = T_0 e^{\mu\left(\frac{3\pi}{4} - \theta\right)} \\ T_0 = T e^{\mu\left(\frac{5\pi}{4} - \theta\right)} \end{cases}$

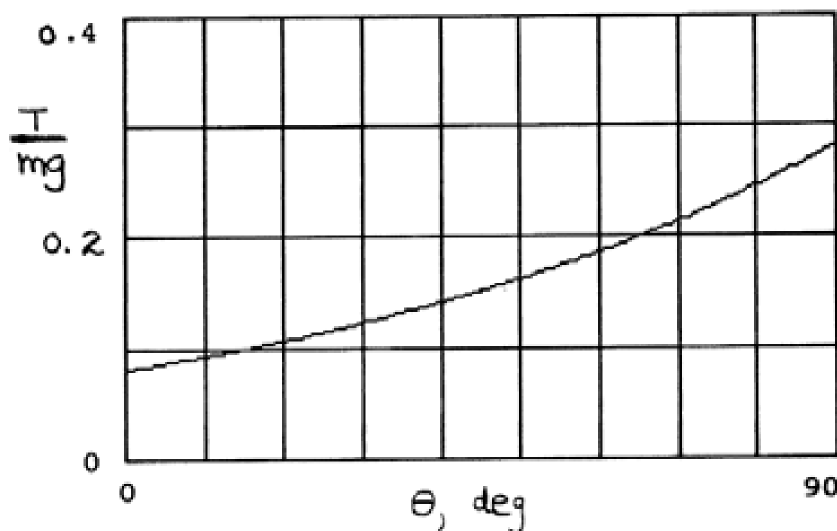
So $mg = Te^{2\mu(\pi - \theta)}$; $\dfrac{T}{mg} = e^{-0.8(\pi - \theta)}$

Device as a whole:

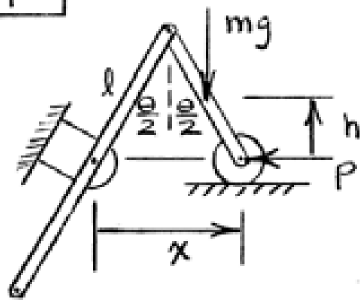
$$\circlearrowleft + \Sigma M_O = 0: \; 3rV - Tr - mg\left(r + 2r\sqrt{2}\cos\theta\right) = 0$$

$$V = \frac{1}{3}\left[T + mg\left(1 + 2\sqrt{2}\cos\theta\right)\right]$$

For $\theta = 60°$, $\dfrac{T}{mg} = 0.1872$, $\underline{V = 0.867mg}$



7/1



$x = 2\ell \sin\frac{\theta}{2}$

$\delta x = \ell \cos\frac{\theta}{2}\,\delta\theta$

$h = \frac{\ell}{2}\cos\frac{\theta}{2}$

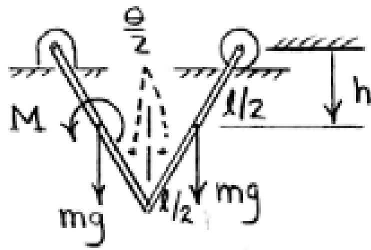
$\delta h = -\frac{\ell}{4}\sin\frac{\theta}{2}\,\delta\theta$

$\delta U = 0: \quad -P\delta x - mg\,\delta h = 0$

$-P\left(\ell\cos\frac{\theta}{2}\,\delta\theta\right) - mg\left(-\frac{\ell}{4}\sin\frac{\theta}{2}\,\delta\theta\right) = 0$

$\Rightarrow \quad \underline{\theta = 2\tan^{-1}\left(\frac{4P}{mg}\right)}$

7/2



$$h = \frac{l}{2}\cos\frac{\theta}{2}$$

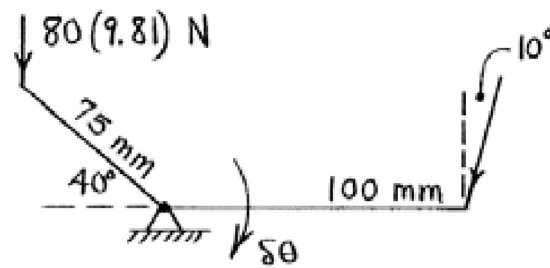
$$\delta h = -\frac{l}{4}\sin\frac{\theta}{2}\,\delta\theta$$

$\delta U = 0$: $\quad M\,\delta\left(\frac{\theta}{2}\right) + 2mg\,\delta h = 0$

$$M\,\frac{\delta\theta}{2} + 2mg\left(-\frac{l}{4}\sin\frac{\theta}{2}\,\delta\theta\right) = 0$$

$$\underline{M = mgl\sin\frac{\theta}{2}}$$

7/3

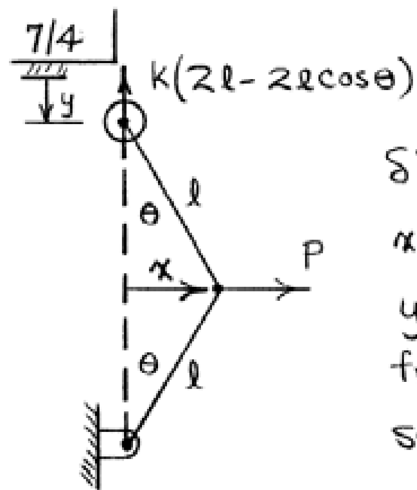


For a virtual displacement $\delta\theta$ of the lever,

$\delta U = 0$: $P\cos 10°(100\,\delta\theta) - 80(9.81)[75\,\delta\theta\cos 40°] = 0$

$\underline{P = 458 \text{ N}}$

7/4



$\delta U = 0: \; P\delta x - k(2l - 2l\cos\theta)\delta y = 0$

$x = l\sin\theta, \quad \delta x = l\cos\theta\,\delta\theta$

$y = 2l - 2l\cos\theta$ (measured from wheel position when spring is unstretched)

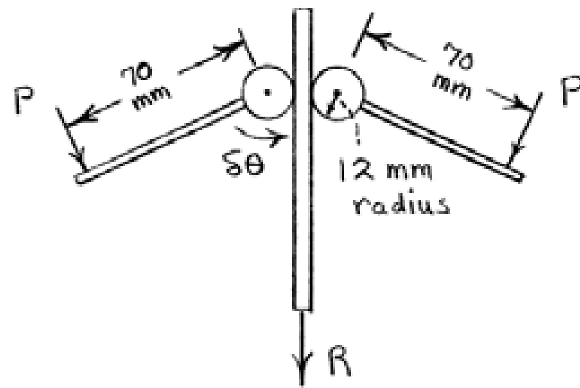
$\delta y = 2l\sin\theta\,\delta\theta$

So $\quad P(l\cos\theta\,\delta\theta) - k(2l - 2l\cos\theta)(2l\sin\theta\,\delta\theta) = 0$

$$\Rightarrow P = \frac{4kl(\sin\theta - \sin\theta\cos\theta)}{\cos\theta}$$

or $\underline{P = 4kl(\tan\theta - \sin\theta)}$

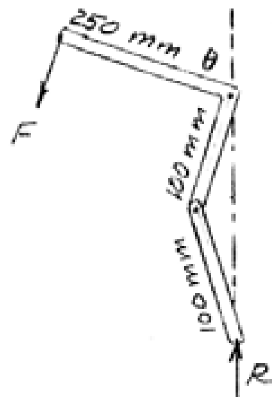
7/5



$$\delta U = 0: \; 2P(70)\delta\theta - R(12)\delta\theta = 0$$

$$\underline{R = 11.67\,P}$$

7/6



$\delta U = 0$

$F(250\,\delta\theta) - R\,\delta(2[100 \sin\theta]) = 0$

$250F\,\delta\theta = 200R\cos\theta\,\delta\theta$

$F = 0.8R\cos\theta$

7/7 $e = \dfrac{\text{output work}}{\text{input work}}$

To raise, $0.75 = \dfrac{250(1/4)}{P(1)}$, $\underline{P = 83.3 \text{ lb}}$

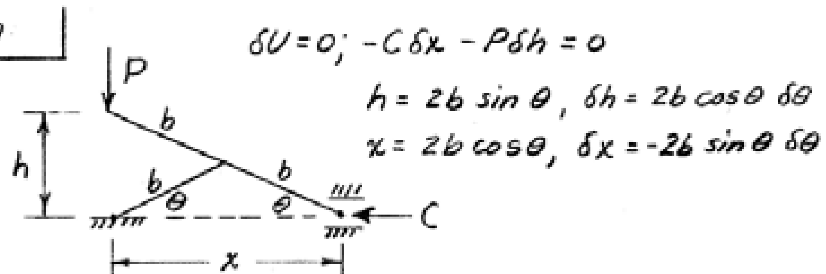
To lower, $0.75 = \dfrac{P'(1)}{250(1/4)}$, $\underline{P' = 46.9 \text{ lb}}$

7/8

$\delta U = 0$; $-mg\,\delta(b\sin\theta + a\cos\theta) + M\delta\theta = 0$

$$\underline{M = mg(b\cos\theta - a\sin\theta)}$$

7/9



$\delta U = 0; \quad -C\delta x - P\delta h = 0$

$h = 2b \sin\theta, \quad \delta h = 2b\cos\theta\,\delta\theta$

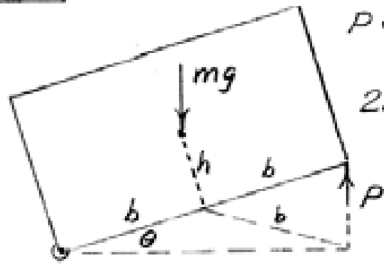
$x = 2b\cos\theta, \quad \delta x = -2b\sin\theta\,\delta\theta$

Thus $\quad -C(-2b\sin\theta\,\delta\theta) - P(2b\cos\theta\,\delta\theta) = 0$

$$C\sin\theta = P\cos\theta, \quad C = P\,\text{ctn}\,\theta$$

But $\quad \text{ctn}\,\theta = \sqrt{4b^2 - h^2}/h = \sqrt{(2b/h)^2 - 1}$, so $\underline{C = P\sqrt{(2b/h)^2 - 1}}$

7/10



$\delta U = 0;$

$$P\,\delta(2b\sin\theta) - mg\,\delta(b\sin\theta + h\cos\theta) = 0$$

$$2Pb\cos\theta\,\delta\theta = mg(b\cos\theta - h\sin\theta)\,\delta\theta$$

$$P = \frac{mg}{2}\left(1 - \frac{h}{b}\tan\theta\right)$$

7/11

$\delta U = 0$: $160 F \delta\theta - 0.4(160 F \delta\theta) - 100(9.81)\left(150\,\delta\frac{\theta}{25}\right) = 0$

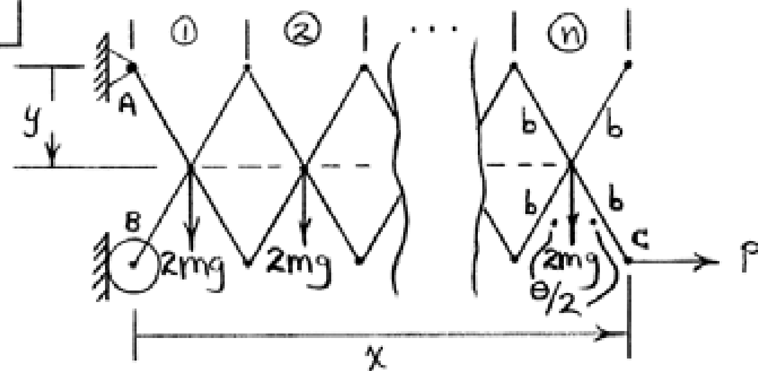
$0.6\,(160)\,F = 981\,(6)$, $\underline{F = 61.3\text{ N}}$

7/12 Let $\delta\theta$ = virtual angle of input rotation

Then $\delta\theta/40$ = " " " output "

$$e = \frac{\text{output work}}{\text{input work}} = \frac{1180\,(\delta\theta/40)}{30\,\delta\theta} = \frac{1180}{30(40)} = \underline{0.983}$$

7/13



$y = b\cos\frac{\theta}{2}$, $\delta y = -\frac{b}{2}\sin\frac{\theta}{2}\,\delta\theta$

$x = n\left(2b\sin\frac{\theta}{2}\right)$, $\delta x = nb\cos\frac{\theta}{2}\,\delta\theta$

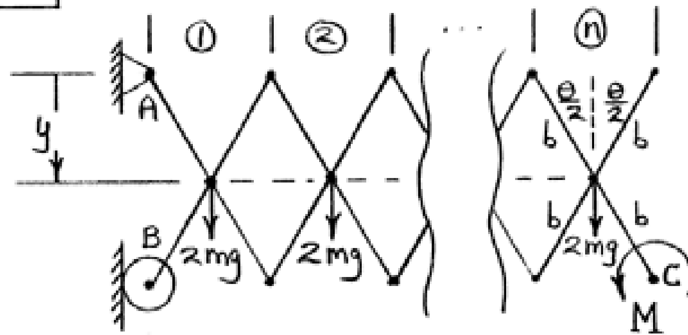
$\delta U = 0:\quad P\delta x + n(2mg)\delta y = 0$

$$P\left(nb\cos\frac{\theta}{2}\,\delta\theta\right) = -2nmg\left(-\frac{b}{2}\sin\frac{\theta}{2}\,\delta\theta\right)$$

$$\underline{P = mg\tan\frac{\theta}{2}}$$

P does <u>not depend</u> on the number n of sections present.

7/14



$y = b\cos\frac{\theta}{2}$, $\delta y = -\frac{b}{2}\sin\frac{\theta}{2}\,\delta\theta$

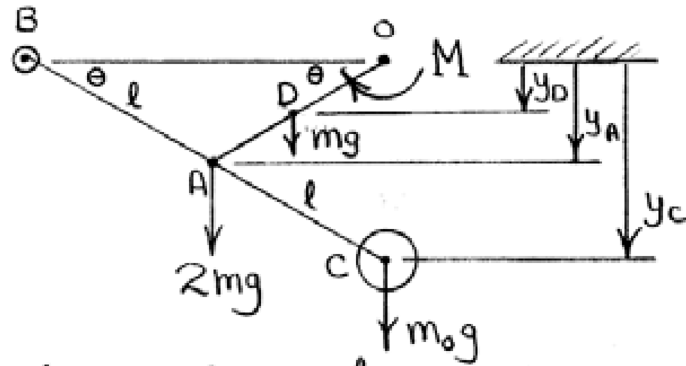
$\delta U = 0$: $M\,\delta\left(\frac{\theta}{2}\right) + n(2mg)\,\delta y = 0$

$\frac{M}{2}\delta\theta = -2nmg\left(-\frac{b}{2}\sin\frac{\theta}{2}\,\delta\theta\right)$

$M = 2nmgb\sin\frac{\theta}{2}$

M does depend on the number n of sections present.

7/15



$y_D = \frac{\ell}{2}\sin\theta$, $\delta y_D = \frac{\ell}{2}\cos\theta\,\delta\theta$

$y_A = \ell\sin\theta$, $\delta y_A = \ell\cos\theta\,\delta\theta$

$y_C = 2\ell\sin\theta$, $\delta y_C = 2\ell\cos\theta\,\delta\theta$

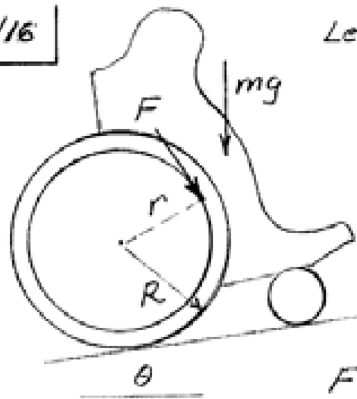
$\delta U = 0$: $-M\delta\theta + mg\,\delta y_D + 2mg\,\delta y_A + m_0g\,\delta y_C = 0$

$$-M\delta\theta + mg\left(\frac{\ell}{2}\cos\theta\,\delta\theta\right) + 2mg\left(\ell\cos\theta\,\delta\theta\right) + m_0g\left(2\ell\cos\theta\,\delta\theta\right) = 0$$

$$\Rightarrow\ M = \left(\frac{5}{2}m + 2m_0\right)g\ell\cos\theta$$

For $\theta = 30°$: $\underline{M = \left(\frac{5}{4}m + m_0\right)g\ell\sqrt{3}}$

7/16



Let β = angle through which wheel turns

s = corresponding displacement along incline.

$s = R\beta$ so $\delta s = R\,\delta\beta$

$\delta U = 0$

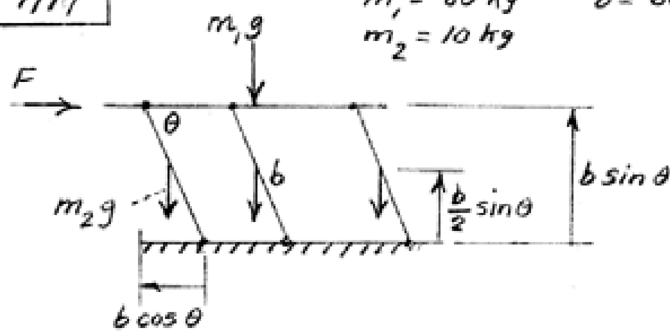
$Fr\,\delta\beta - mg\,\delta s \sin\theta = 0$

$Fr\,\delta\beta = mg\,R\sin\theta\,\delta\beta$

$F = mg\,\dfrac{R}{r}\sin\theta$

7/17

$m_1 = 80$ kg $b = 600$ mm
$m_2 = 10$ kg



$$\delta U = 0; \quad -F\,\delta(b\cos\theta) - m_1 g\,\delta(b\sin\theta) - 3m_2 g\,\delta\left(\frac{b}{2}\sin\theta\right) = 0$$

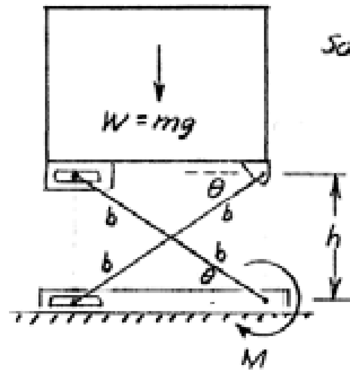
$$Fb\sin\theta\,\delta\theta = m_1 g b\cos\theta\,\delta\theta + \frac{3}{2}m_2 g b\cos\theta\,\delta\theta$$

$$F = g\,\text{ctn}\,\theta\left(m_1 + \frac{3}{2}m_2\right)$$

$$= 9.81\left(80 + \frac{3}{2}10\right)\text{ctn}\,\theta = \underline{932\,\text{ctn}\,\theta \text{ N}}$$

Solution by force and moment equilibrium would require dismemberment with four FBD's and eventual elimination of unwanted forces and dimensions

7/18 $\delta U = 0; \quad M\delta\theta - mg\,\delta h = 0$



$h = 2b\sin\theta, \quad \delta h = 2b\cos\theta\,\delta\theta$

So $M\delta\theta = mg\,(2b\cos\theta)\,\delta\theta$

$M = 2mgb\cos\theta$

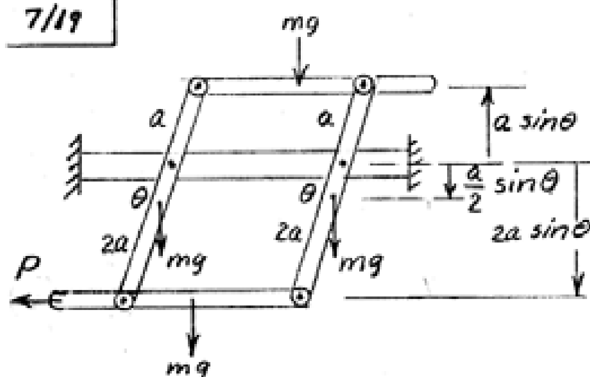
But since $\sin\theta = \dfrac{h}{2b}$,

$\cos\theta = \sqrt{1 - \left(\dfrac{h}{2b}\right)^2}$

Thus

$M = 2mgb\sqrt{1 - \left(\dfrac{h}{2b}\right)^2}$

7/19



$\delta U = 0;\quad P\,\delta(2a\cos\theta) + mg\,\delta\left(2a\sin\theta + 2\left[\frac{a}{2}\sin\theta\right] - a\sin\theta\right) = 0$

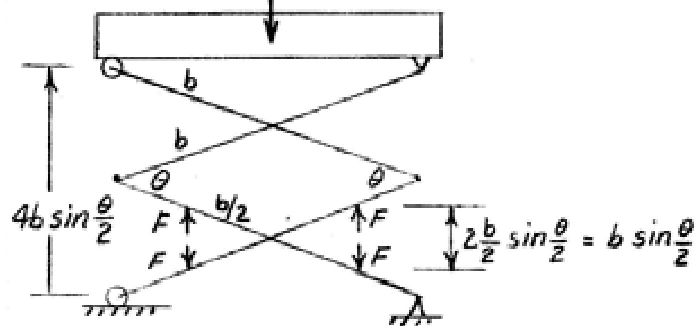
$-2Pa\sin\theta\,\delta\theta + 2mga\cos\theta\,\delta\theta = 0$

$P\sin\theta = mg\cos\theta\,,\quad \underline{\theta = \tan^{-1}\frac{mg}{P}}$

If P is replaced by a couple M no work by M can be done since the lower bar remains horizontal & cannot rotate. Thus M could not produce equilibrium.

7/20

$F = pA$



Work done by each cylinder is $F\delta(b\sin\frac{\theta}{2})$

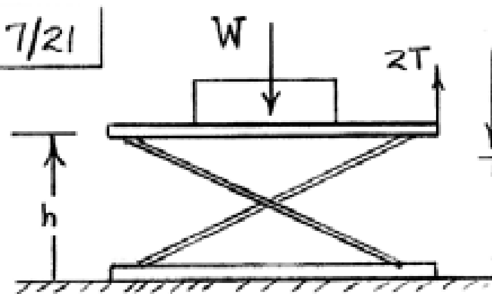
$$= \frac{Fb}{2}\cos\frac{\theta}{2}\,\delta\theta$$

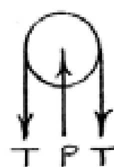
$\delta U = 0$; $\quad -mg\,\delta(4b\sin\frac{\theta}{2}) + 2\,\frac{Fb}{2}\cos\frac{\theta}{2}\,\delta\theta = 0$

$$Fb\cos\frac{\theta}{2} = 2mgb\cos\frac{\theta}{2}\,, \quad F = pA = 2mg$$

so $\underline{p = 2mg/A}$ independent of b & θ

7/21



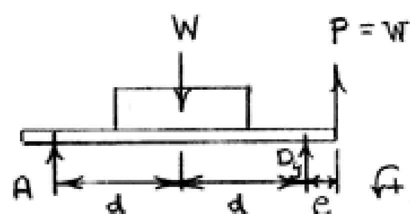


$\delta U = 0$: $2T\delta h - W\delta h = 0$

$T = W/2$

Pulley: $P = 2T$, so $\underline{P = W}$

(independent of h)



D_y and A form a clockwise couple

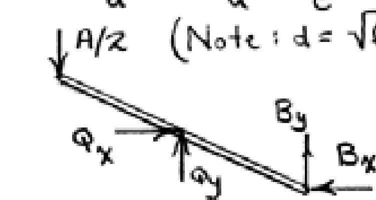
$D_y = -A$ (Note $D_x = 0$)

$\circlearrowleft \Sigma M = 0$: $W(d+e) - A(2d) = 0$

(Note: $d = \sqrt{l^2 - h^2}/2$)

$A = \frac{W}{2}\left(1 + \frac{e}{d}\right)$

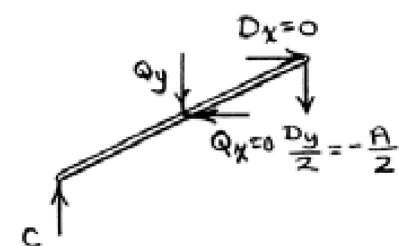
$= \frac{W}{2}\left(1 + \frac{2e}{\sqrt{l^2 - h^2}}\right)$



CD: $\Sigma M_C = 0$:

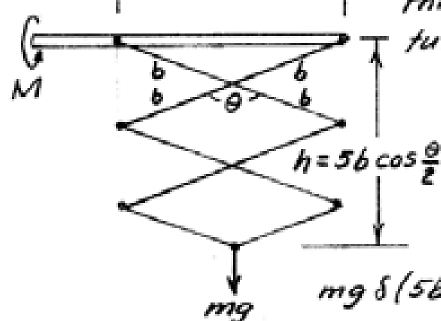
$Q_y d - \frac{A}{2}(2d) = 0$

$\underline{Q = Q_y = A = \frac{W}{2}\left(1 + \frac{2e}{\sqrt{l^2 - h^2}}\right)}$



(From drawing, $d > e$, so $Q \cong \frac{W}{2}$)

7/22 L = lead of screw; $\delta\left(2b \sin\frac{\theta}{2}\right) = \frac{\delta\alpha}{2\pi}L$



where α is the angle through which the screw turns. Thus

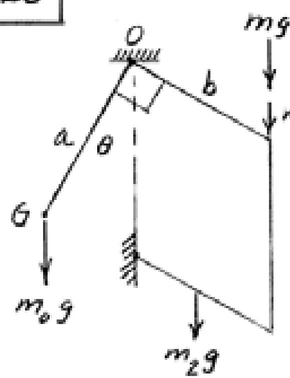
$$\delta\alpha = \frac{2\pi b}{L}\cos\frac{\theta}{2}\,\delta\theta$$

$\delta U = 0$; $mg\,\delta h + M\delta\alpha = 0$

$$mg\,\delta\left(5b\cos\frac{\theta}{2}\right) + M\left(\frac{2\pi b}{L}\cos\frac{\theta}{2}\,\delta\theta\right) = 0$$

$$-\frac{5}{2}mgb\sin\frac{\theta}{2}\,\delta\theta + \frac{2\pi bM}{L}\cos\frac{\theta}{2}\,\delta\theta = 0, \quad \underline{M = \frac{5mgL}{4\pi}\tan\frac{\theta}{2}}$$

7/23



$\delta U = 0;\ m_0 g\,\delta(a\cos\theta)$

$+(m+m_1)g\,\delta(b\sin\theta)$

$+ m_2 g\,\delta\left(\frac{b}{2}\sin\theta\right) = 0$

$-m_0 a\sin\theta\,\delta\theta + (m+m_1)b\cos\theta\,\delta\theta$

$+\frac{m_2 b}{2}\cos\theta\,\delta\theta = 0$

$m_0 a\tan\theta = mb + \left(m_1 + \frac{m_2}{2}\right)b$ ---(1)

when $\theta = \theta_0$, $mg = 0$ so

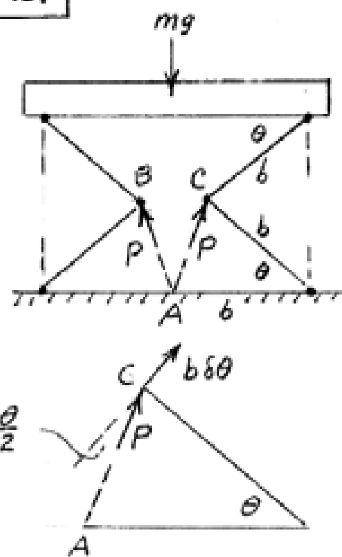
$m_0 a\tan\theta_0 = \left(m_1 + \frac{m_2}{2}\right)b$ ------(2)

Eliminate m_1 & m_2 from Eqs. (1) & (2) & get

$m_0 a\tan\theta = mb + m_0 a\tan\theta_0$

$m = \frac{a}{b}m_0(\tan\theta - \tan\theta_0)$

7/24



Work done by each P is

$$\left(P\cos\frac{\theta}{2}\right)b\delta\theta$$

Work done by mg is

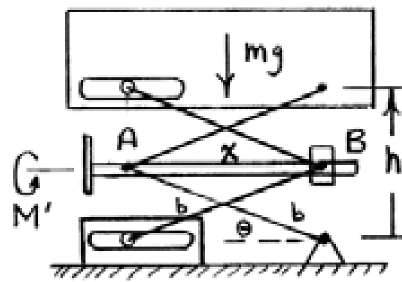
$$-mg\,\delta(2b\sin\theta)$$

$\delta U = 0$; $2Pb\cos\frac{\theta}{2}\delta\theta - 2bmg\cos\theta\,\delta\theta = 0$

$$P\cos\frac{\theta}{2} = mg\cos\theta$$

$$P = mg\frac{\cos\theta}{\cos\theta/2}$$

7/25 M' = necessary moment without friction



Let β = angle through which screw turns

$\delta U = 0: \quad M'\delta\beta - mg\,\delta h = 0$

$\frac{L}{2\pi} = \frac{-\delta(x)}{\delta\beta}, \quad \delta\beta = \frac{2\pi}{L}(-\delta x)$

$x = 2b\cos\theta, \quad \delta x = -2b\sin\theta\,\delta\theta$

$\delta\beta = \frac{4\pi b}{L}\sin\theta\,\delta\theta$

$h = 4b\sin\theta, \quad \delta h = 4b\cos\theta\,\delta\theta$

Thus $M'\frac{4\pi b}{L}\sin\theta\,\delta\theta - mg\,(4b\cos\theta\,\delta\theta) = 0$

$M' = \frac{mgL}{\pi}\cot\theta$

$M = M_f + \frac{mgL}{\pi}\cot\theta$

7/26 $\delta U = 0$; $2C\,\delta a + P\,\delta c = 0$

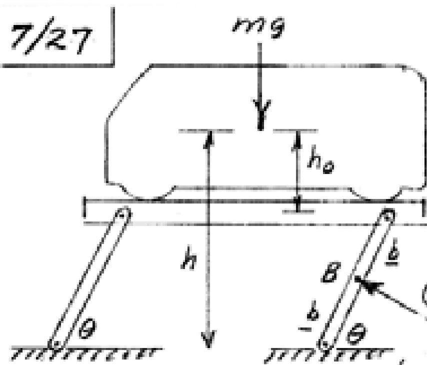
From rotation of jaw about fixed pivot

$\delta a = \frac{b}{d+c}\delta e$; also if l = length of connecting link, $l^2 = e^2 + c^2$

$0 = 2e\,\delta e + 2c\,\delta c$, $\delta c = \frac{-e}{c}\delta e$

Thus $2C\frac{b}{d+c}\delta e + P\left(-\frac{e}{c}\delta e\right) = 0$, $\underline{C = \frac{P}{2}\frac{e(d+c)}{bc}}$

7/27



Let $\overline{AB} = l$

$\delta U = 0$; $C\delta l - mg\,\delta h = 0$

$l^2 = (b\sin\theta)^2 + (L - b\cos\theta)^2$

$2l\delta l = 2b^2 \sin\theta\cos\theta\,\delta\theta + 2(L - b\cos\theta)(b\sin\theta\,\delta\theta)$

$= 2Lb\sin\theta\,\delta\theta$

$\delta l = \frac{Lb}{l}\sin\theta\,\delta\theta$

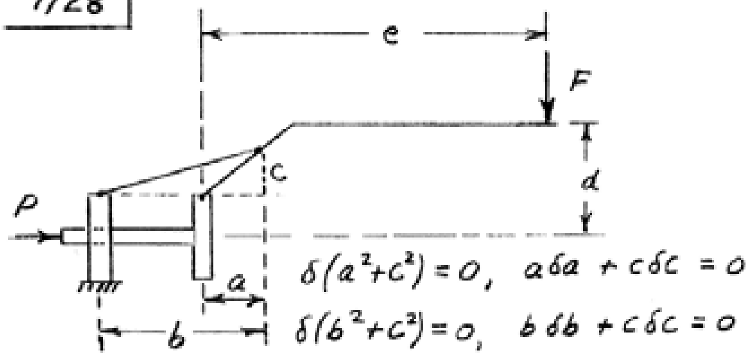
$h = 2b\sin\theta + h_0$, $\delta h = 2b\cos\theta\,\delta\theta + 0$

Thus $C\frac{Lb}{l}\sin\theta\,\delta\theta - mg(2b\cos\theta\,\delta\theta) = 0$

$C = 2mg\frac{l}{L}\text{ctn}\,\theta = \frac{2mg}{L}\sqrt{(b\sin\theta)^2 + (L - b\cos\theta)^2}\,\text{ctn}\,\theta$

$C = 2mg\sqrt{1 + \left(\frac{b}{L}\right)^2 - 2\frac{b}{L}\cos\theta}\;\text{ctn}\,\theta$

7/28



$\delta(a^2+c^2)=0, \quad a\delta a + c\delta c = 0$

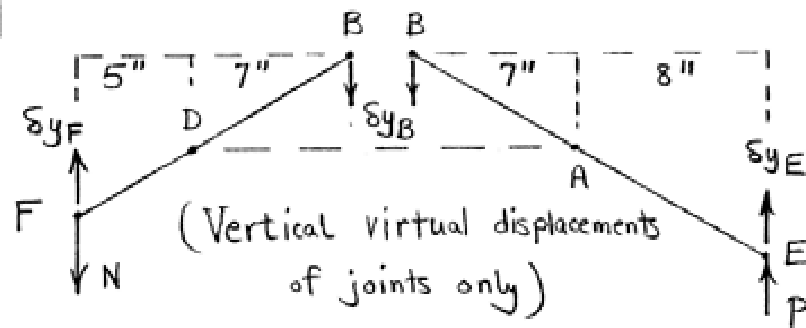
$\delta(b^2+c^2)=0, \quad b\delta b + c\delta c = 0$

$\delta U = 0; \; P\delta(b-a) - 2F\delta d = 0$ where $\delta d = \frac{e}{a}\delta c$

so $P\left(-\frac{c}{b}\delta c + \frac{c}{a}\delta c\right) - 2F\frac{e}{a}\delta c = 0$

$\frac{Pc}{ab}(b-a) = \frac{2Fe}{a}, \quad P = \frac{2Feb}{c(b-a)}$

7/29



$\delta y_B = \frac{7}{8}\delta y_E$, $\delta y_F = \frac{5}{7}\delta y_B$

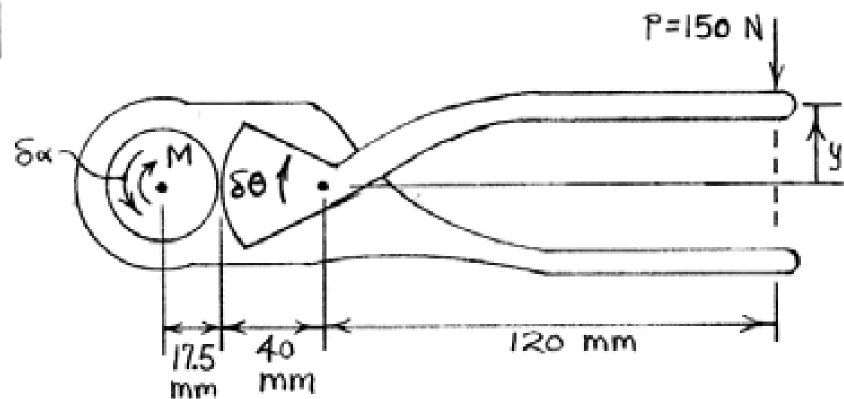
So $\delta y_F = \frac{5}{7}\cdot\frac{7}{8}\delta y_E = \frac{5}{8}\delta y_E$

$\delta U = 0$: $P\delta y_E - N\delta y_F = 0$

$P\delta y_E = N\left(\frac{5}{8}\delta y_E\right)$

$N = \frac{8}{5}P = \underline{1.6P}$

7/30



$\delta\alpha$ = rotation of socket on bolt head

$\delta\theta$ = rotation of upper handle (lower handle and frame taken as fixed)

$17.5\,\delta\alpha = 40\,\delta\theta$, $\delta y = -120\,\delta\theta$

$\delta U = 0: \; -M\,\delta\alpha + P(-\delta y) = 0$

$$M\left(\frac{40}{17.5}\,\delta\theta\right) = 150\,(120\,\delta\theta)$$

$M = 7880$ N·mm or $\underline{M = 7.88 \text{ N·m}}$

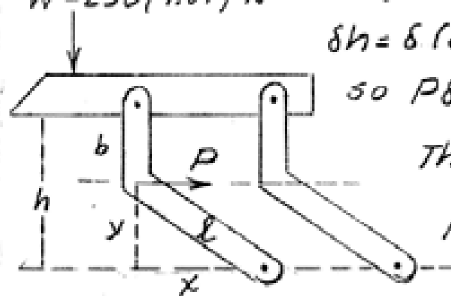
7/31 $\delta U = 0;\quad -P\delta x - W\delta h = 0$

$W = 250(9.81)$ N

$x^2 + y^2 = \ell^2,\quad x\,\delta x = -y\,\delta y$

$\delta h = \delta(b + y) = \delta y$

so $P\delta x = -W\delta y = -W\left(-\frac{x}{y}\right)\delta x$



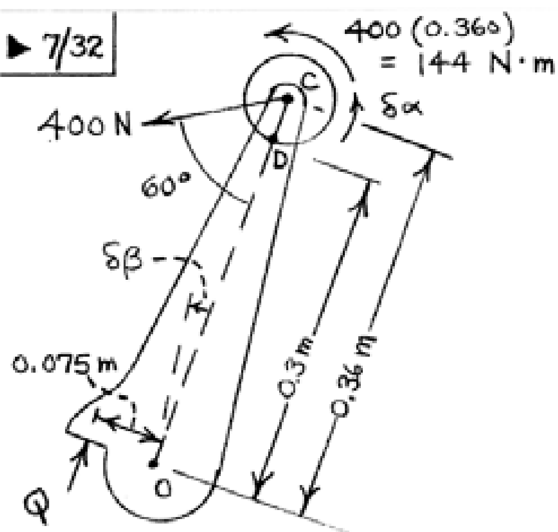
Thus $P = W\frac{x}{y}$

$P = 250(9.81)\,\frac{500}{350} = 3500$ N

or $\underline{P = 3.5\text{ kN}}$

$h = 650$ mm, $b = 300$ mm

$y = 350$ mm, $x = 500$ mm



During rotation $\delta\beta$, CD moves to C'D' & rotates through the absolute angle $\delta\alpha = \delta\gamma + \delta\beta = \dfrac{\text{arc}}{r_1} + \delta\beta$

$$= \frac{r_2\,\delta\beta}{r_1} + \delta\beta = \left(\frac{r_2}{r_1} + 1\right)\delta\beta = \left(\frac{300}{60} + 1\right)\delta\beta = 6\,\delta\beta$$

$$\delta U = 0: \quad -Q(0.075)\delta\beta + 400\cos 30^\circ(0.360\,\delta\beta) + 144(6\,\delta\beta) = 0 \quad , \quad \underline{Q = 13.18\text{ kN}}$$

7/33 $V = 6x^4 - 3x^2 + 5$

$\frac{dV}{dx} = 24x^3 - 6x = 0$ for equilibrium

$6x(4x^2 - 1) = 0 \Rightarrow x = 0, \ x = \pm \frac{1}{2}$

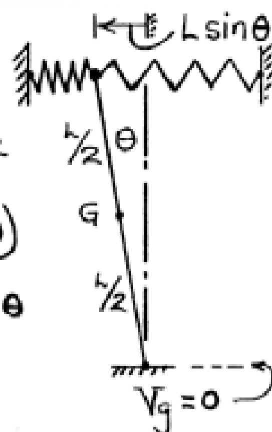
$\frac{d^2V}{dx^2} = 72x^2 - 6$

For $\underline{x = 0}$, $d^2V/d^2x = -6$ <u>unstable</u>

$\underline{x = \frac{1}{2}}$, $d^2V/d^2x = 18 - 6 = 12$ <u>stable</u>

$\underline{x = -\frac{1}{2}}$, $d^2V/d^2x = 18 - 6 = 12$ <u>stable</u>

7/34 | δ = initial spring compression



$$V = V_g + V_e = mg\frac{L}{2}\cos\theta + \frac{1}{2}k(\delta + L\sin\theta)^2 + \frac{1}{2}k(\delta - L\sin\theta)^2$$

$$= \frac{1}{2}mgL\cos\theta + k(\delta^2 + L^2\sin^2\theta)$$

$$\frac{dV}{d\theta} = -\frac{1}{2}mgL\sin\theta + 2kL^2\sin\theta\cos\theta$$

$$\frac{d^2V}{d\theta^2} = -\frac{1}{2}mgL\cos\theta + 2kL^2\cos 2\theta$$

For equilibrium, $\frac{dV}{d\theta} = 0$, so

$$\left(-\frac{1}{2}mg + 2kL\cos\theta\right)\sin\theta = 0$$

$$\Rightarrow \quad \theta = 0 \quad \text{or} \quad \cos\theta = \frac{mg}{4kL}$$

For $\theta = 0$, $\frac{d^2V}{d\theta^2} = -\frac{1}{2}mgL + 2kL^2$

> 0 (Stable) if $2kL^2 > \frac{1}{2}mgL$

So $k_{min} = \frac{mg}{4L}$

7/35

$V = V_g = mgb\cos\theta$

$\frac{dV}{d\theta} = -mgb\sin\theta = 0$ for equil.

$\theta = 0°$ or $\theta = 180°$

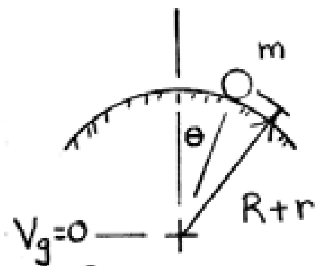
G, θ, b, $V_g = 0$, O

$\frac{d^2V}{d\theta^2} = -mgb\cos\theta$

$\theta = 0$, $\frac{d^2V}{d\theta^2} = -mgb$ so unstable

$\theta = 180°$, $\frac{d^2V}{d\theta^2} = -mgb(-1) = +mgb$ stable

7/36

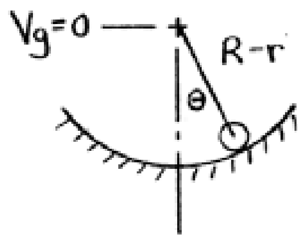


$V = V_g = (R+r)\cos\theta$

$\frac{dV}{d\theta} = -(R+r)\sin\theta = 0$ (for equil.

$\theta = 0, \pi$ (reject)

$\frac{d^2V}{d\theta^2} = -(R+r)\cos\theta < 0$ @ $\theta = 0$: unstable



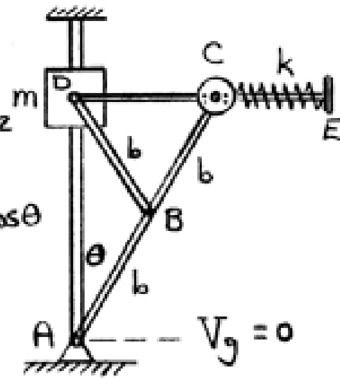
$V = V_g = -(R-r)\cos\theta$

$\frac{dV}{d\theta} = (R-r)\sin\theta = 0$ (for equil.

$\theta = 0, \pi$ (reject)

$\frac{d^2V}{d\theta^2} = (R-r)\cos\theta > 0$ @ $\theta = 0$: stable

7/37



$$V = V_g + V_e$$

$$= mg(2b\cos\theta) + \frac{1}{2}k(2b\sin\theta)^2$$

$$\frac{dV}{d\theta} = -2mgb\sin\theta + 4kb^2\sin\theta\cos\theta$$

$$= 2b\sin\theta(-mg + 2kb\cos\theta)$$

$$= 0 \text{ for equilibrium}$$

So $\sin\theta = 0$ or $\underline{\theta = \cos^{-1}\frac{mg}{2kb}}$

For $\theta_{max} = 30°$, $k_{min} = \frac{mg}{2b\cos 30°} = \underline{\frac{mg}{b\sqrt{3}}}$

7/38 | Take $V_g = 0$ through AO & $V_e = 0$ when $\theta = 0$

So $V_g = -mgh = -60(9.81)(0.7 \sin\theta) = -412.0 \sin\theta$

$V_e = \frac{1}{2}kx^2 = \frac{1}{2}(160)\left[2(1.4)\sin\frac{\theta}{2}\right]^2 = 627.2 \sin^2\frac{\theta}{2}$

$V = V_e + V_g = 627.2 \sin^2\frac{\theta}{2} - 412.0 \sin\theta$

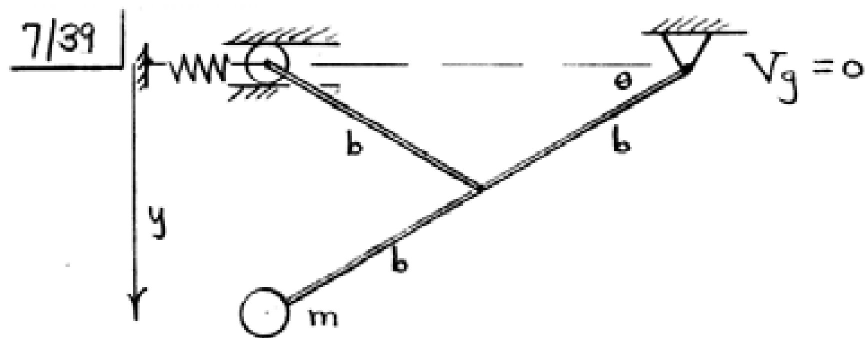
$\frac{dV}{d\theta} = \frac{2}{2}(627.2)\sin\frac{\theta}{2}\cos\frac{\theta}{2} - 412.0\cos\theta$

$= 313.6 \sin\theta - 412.0 \cos\theta = 0$ for equil.

or $\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{412.0}{313.6} = 1.314$

$\underline{\theta = 52.7°}$

7/39



Spring stretch $= 2b - 2b\cos\theta = 2b(1-\cos\theta)$

$V_e = \frac{1}{2}k[2b(1-\cos\theta)]^2 = 2kb^2(1-\cos\theta)^2$

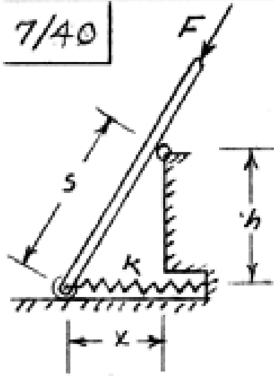
$V_g = -mg(2b\sin\theta) = -2mgb\sin\theta$

$V = 2kb^2(1-\cos\theta)^2 - 2mgb\sin\theta$

$\frac{dV}{d\theta} = 4kb^2(1-\cos\theta)\sin\theta - 2mgb\cos\theta = 0$
(for equilibrium)

$2kb(1-\cos\theta)\sin\theta = mg\cos\theta$, $k = \frac{mg}{2b}\frac{\cot\theta}{1-\cos\theta}$

7/40



$$\delta U = F\,\delta s = F\,\delta\left(\sqrt{h^2+x^2}\right)$$

$$= \frac{F x\,\delta x}{\sqrt{h^2+x^2}}$$

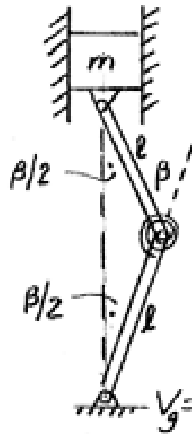
$$\delta V_e = kx\,\delta x$$

$$\delta U = \delta V_e\,;\quad \frac{F x\,\delta x}{\sqrt{h^2+x^2}} = kx\,\delta x$$

Sol. is $x = 0$ or $\dfrac{F}{\sqrt{h^2+x^2}} = k$, $\left(\dfrac{F}{k}\right)^2 = h^2 + x^2$

$$\underline{x = \sqrt{(F/k)^2 - h^2} \quad \text{provided } k < F/h}$$

7/41



$V_g = 2mgl\cos\frac{\beta}{2}$, $V_e = \frac{1}{2}K\beta^2$

$V = V_g + V_e = 2mgl\cos\frac{\beta}{2} + \frac{1}{2}K\beta^2$

$\frac{dV}{d\beta} = -mgl\sin\frac{\beta}{2} + K\beta$, $\frac{dV}{d\beta} = 0$ for $\beta = 0$

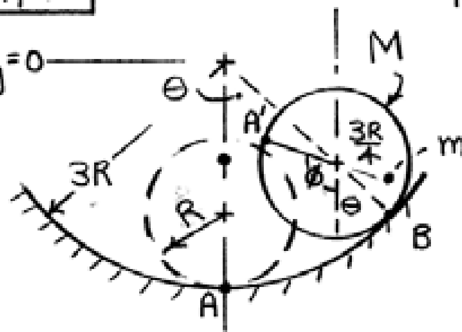
$\frac{d^2V}{d\beta^2} = -\frac{1}{2}mgl\cos\frac{\beta}{2} + K$

$= -\frac{1}{2}mgl + K$ for $\beta = 0$

$= (+)$ stable if $K > \frac{1}{2}mgl$

Thus $\underline{K_{min} = \frac{1}{2}mgl}$

7/42



Arc $\overline{AB}$ = Arc $\overline{A'B}$

$3R\theta = R(\theta + \phi)$

$\phi = 2\theta$

$$V = V_g = -Mg2R\cos\theta - mg\left(2R\cos\theta - \frac{3R}{4}\cos\phi\right)$$

$$= -2(M+m)gR\cos\theta + \frac{3}{4}mgR\cos 2\theta$$

$$\frac{dV}{d\theta} = 2(M+m)gR\sin\theta - \frac{3}{2}mgR\sin 2\theta$$

$= 0$ for equilibrium; $\theta = 0$ is desired solution.

$$\frac{d^2V}{d\theta^2} = 2(M+m)gR\cos\theta - 3mgR\cos 2\theta$$

For $\theta = 0$: $2(M+m)gR - 3mgR > 0$ for stability

or $\underline{M > \frac{m}{2}}$

7/43 Take $V_g = 0$ through bearing

$V = V_g = mg(2a\cos\theta) + mg(a\cos 2\theta)$

$= mga(2\cos\theta + \cos 2\theta)$

$\frac{dV}{d\theta} = mga(-2\sin\theta - 2\sin 2\theta) = -2mga(\sin\theta + \sin 2\theta)$

$\frac{d^2V}{d\theta^2} = -2mga(\cos\theta + 2\cos 2\theta)$

For equil. $\frac{dV}{d\theta} = 0$ so $\sin\theta = -\sin 2\theta$

or $\sin\theta(1 + 2\cos\theta) = 0$; $\sin\theta = 0$, $\cos\theta = -1/2$

so sols. of interest are $\theta = 0$, $\theta = 180°$, $\theta = 120°$, $\theta = 240°$

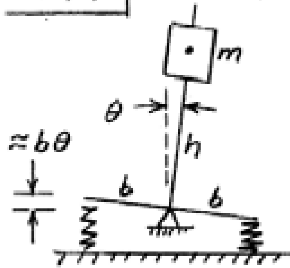
$\theta = 0$, $d^2V/d\theta^2 = -2mga(1 + 2) = (-)$ unstable

$\theta = 120°$, $d^2V/d\theta^2 = -2mga(-\frac{1}{2} - 2[\frac{1}{2}]) = (+)$ stable

$\theta = 180°$, $d^2V/d\theta^2 = -2mga(-1 + 2) = (-)$ unstable

$\theta = 240°$, $d^2V/d\theta^2 = -2mga(-1 + 2[-\frac{1}{2}]) = (+)$ stable

7/44



$V = V_e + V_g = \frac{1}{2}k(\Delta + b\theta)^2 + \frac{1}{2}k(\Delta - b\theta)^2 + mgh\cos\theta$

for θ small

$V = k(\Delta^2 + b^2\theta^2) + mgh\cos\theta$

$\frac{dV}{d\theta} = 2kb^2\theta - mgh\sin\theta$

$\frac{d^2V}{d\theta^2} = 2kb^2 - mgh\cos\theta$

Let preset of springs be Δ when θ = 0

For $\theta \to 0$, $\frac{d^2V}{d\theta^2}$ is (+) if $2kb^2 > mgh$

Thus θ = 0 is stable if $h < \frac{2kb^2}{mg}$

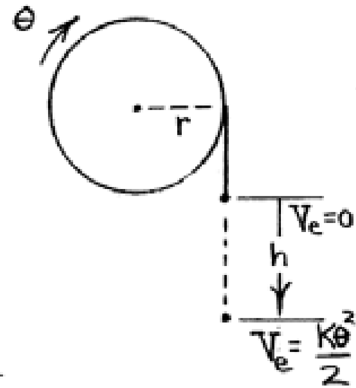
7/45

$\delta U' = \delta V: 0 - mgr\,\delta\theta + \delta\left(\frac{1}{2}K\theta^2\right)$

$mgr = K\theta$

With $h = r\theta$: $mgr = K\left(\frac{h}{r}\right)$

$h = \dfrac{mgr^2}{K}$



7/46 $\delta U' = \delta V_g + \delta V_e$

$\delta U' = -P\delta\left(2b\sin\frac{\theta}{2}\right) = -Pb\cos\frac{\theta}{2}\,\delta\theta$

$\delta V_g = mg\,\delta\left(b\sin\frac{\theta}{2}\right) = mg\,\frac{b}{2}\cos\frac{\theta}{2}\,\delta\theta$

$\delta V_e = \delta\left[\frac{1}{2}k\left(2b\cos\frac{\theta}{2} - 0\right)^2\right]$

$= -2kb^2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\,\delta\theta$

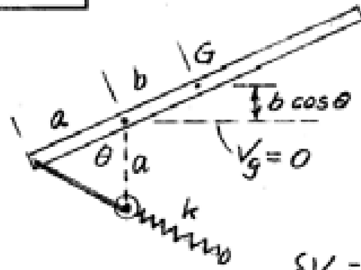
Thus $-Pb\cos\frac{\theta}{2}\,\delta\theta = mg\frac{b}{2}\cos\frac{\theta}{2}\delta\theta - 2kb^2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\,\delta\theta$

or $\left(P + \frac{mg}{2}\right)\cos\frac{\theta}{2} = 2kb\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$\cos\frac{\theta}{2} = 0$, $\theta = 180°$ (collapsed position)

$\sin\frac{\theta}{2} = \left(P + \frac{mg}{2}\right)/2kb$ or $\theta = 2\sin^{-1}\dfrac{P + \frac{mg}{2}}{2kb}$

7/47



Spring compression is

$x = 2a \sin\frac{\theta}{2}$

$\delta V_e = kx\delta x = 2ka \sin\frac{\theta}{2} \delta(2a \sin\frac{\theta}{2})$

$= 2ka^2 \sin\frac{\theta}{2} \cos\frac{\theta}{2} \delta\theta$

$= ka^2 \sin\theta \, \delta\theta$

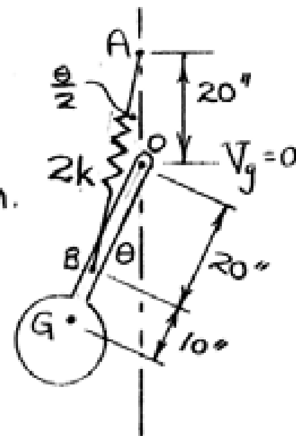
$\delta V_g = \delta(mgb \cos\theta) = -mgb \sin\theta \, \delta\theta$

$\delta V_e + \delta V_g = 0$; $ka^2 \sin\theta \, \delta\theta - mgb \sin\theta \, \delta\theta = 0$

$(ka^2 - mgb) \sin\theta = 0$

Equil. for any θ if $k = \frac{mgb}{a^2}$

7/48



Length $AB = 2(20)\cos\frac{\theta}{2}$ (in.)

Unstretched length = 40 − 4 = 36 in.

Spring stretch for arbitrary θ is $x = 40\cos\frac{\theta}{2} - 36$ in.

$$V_e = 2\,\tfrac{1}{2}k\left(40\cos\frac{\theta}{2} - 36\right)^2$$

$$= k\left[1600\cos^2\frac{\theta}{2} - 2880\cos\frac{\theta}{2} + 1296\right] \text{ in.-lb}$$

$$V_g = -3(20\cos\theta) = -60\cos\theta \text{ in.-lb}$$

$$V = V_e + V_g = k\left[1600\cos^2\frac{\theta}{2} - 2880\cos\frac{\theta}{2} + 1296\right] - 60\cos\theta \text{ in.-lb}$$

$$\frac{dV}{d\theta} = k\left[-1600\sin\frac{\theta}{2}\cos\frac{\theta}{2} + 1440\sin\frac{\theta}{2}\right] + 60\sin\theta$$

$$= k\left[-800\sin\theta + 1440\sin\frac{\theta}{2}\right] + 60\sin\theta$$

$$\frac{d^2V}{d\theta^2} = k\left[-800\cos\theta + 720\cos\frac{\theta}{2}\right] + 60\cos\theta$$

$$\left(\frac{d^2V}{d\theta^2}\right)_{\theta=0} = k[-800 + 720] + 60 > 0 \text{ (Stable) if } k \text{ does}$$

not exceed $\frac{60}{80}$ = $\underline{0.75 \text{ lb/in.} = k_{max}}$

7/49 For solid hemisphere $\overline{OG_1} = 3b/8$

ρ = density

$$V_{g_I} = -m_I g \frac{3b}{8}\cos\theta = -\frac{2}{3}\pi b^3 \rho g \frac{3b}{8}\cos\theta$$

$$= -\pi\rho g \frac{b^4}{4}\cos\theta$$

$$V_{g_{II}} = +m_{II} g \frac{h}{2}\cos\theta$$

$$= \pi\left(\frac{b}{2}\right)^2 h \rho g \frac{h}{2}\cos\theta$$

$$= \pi\rho g \frac{b^2 h^2}{8}\cos\theta$$

$$V = V_{II} + V_I = \pi\rho g \frac{b^2}{4}\left(\frac{h^2}{2} - b^2\right)\cos\theta$$

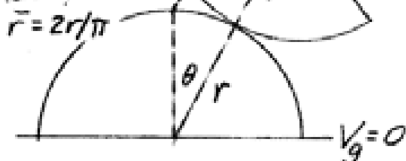
$$\frac{d^2V}{d\theta^2} = -\pi\rho g \frac{b^2}{4}\left(\frac{h^2}{2} - b^2\right)\cos\theta = +\pi\rho g \frac{b^2}{4}\left(b^2 - \frac{h^2}{2}\right)\cos\theta$$

$= (+)$ stable if $\frac{h^2}{2} < b^2$ or $\underline{h < b\sqrt{2}}$

7/50

Cylinder $\bar{r} = \frac{4r}{3\pi}$

Shell $\bar{r} = 2r/\pi$



$$V = V_g = mg(2r\cos\theta - \bar{r}\cos 2\theta)$$

$$\frac{dV}{d\theta} = mg(-2r\sin\theta + 2\bar{r}\sin 2\theta)$$

$$= 2mg(-r\sin\theta + \bar{r}\sin 2\theta)$$

$$\frac{d^2V}{d\theta^2} = 2mg(-r\cos\theta + 2\bar{r}\cos 2\theta)$$

For $\theta = 0$, $\frac{d^2V}{d\theta^2} = 2mrg\left(-1 + \frac{2\bar{r}}{r}\right)$

For cylinder $\bar{r}/r = \frac{4}{3\pi}$, $\frac{d^2V}{d\theta^2} = 2mrg\left(-1 + \frac{8}{3\pi}\right) = (-)$

<u>unstable</u>

For shell $\bar{r}/r = \frac{2}{\pi}$, $\frac{d^2V}{d\theta^2} = 2mrg\left(-1 + \frac{4}{\pi}\right) = (+)$

<u>stable</u>

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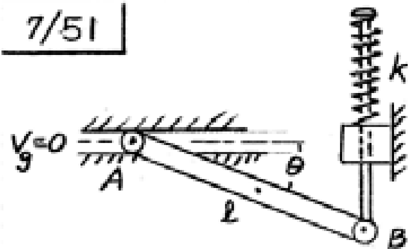


Spring compressed $\ell \sin\theta$

so $V_e = \frac{1}{2} k (\ell \sin\theta)^2$

$V_g = -mg\frac{\ell}{2}\sin\theta$

$V = V_e + V_g = \frac{k}{2}\ell^2 \sin^2\theta - mg\frac{\ell}{2}\sin\theta$

$$\frac{dV}{d\theta} = k\ell^2 \sin\theta\cos\theta - mg\frac{\ell}{2}\cos\theta = \frac{k\ell^2}{2}\sin 2\theta - mg\frac{\ell}{2}\cos\theta$$

$$= \ell\cos\theta\,(k\ell\sin\theta - mg/2) = 0 \text{ for equil.}$$

(1) $\cos\theta = 0$, $\theta = \pi/2$

(2) $\sin\theta = \dfrac{mg}{2k\ell}$

$$\frac{d^2V}{d\theta^2} = k\ell^2\cos 2\theta + mg\frac{\ell}{2}\sin\theta$$

$$\left(\frac{d^2V}{d\theta^2}\right)_{(1)} = k\ell^2\cos\pi + mg\frac{\ell}{2}(1) = k\ell^2\left(-1 + \left[\frac{mg}{2k\ell}\right]\right)$$

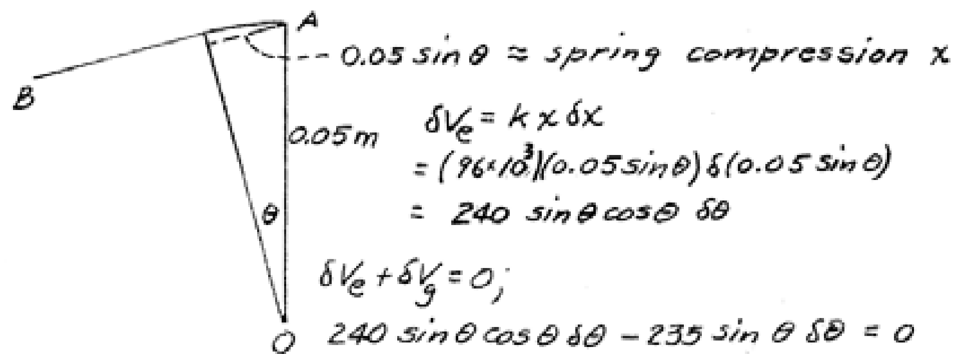
$= (+)$ stable if $k < \dfrac{mg}{2\ell}$

$$\left(\frac{d^2V}{d\theta^2}\right)_{(2)} = k\ell^2\left(1 - 2\left[\frac{mg}{2k\ell}\right]^2\right) + \frac{mg\ell}{2}\frac{mg}{2k\ell} = k\ell^2\left[1 - \left(\frac{mg}{2k\ell}\right)^2\right]$$

$= (+)$ stable if $k > \dfrac{mg}{2\ell}$

7/52

$$\delta V_g = \delta(0.3\,mg\cos\theta) = -0.3(80)(9.81)\sin\theta\,\delta\theta$$
$$= -235\sin\theta\,\delta\theta$$

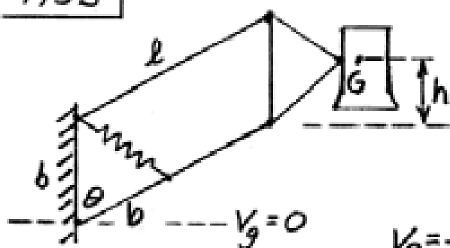


$$\delta V_e = kx\,\delta x$$
$$= (96\times10^3)(0.05\sin\theta)\,\delta(0.05\sin\theta)$$
$$= 240\sin\theta\cos\theta\,\delta\theta$$

$$\delta V_e + \delta V_g = 0;$$
$$240\sin\theta\cos\theta\,\delta\theta - 235\sin\theta\,\delta\theta = 0$$

$\sin\theta = 0$ or $\cos\theta = \frac{235}{240} = 0.9810$, $\underline{\theta = 11.19°}$

7/53



$V_g = mg(l\cos\theta + h)$

Spring length $= 2b\sin\frac{\theta}{2}$

" stretch $x = 2b\sin\frac{\theta}{2} - \frac{b}{2}$

$= \frac{b}{2}(4\sin\frac{\theta}{2} - 1)$

$V_e = \frac{1}{2}kx^2 = \frac{kb^2}{8}(4\sin\frac{\theta}{2} - 1)^2$

$V = V_e + V_g$; $\frac{dV}{d\theta} = -mgl\sin\theta + \frac{kb^2}{4}(4\sin\frac{\theta}{2} - 1)\,2\cos\frac{\theta}{2}$

$= (kb^2 - mgl)\sin\theta - \frac{kb^2}{2}\cos\frac{\theta}{2} = 0$ for equil.

Thus $\left[2(kb^2 - mgl)\sin\frac{\theta}{2} - \frac{kb^2}{2}\right]\cos\frac{\theta}{2} = 0$

& [ ] = 0 gives $k = \frac{mgl}{b^2}\,\frac{1}{1 - \frac{1}{4}\csc\frac{\theta}{2}}$

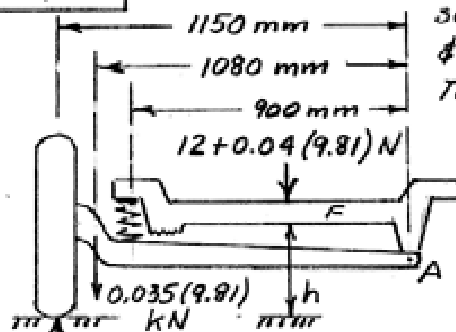
$\frac{d^2V}{d\theta^2} = (kb^2 - mgl)\cos\theta + \frac{kb^2}{4}\sin\frac{\theta}{2}$; substitute $\sin\frac{\theta}{2} = \frac{kb^2}{4(kb^2 - mgl)}$

& get $\frac{d^2V}{d\theta^2} = +$ (stable) within range of

$\theta = 29°, k = \infty$ to $\theta = 180°, k = \frac{4mgl}{3b^2}$

▶7/54 Let x = compression of spring (most easily seen by considering A fixed & wheels moving up)

Thus $x = \frac{900}{1150}(0.35 - h)$ meters



With F & hence A fixed,

$\delta U' = -2(6.54)\delta h + 2(0.035)9.81\frac{1080}{1150}\delta h$

$= (-13.08 + 0.645)\delta h$

$= -12.43\,\delta h$

$P = 6 + 0.035(9.81) + 0.02(9.81)$
$= 6.54$ kN

$\delta V_e = 2(kx\,\delta x) = 2(120)\frac{900}{1150}(0.35 - h)\frac{900}{1150}(-\delta h)$

$= -147.0(0.35 - h)\delta h$

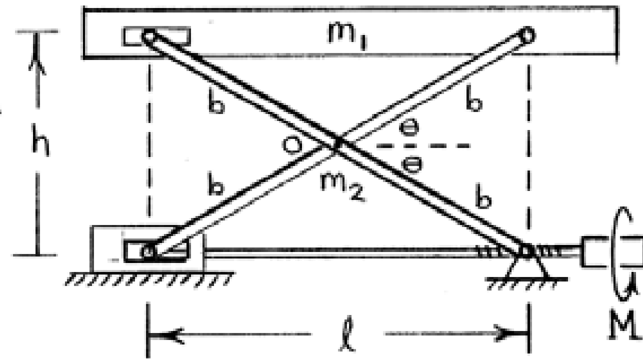
$\delta U' = \delta V_e$; $-12.43\,\delta h = -147.0(0.35 - h)\delta h$

$h = 0.35 - \frac{12.43}{147.0} = 0.265$ m or $\underline{h = 265 \text{ mm}}$

▶7/55

Let β = rotation angle of screw

p = screw pitch



$\delta U' = \delta V_g$

$\delta U' = M\delta\beta$

$\delta V_g = m_1 g\,\delta h + m_2 g\,\delta\left(\frac{h}{2}\right) = \left(m_1 + \frac{m_2}{2}\right) g\,\delta h$

$\delta h = \delta(2b\sin\theta) = 2b\cos\theta\,\delta\theta$

$l = 2b\cos\theta$, $\delta l = -2b\sin\theta\,\delta\theta$

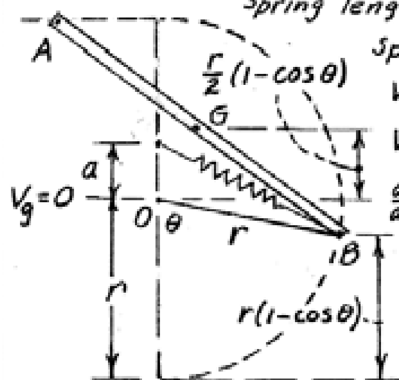
$\left|\frac{\delta l}{\delta\beta}\right| = \frac{p}{2\pi}$, so $|\delta\beta| = \frac{2\pi}{p}|\delta l| = \frac{4\pi b}{p}\sin\theta\,\delta\theta$

So $M\frac{4\pi b}{p}\sin\theta\,\delta\theta = \left(m_1 + \frac{m_2}{2}\right) g\,(2b\cos\theta\,\delta\theta)$

$M = \frac{(2m_1 + m_2)pg}{4\pi}\cot\theta$

▶ 7/56 | For $\frac{1}{2}$ of door $V_g = \frac{mg}{2}\frac{r}{2}(1-\cos\theta)$

Spring length $\ell = \sqrt{r^2+a^2+2ra\cos\theta}$



Spring stretch is $\ell - (r-a)$

$V_e = \frac{1}{2}k(\ell - [r-a])^2$

$V = V_g + V_e = \frac{mgr}{4}(1-\cos\theta) + \frac{k}{2}(\ell - [r-a])^2$

$\frac{dV}{d\theta} = \frac{mgr}{4}\sin\theta + k(\ell - [r-a])\frac{-2ra\sin\theta}{2\ell}$

$= \left(\frac{mgr}{4} - kar\right)\sin\theta + \frac{kar(r-a)\sin\theta}{\sqrt{r^2+a^2+2ra\cos\theta}}$

$$\frac{d^2V}{d\theta^2} = \left(\frac{mgr}{4} - kar\right)\cos\theta + kar(r-a)\frac{\ell\cos\theta - \sin\theta(-ra\sin\theta)/\ell}{\ell^2}$$

For $\theta = 0$, $\frac{d^2V}{d\theta^2} = \frac{mgr}{4} - kar + kar(r-a)\left(\frac{1}{r+a} + 0\right)$

$= 0$ for neutral equil. (insensitive response)

Thus $\frac{mgr}{4} = kar\left(1 - \frac{r-a}{r+a}\right)$, $\underline{k = \frac{mg(r+a)}{8a^2}}$

7/57 | $x = k\theta$, $0.060 = k(2\pi)$, $k = \frac{0.030}{\pi} \frac{m}{rad}$

$\delta U = 0$; $M\delta\theta - P\delta x = 0$, $P = M\frac{\delta\theta}{\delta x} = M/k$

so $P = \frac{\pi}{0.030} 10 = \underline{1047 \text{ N}}$

7/58 Force & moment equilibrium (A)

(a), (b), (d)

Virtual work (B)

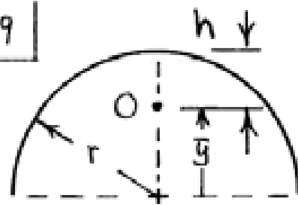
(c), (e), (f)

(c) $\delta U' = \delta V_g$

(e) $\delta U' = \delta V_g + \delta V_e$

(f) $d^2V/d\theta^2$ must be (+)

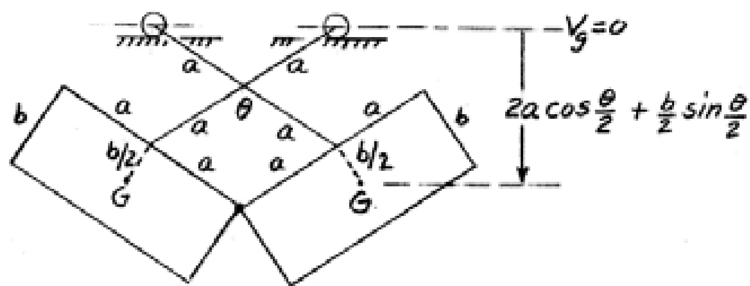
7/59



The pivot O must be at or above the mass center G of the shell, which is located at $\bar{y} = \frac{2r}{\pi}$. So

$$h_{max} = r - \bar{y} = r\left(1 - \frac{2}{\pi}\right) = \underline{0.363r}$$

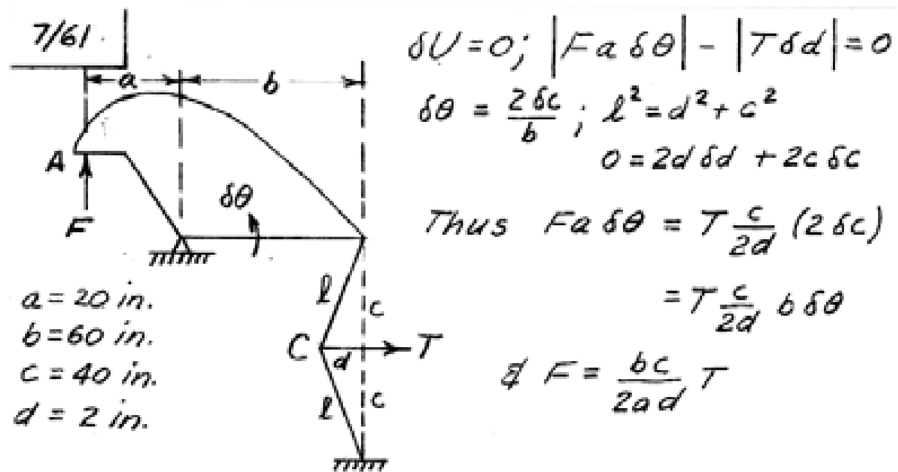
7/60 $V = V_g = -2mg\left[2a\cos\frac{\theta}{2} + \frac{b}{2}\sin\frac{\theta}{2}\right]$



$$\frac{dV}{d\theta} = -2mg\left[-a\sin\frac{\theta}{2} + \frac{b}{4}\cos\frac{\theta}{2}\right] = 0 \text{ for equil.}$$

$\tan\frac{\theta}{2} = \frac{b}{4a}$, $\theta = 2\tan^{-1}\frac{b}{4a}$; For $b = a$, $\theta = 2\tan^{-1}1/4$

$= 28.1°$



$\delta U = 0; \quad |F a\,\delta\theta| - |T\,\delta d| = 0$

$\delta\theta = \dfrac{2\,\delta c}{b}\ ; \quad \ell^2 = d^2 + c^2$

$0 = 2d\,\delta d + 2c\,\delta c$

Thus $F a\,\delta\theta = T\dfrac{c}{2d}(2\,\delta c)$

$= T\dfrac{c}{2d}\,b\,\delta\theta$

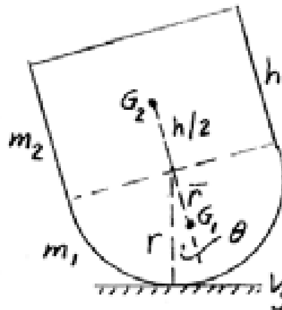
$\therefore\ F = \dfrac{bc}{2ad}\,T$

So $F = \dfrac{60(40)}{2(20)(2)}(2000)(16) = \underline{960{,}000\ \text{lb}}$

7/62 Let ρ = mass per unit area of shell

$m_1 = 2\pi r^2\rho$, $m_2 = 2\pi r h\rho$

$\bar{r} = r/2$ for hemispherical shell



$V = V_{g_1} + V_{g_2}$

$= 2\pi r^2\rho g\,(r - \bar{r}\cos\theta) + 2\pi r h\rho g\,(r + \frac{h}{2}\cos\theta)$

$= 2\pi r\rho g\left[(r^2 + hr) - \frac{1}{2}(r^2 - h^2)\cos\theta\right]$

$\frac{dV}{d\theta} = 2\pi r\rho g\left[0 + \frac{1}{2}(r^2 - h^2)\sin\theta\right]$

$\frac{d^2V}{d\theta^2} = \pi r\rho g\,(r^2 - h^2)\cos\theta$

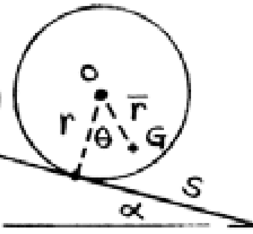
For equil. $\frac{dV}{d\theta} = 0$ gives $\theta = 0$ & $h = r$

For $\theta = 0$, $\frac{d^2V}{d\theta^2} = (+)$ if $\underline{h < r}$

For $h = r$, neutral equilibrium

7/63

$V_g = W(s\sin\alpha + r\cos\alpha - \bar{r}\cos(\theta-\alpha))$



But $s = s_0 + r\theta$, $\frac{ds}{d\theta} = r$

$V_g = 0$

So $\frac{dV_g}{d\theta} = W(r\sin\alpha + \bar{r}\sin(\theta-\alpha))$

For $\alpha = 10°$, $r = 100$ mm, $\bar{r} = 60$ mm:

$\frac{dV_g}{d\theta} = W[0.1\sin 10° + 0.060\sin(\theta - 10°)]$

$= 0$ for equilibrium

$-0.1\sin 10° = 0.060\sin(\theta - 10°)$

$\theta - 10° = \sin^{-1}\left(-\frac{0.1}{0.06}\sin 10°\right) = -16.82°$ or $196.8°$

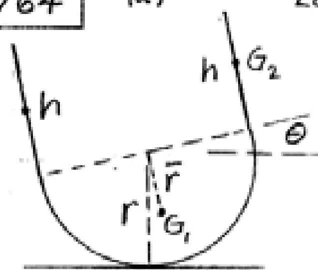
$\Rightarrow$ $\underline{\theta = -6.82°}$ or $\underline{\theta = 207°}$

$\frac{d^2V_g}{d\theta^2} = W[0 + \bar{r}\cos(\theta - \alpha)]$

$\theta = -6.82°$: $d^2V_g/d\theta^2 = W\bar{r}\cos(-16.82°) > 0$ Stable

$\theta = 207°$: $d^2V_g/d\theta^2 = W\bar{r}\cos(196.8°) < 0$ Unstable

7/64 (a)



Let ρ = mass per unit periphery of shell

$\bar{r} = 2r/\pi$

$$V_g = \rho\left[\pi r\left(r - \frac{2r}{\pi}\cos\theta\right) + h\left(r + r\sin\theta + \frac{h}{2}\cos\theta\right) + h\left(r - r\sin\theta + \frac{h}{2}\cos\theta\right)\right]$$

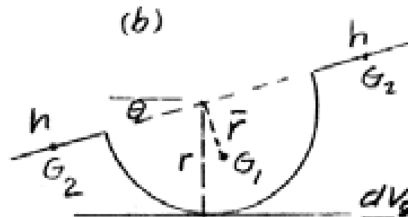
$$= \rho\left[\pi r^2 - 2r^2\cos\theta + 2hr + h^2\cos\theta\right]$$

$$\frac{dV_g}{d\theta} = \rho\left[2r^2 - h^2\right]\sin\theta\,, \quad \frac{d^2V_g}{d\theta^2} = \rho\left[2r^2 - h^2\right]\cos\theta$$

Equil. at $\theta = 0$ stable if $h < r\sqrt{2}$
unstable if $h > r\sqrt{2}$

Neutral equil. if $h = r\sqrt{2}$ for any θ

(b)



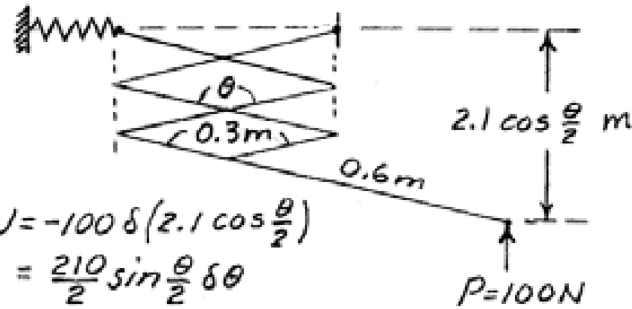
$$V_g = \rho\left[\pi r\left(r - \frac{2r}{\pi}\cos\theta\right) + h\left(r + \left[r + \frac{h}{2}\sin\theta\right]\right) + h\left(r - \left[r + h/2\sin\theta\right]\right)\right]$$

$$= \rho\left[\pi r^2 - 2r^2\cos\theta + 2hr\right]$$

$\frac{dV_g}{d\theta} = 2\rho r^2\sin\theta$, $\theta = 0$ for stable equil. independent of θ

7/65 | Spring compression $x = 0.6(\sin\frac{\theta}{2} - \sin 15^\circ)$

$\delta V_e = kx\,\delta x$; $\delta V_e = k(0.6)(\sin\frac{\theta}{2} - \sin 15^\circ)\frac{0.6}{2}\cos\frac{\theta}{2}\,\delta\theta$



$\delta U = -100\,\delta(2.1\cos\frac{\theta}{2})$

$= \frac{210}{2}\sin\frac{\theta}{2}\,\delta\theta$

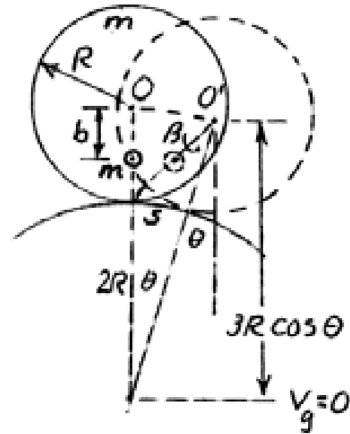
P=100N

$\delta U = \delta V_e$; $105\sin\frac{\theta}{2}\,\delta\theta = 0.18k\left(\sin\frac{\theta}{2} - \sin 15^\circ\right)\cos\frac{\theta}{2}\,\delta\theta$

For $\frac{\theta}{2} = 60^\circ$, $105\frac{\sqrt{3}}{2} = 0.18k\left(\frac{\sqrt{3}}{2} - 0.2588\right)\frac{1}{2}$

$k = 1664$ N/m or $\underline{k = 1.664\text{ kN/m}}$

7/66 In displaced position



$V_g = mg(3R\cos\theta)$
$\quad + mg(3R\cos\theta - b\cos[\theta+\beta])$

But $s = 2R\theta = R\beta$

so $\beta = 2\theta$

& $V_g = 6mgR\cos\theta - mgb\cos 3\theta$

$\dfrac{dV_g}{d\theta} = -6mgR\sin\theta + 3mgb\sin 3\theta$

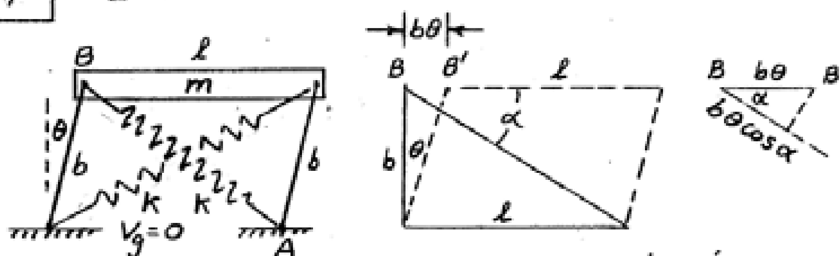
$(=0 \text{ for } \theta = 0)$

$\dfrac{d^2V_g}{d\theta^2} = -6mgR\cos\theta + 9mgb\cos 3\theta$

$\left(\dfrac{d^2V_g}{d\theta^2}\right)_{\theta=0} = 3mg(-2R + 3b) = +$ stable if $3b > 2R$

so $\underline{b_{min} = \frac{2}{3}R}$

▶ 7/67 Δ = initial tensile deflection of each spring



For small θ, change in length of each spring is approx. $\pm b\theta\cos\alpha$

$$V = V_e + V_g = \frac{1}{2}k(\Delta + b\theta\cos\alpha)^2 + \frac{1}{2}k(\Delta - b\theta\cos\alpha)^2 + mgb\cos\theta$$

$$= k(\Delta^2 + b^2\theta^2\cos^2\alpha) + mgb\cos\theta$$

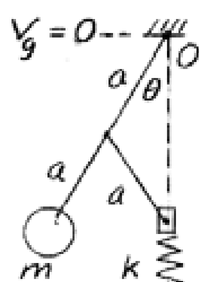
$$\frac{dV}{d\theta} = 2kb^2\theta\cos^2\alpha - mgb\sin\theta$$

$$\frac{d^2V}{d\theta^2} = 2kb^2\cos^2\alpha - mgb\cos\theta;\ \text{for } \theta = 0,\ \frac{d^2V}{d\theta^2} = 2kb^2\cos^2\alpha - mgb$$

Thus $\left(\frac{d^2V}{d\theta^2}\right)_{\theta=0}$ is (+) stable if $2kb^2\cos^2\alpha > mgb$

or $k_{min} = \frac{mg}{2b\cos^2\alpha} = \underline{\frac{mg}{2b}\left(1 + \frac{b^2}{\ell^2}\right)}$ where $\cos^2\alpha = \frac{\ell^2}{b^2 + \ell^2}$

▶7/68 Take $V_g = 0$ through pt. O



$V_g = -2mga\cos\theta$

Compression in spring $x = 2a\cos\theta - a$

$= 0$ for $\theta = 60°$

$V_e = \frac{1}{2}kx^2 = \frac{ka^2}{2}(2\cos\theta - 1)^2$

$V = V_e + V_g = \frac{ka^2}{2}(2\cos\theta - 1)^2 - 2mga\cos\theta$

$\frac{dV}{d\theta} = 2a\left[(mg + ka)\sin\theta - 2ka\sin\theta\cos\theta\right]$

$\frac{d^2V}{d\theta^2} = 2a\left[(mg + ka)\cos\theta - 2ka(2\cos^2\theta - 1)\right]$

For equil. $\frac{dV}{d\theta} = 0$, $\left[(mg + ka) - 2ka\cos\theta\right]\sin\theta = 0$

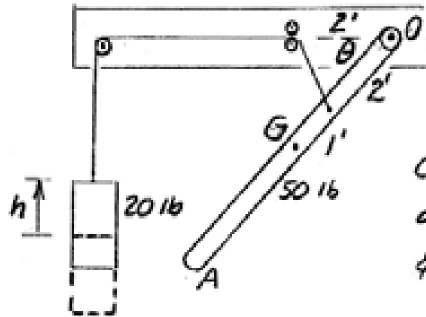
$\sin\theta = 0$ & $\cos\theta = \frac{mg + ka}{2ka} = \frac{1}{2}\left(1 + \frac{mg}{ka}\right)$

For $\sin\theta = 0$, $\underline{\theta = 0}$; $\frac{d^2V}{d\theta^2} = 2a(mg - ka) = +$ stable if $\underline{k < \frac{mg}{a}}$

$= -$ unstable if $\underline{k > \frac{mg}{a}}$

For $\underline{\theta = \cos^{-1}\frac{1}{2}\left(1 + \frac{mg}{ka}\right)}$, $\underline{k > \frac{mg}{a}}$ $\underline{\text{stable}}$

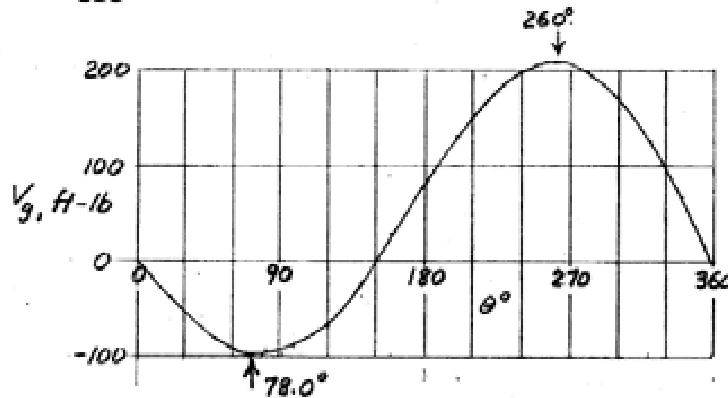
***7/69** $h = 2(2\sin\frac{\theta}{2}) = 4\sin\frac{\theta}{2}$



$$V_g = 20(4\sin\frac{\theta}{2}) - 50(3\sin\theta)$$

$$= 80\sin\frac{\theta}{2} - 150\sin\theta \quad \text{ft-lb}$$

Compute V_g as a function of θ from $\theta = 0$ to $\theta = 360°$ & plot



$\theta = 78.0°$ stable

$\theta = 260°$ unstable

*7/70

$y = 2(150)\cos\theta$

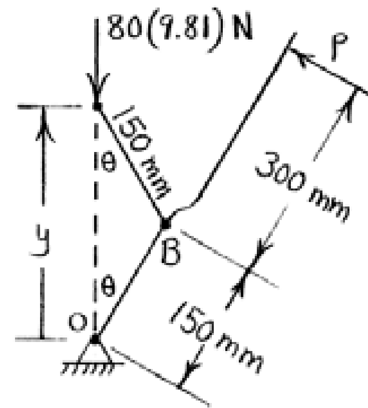
$\delta y = -300 \sin\theta \, \delta\theta$

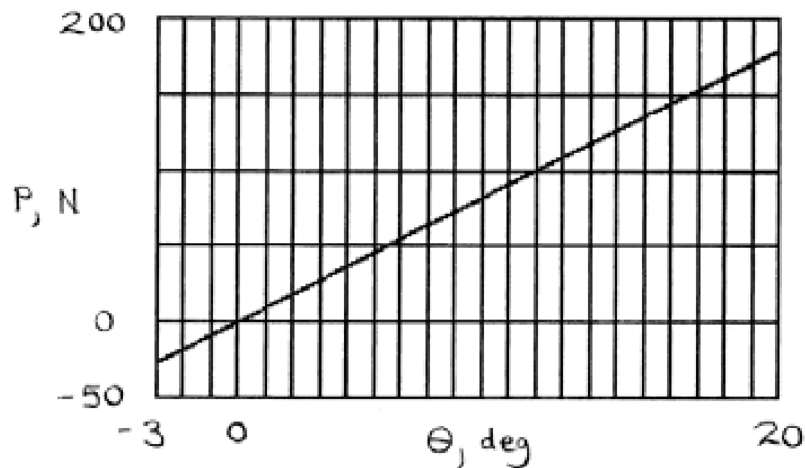
$\delta U = 0$:

$-P(450\,\delta\theta) - 80(9.81)\,\delta y = 0$

$P = \dfrac{80(9.81)(300 \sin\theta)}{450}$

$= \underline{523 \sin\theta \text{ (in newtons)}}$

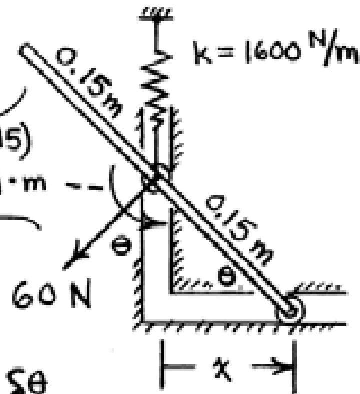




(At $\theta = -3°$, $P = -27.4$ N

At $\theta = 20°$, $P = 178.9$ N)

∗ 7/71



Spring stretch = 0.15(1 − sin θ) m

$V_e = \frac{1}{2}(1600)[0.15(1-\sin\theta)]^2$

$= 18(1-\sin\theta)^2$ J

$\delta V_e = 36(1-\sin\theta)(-\cos\theta)\delta\theta$

$\delta U' = -60\cos\theta\,\delta(0.15\sin\theta) - 9\delta\theta$

$= -9\cos^2\theta\,\delta\theta - 9\delta\theta = -9[1+\cos^2\theta]\,\delta\theta$

$\delta U' = \delta V_e$: $-9[1+\cos^2\theta]\delta\theta = -36\cos\theta\,(1-\sin\theta)\,\delta\theta$

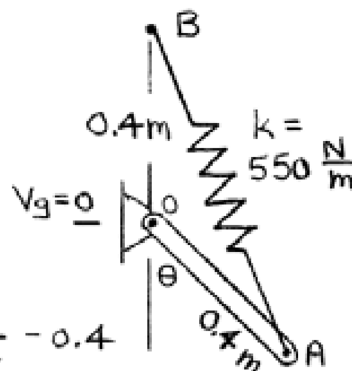
$(1+\cos^2\theta) - 4\cos\theta\,(1-\sin\theta) = 0$

Numerical solution : $\theta = 29.7°$

So that $x = 0.15\cos 29.7° = 0.1303$ m

or $x = \underline{130.3 \text{ mm}}$

*7/72



$V_g = 20(9.81)(-0.2\cos\theta)$

$= -39.24 \cos\theta$ J

$\overline{AB} = 2(0.4)\sin\left(\frac{180-\theta}{2}\right)$

$= 0.8\cos\frac{\theta}{2}$

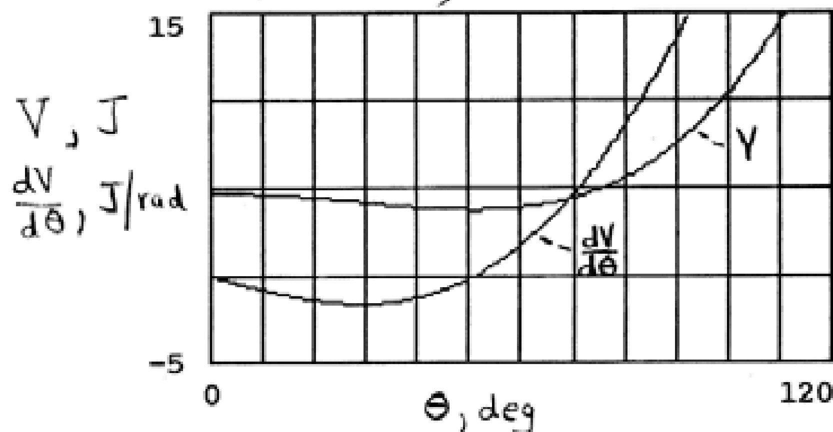
Stretch of spring $= 0.8\cos\frac{\theta}{2} - 0.4$

$= 0.4\left(2\cos\frac{\theta}{2} - 1\right)$

$V_e = \frac{1}{2}550\left[0.4\left(2\cos\frac{\theta}{2}-1\right)\right]^2 = 44\left(2\cos\frac{\theta}{2}-1\right)^2$

$V = V_e + V_g = 44\left(2\cos\frac{\theta}{2}-1\right)^2 - 39.24\cos\theta$

$dV/d\theta = 88\left(2\cos\frac{\theta}{2}-1\right)\left(-\sin\frac{\theta}{2}\right) + 39.24\sin\theta$



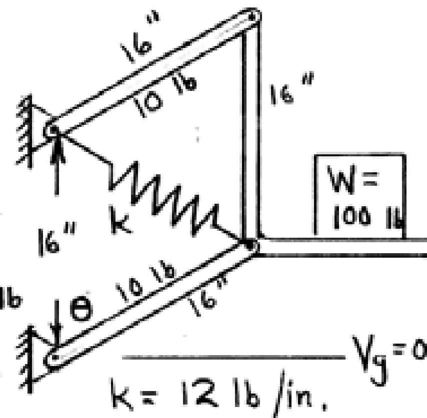
$\theta = 0$, unstable

$\theta = 51.1°$, stable

*7/73

Spring stretch $= 2(16 \sin \frac{\theta}{2}) - 8$

$= 8(4 \sin \frac{\theta}{2} - 1)$ in.



$V_e = \frac{1}{2}(12)(8)^2(4 \sin \frac{\theta}{2} - 1)^2$

$= 384(4 \sin \frac{\theta}{2} - 1)^2$ in.-lb

$V_g = 10(8 \cos\theta) + 10(16 + 8\cos\theta)$

$+ 100(16 \cos\theta) = 1760 \cos\theta + 160$ in.-lb

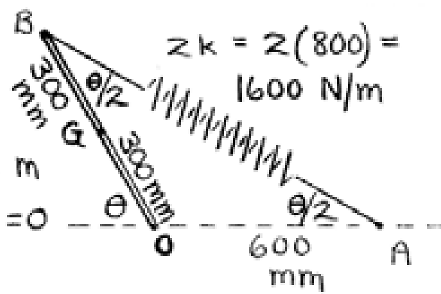
$V = V_e + V_g = 384(4 \sin \frac{\theta}{2} - 1)^2 + 1760 \cos\theta + 160$ in.-lb

$\frac{dV}{d\theta} = 768(4 \sin \frac{\theta}{2} - 1) 2 \cos \frac{\theta}{2} - 1760 \sin\theta$

$= 0$ for equilibrium

Numerical solution : $\underline{\theta = 71.7^\circ}$

*7/74



Springs undeflected @ $\theta = \frac{\pi}{2}$

Deflection $x = \overline{AB} - 0.6\sqrt{2}$ m

$\overline{AB} = 2(0.6)\cos\theta/2$

So $x = 0.6\left(2\cos\frac{\theta}{2} - \sqrt{2}\right)$ m

$$V = V_g + V_e = 25(9.81)(0.3\sin\theta) + \frac{1}{2}2k\left[0.6\left(2\cos\frac{\theta}{2} - \sqrt{2}\right)\right]^2$$

$$= 73.6\sin\theta + 800(0.36)\left(4\cos^2\frac{\theta}{2} - 4\sqrt{2}\cos\frac{\theta}{2} + 2\right)$$

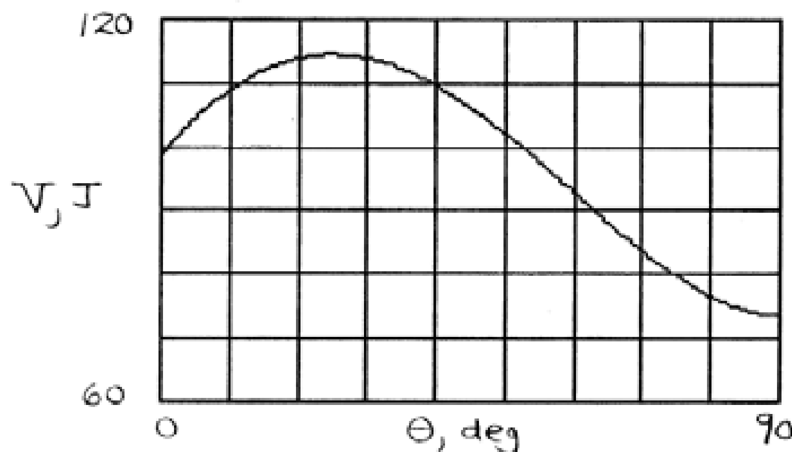
$$\frac{dV}{d\theta} = 73.6\cos\theta + 288\left(-4\cos\frac{\theta}{2}\sin\frac{\theta}{2} + 2\sqrt{2}\sin\frac{\theta}{2}\right)$$

$$= 73.6\cos\theta - 576\sin\theta + 815\sin\frac{\theta}{2}$$

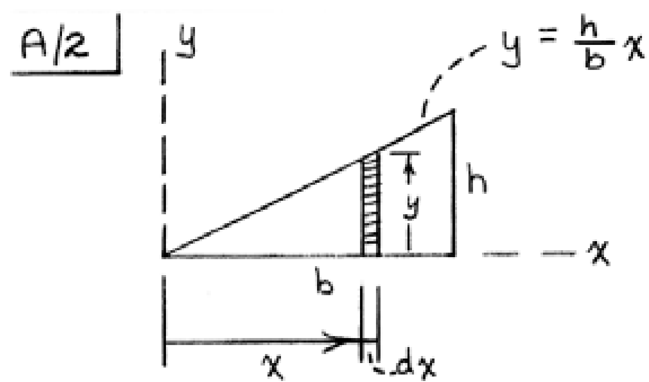
$$\frac{d^2V}{d\theta^2} = -73.6\sin\theta - 576\cos\theta + 407\cos\frac{\theta}{2}$$

Set $\frac{dV}{d\theta} = 0$ & solve numerically : $\underline{\theta = 24.8°}$

$\left(\frac{d^2V}{d\theta^2}\right)_{\theta = 24.8°} = -156 < 0$ so $\underline{\text{unstable}}$.



A/1 $I_x \cong Ay^2$, $A = \dfrac{2.56(10^6)}{40^2} = \underline{1600 \text{ mm}^2}$



$$I_y = \int x^2 \, dA = \int x^2 y \, dx = \int x^2 \left(\frac{h}{b} x\right) dx$$

$$= \frac{h}{b} \int_0^b x^3 \, dx = \frac{h}{4b} x^4 \Big|_0^b = \underline{\frac{hb^3}{4}}$$

A/3 $I_p = I_c + A(3)^2$, $I_{p'} = I_c + A(2)^2$

$I_p - I_{p'} = 50 = A(3^2 - 2^2)$, $\underline{A = 10 \text{ in.}^2}$

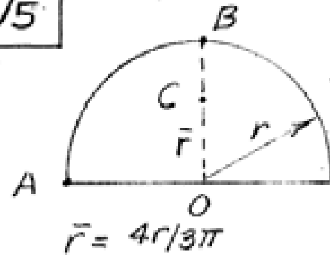
A/4

$I_x + I_y = I_z$, but $I_x = I_y$ so $I_x = \frac{I_z}{2}$

where $I_z \cong Ar^2 = 1600\,(100)^2 = 16(10^6)\ \text{mm}^4$

So $\underline{I_x = 8\,(10^6)\ \text{mm}^4}$

A/5



$\bar{r} = 4r/3\pi$

For complete circle

$$I_A = I_O + Ar^2 = \frac{1}{2}Ar^2 + Ar^2 = \frac{3}{2}Ar^2$$

For half circle

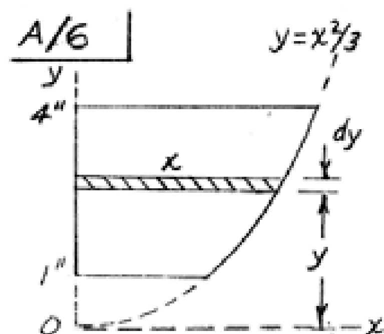
$$I_A = \frac{1}{2}\left(\frac{3}{2}\pi r^4\right) = \underline{\frac{3}{4}\pi r^4}$$

For half circle, $I_O = \frac{1}{4}\pi r^4$

$$I_B = I_C + A(r-\bar{r})^2 = I_O - A\bar{r}^2 + A(r-\bar{r})^2$$

$$= I_O + A(r^2 - 2r\bar{r})$$

$$= \frac{1}{4}\pi r^4 + \frac{\pi r^4}{2}\left(1 - \frac{8}{3\pi}\right) = \underline{r^4\left(\frac{3\pi}{4} - \frac{4}{3}\right)}$$



$dI_y = \frac{1}{3}x^3\,dy = \frac{1}{3}(3y)^{3/2}dy$

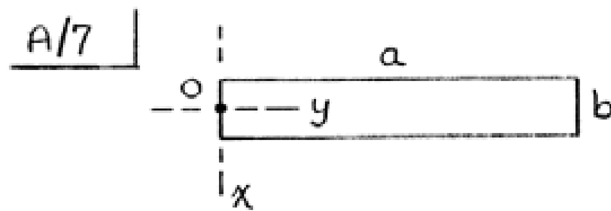
$= \sqrt{3}\,y^{3/2}dy$

$I_y = \sqrt{3}\int_1^4 y^{3/2}dy = \sqrt{3}\,\frac{2}{5}\left(4^{5/2} - 1^{5/2}\right)$

$= \frac{2\sqrt{3}}{5}(32-1)$

$= 21.5 \text{ in.}^4$

A/7



$I_x = \frac{1}{3}Aa^2 = \frac{1}{3}a^3b$, $I_y = \frac{1}{12}ab^3$

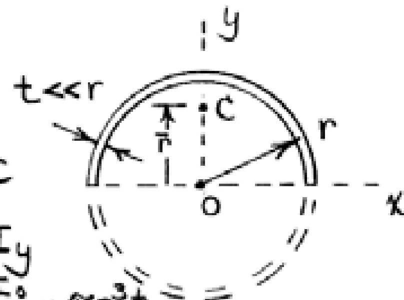
$I_o = I_x + I_y = \frac{1}{3}ab\left(a^2 + \frac{b^2}{4}\right)$

$n = \%\ \text{error} = \frac{I_x - I_o}{I_o}(100) = \frac{-\frac{1}{12}ab^3}{\frac{1}{3}ab\left(a^2+\frac{b^2}{4}\right)}100$

$= -\frac{1}{4}\frac{b^2}{a^2+\frac{b^2}{4}}100$

For $\frac{b}{a} = \frac{1}{10}$, $n = -25\frac{1}{10^2+\frac{1}{4}} = \underline{-0.249\ \%}$

A/8



For complete ring,

$I_o = Ar^2 = 2\pi r t r^2 = 2\pi r^3 t$

and $I_o = I_x + I_y$, $I_x = I_y$

So for complete ring, $I_x = \frac{I_o}{2} = \pi r^3 t$

For half-ring, $\underline{I_x = \frac{1}{2}\pi r^3 t}$ and $I_y = I_x$

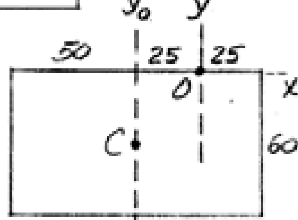
by symmetry so $\underline{I_y = \frac{1}{2}\pi r^3 t}$

For half-ring, $I_o = \frac{1}{2}(2\pi r^3 t) = \pi r^3 t$

$I_C = I_o - A\bar{r}^2 = \pi r^3 t - \pi r t\left(\frac{2r}{\pi}\right)^2$

$= \underline{\pi r^3 t\left(1 - \frac{4}{\pi^2}\right)}$

A/9



Dimensions in mm

$$I_{y_o} = \frac{1}{12}bd^3 = \frac{1}{12}(60)(100)^3 = 5(10^6)\ mm^4$$

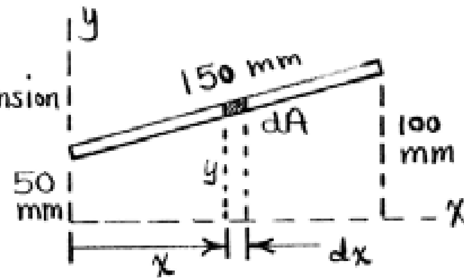
$$I_y = I_{y_o} + Ad^2 = 5(10^6) + 6(10^3)(25)^2 = 8.75(10^6)\ mm^4$$

$$I_x = \frac{1}{3}bd^3 = \frac{1}{3}(100)(60^3) = \underline{7.2(10^6)\ mm^4}$$

$$I_O = I_x + I_y = (7.2 + 8.75)10^6 = \underline{15.95(10^6)}\ mm^4$$

A/10

Area per unit x-dimension is $\dfrac{750}{\sqrt{150^2-(100-50)^2}}$



$= \dfrac{750}{141.4} = 5.30 \ \dfrac{mm^2}{mm}$

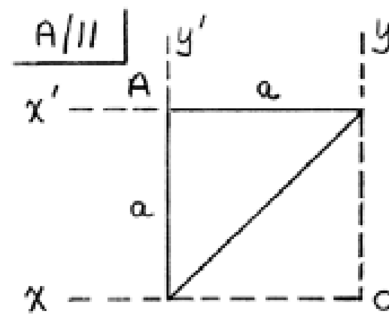
$dA = 5.30\,dx$, $y = 50 + \dfrac{50}{141.4}x = 50\left(1+\dfrac{x}{141.4}\right)$

$dI_x = y^2 dA = 50^2\left(1+\dfrac{x}{141.4}\right)^2 \dfrac{750}{141.4}\,dx$

$I_x = 50^2\,\dfrac{750}{141.4}\displaystyle\int_0^{141.4}\left(1+\dfrac{2x}{141.4}+\dfrac{x^2}{20\,000}\right)dx$

$= \underline{4.38(10^6)\ mm^4}$

Erroneous result $I_x \neq A\bar{y}^2 = 750(75)^2 = \underline{4.22(10^6)\,mm^4}$



From Sample Problem A/2,

$I_x = I_y = \frac{bh^3}{4} = \frac{aa^3}{4} = \frac{a^4}{4}$

$I_z = I_x + I_y = \frac{a^4}{4} + \frac{a^4}{4} = \frac{a^4}{2}$

$k_o = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{a^4/2}{a^2/2}} = \underline{a}$

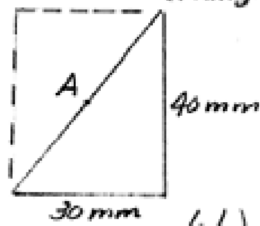
Also, $I_{y'} = I_{x'} = \frac{bh^3}{12} = \frac{a^4}{12}$

$I_{z'} = I_{x'} + I_{y'} = \frac{a^4}{6}$

$k_A = \sqrt{I_{z'}/A} = \sqrt{\frac{a^4/6}{a^2/2}} = \underline{a/\sqrt{3}}$

A/12

$(J_A)_{triangle} = \frac{1}{2}(J_A)_{rectangle}$



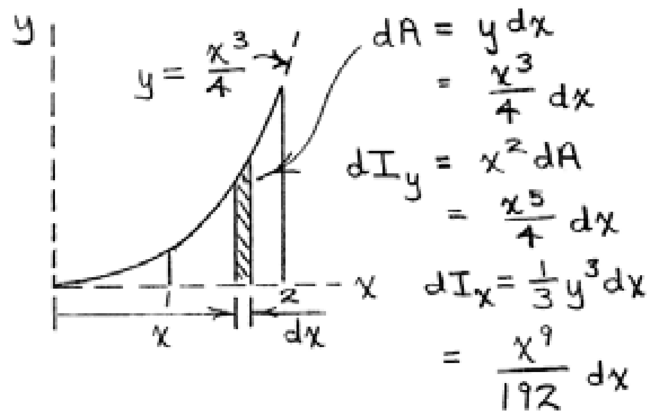
$= \frac{1}{2}\left[\frac{1}{12}A(b^2+h^2)\right]$ from Sample Prob. A/1

$= \frac{1}{24}(30)(40)(\overline{30}^2 + \overline{40}^2) = 12.5(10^4)$ mm^4

$(J_A)_{triangle} = k_A^2 A$

So $k_A = \sqrt{\frac{12.5(10^4)}{30(40)/2}} = \sqrt{208.4} = \underline{14.43 \text{ mm}}$

A/13



$dA = y\,dx = \frac{x^3}{4}\,dx$

$dI_y = x^2\,dA = \frac{x^5}{4}\,dx$

$dI_x = \frac{1}{3}y^3\,dx = \frac{x^9}{192}\,dx$

$$A = \int dA = \int_1^2 \frac{x^3}{4}\,dx = \frac{1}{4}\frac{x^4}{4}\Big|_1^2 = \frac{15}{16}$$

$$I_y = \int dI_y = \int_1^2 \frac{x^5}{4}\,dx = \frac{1}{4}\frac{x^6}{6}\Big|_1^2 = \frac{63}{24}$$

$$I_x = \int dI_x = \int_1^2 \frac{x^9}{192}\,dx = \frac{1}{192}\frac{x^{10}}{10}\Big|_1^2 = \frac{1023}{1920}$$

$$k_y = \sqrt{I_y/A} = \sqrt{\frac{63/24}{15/16}} = \sqrt{14/5} = \underline{1.673}$$

$$k_x = \sqrt{I_x/A} = \sqrt{\frac{1023/1920}{15/16}} = \underline{0.754}$$

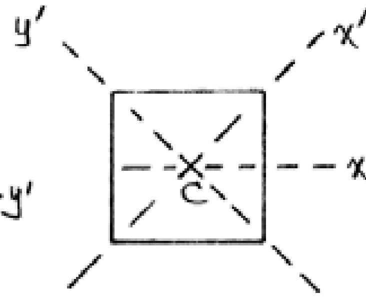
$$k_z = \sqrt{k_x^2 + k_y^2} = \sqrt{1.673^2 + 0.754^2} = \underline{1.835}$$

A/14

$I_x + I_y = I_z = I_{x'} + I_{y'}$

Because $I_x = I_y$ & $I_{x'} = I_{y'}$

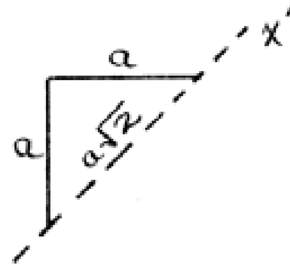
Therefore $\underline{I_x = I_{x'}}$



Triangle: From Sample Prob.

A/2: $I_{x'} = \frac{bh^3}{12} = \frac{a\sqrt{2}\,(a/\sqrt{2})^3}{12}$

$= \underline{\frac{a^4}{24}}$

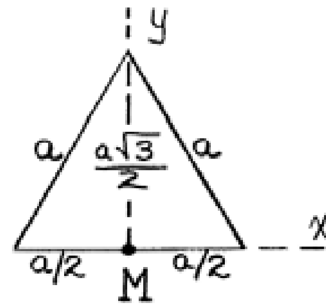


Rectangle: From Sample Prob.

A/1: $I_x = \frac{1}{3}Ah^2 = \frac{1}{3}\frac{a^2}{2}\left(\frac{a}{2}\right)^2$

$= \underline{\frac{a^4}{24}}$

A/15



$I_z = I_x + I_y$, $I_z = Ak_z^2$

$\therefore \quad k_M = \sqrt{(I_x + I_y)/A}$

$I_x = \frac{1}{12} bh^3 = \frac{1}{12} a \left(\frac{a\sqrt{3}}{2}\right)^3$

$= \frac{\sqrt{3}}{32} a^4$

$I_y = 2\left(\frac{1}{12} \frac{a\sqrt{3}}{2} \left(\frac{a}{2}\right)^3\right) = \frac{\sqrt{3}}{96} a^4$

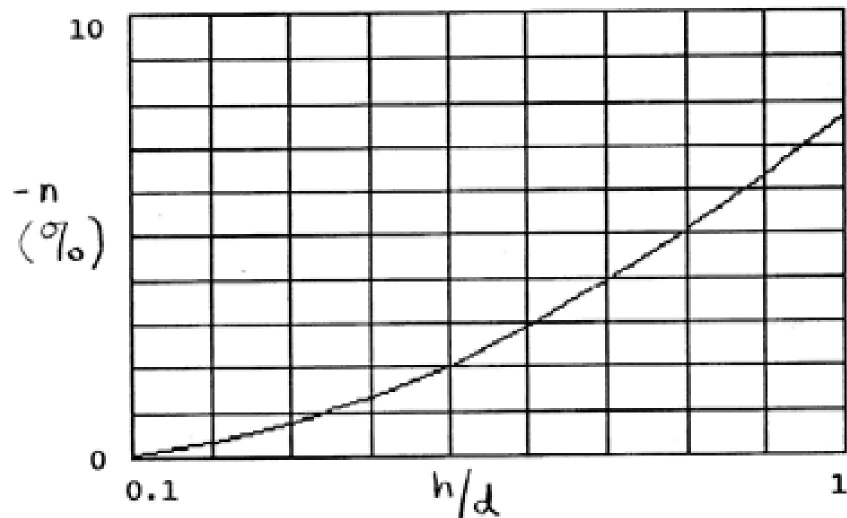
$k_M = \left[\frac{\frac{\sqrt{3}}{32} a^4 + \frac{\sqrt{3}}{96} a^4}{\frac{a}{2} a \frac{\sqrt{3}}{2}}\right]^{1/2} = \frac{a}{\sqrt{6}}$

A/16 Exact : $I_x = I_{x_0} + Ad^2 = \frac{1}{12}bh^3 + bhd^2$

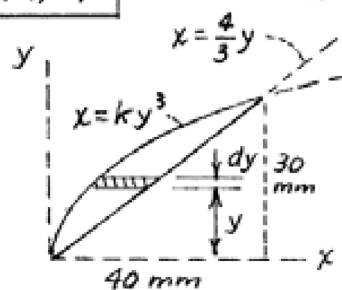
Approximate : $I_x' = Ad^2 = bhd^2$

$$n = \frac{I_x' - I_x}{I_x} 100 = \frac{-\frac{1}{12}bh^3}{\frac{1}{12}bh^3 + bhd^2} 100 = \frac{-1}{1 + 12/\left(\frac{h}{d}\right)^2} 100$$

For $h = \frac{d}{4}$, $\underline{n = -0.518\ \%}$



A/17



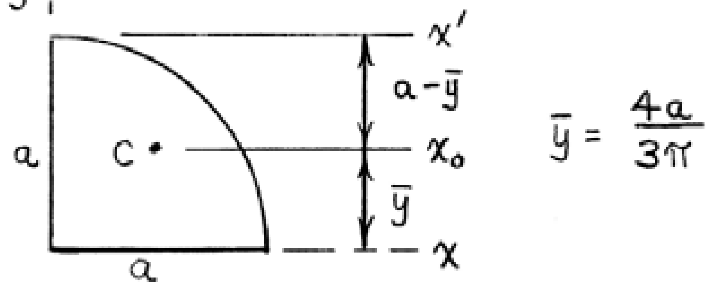
For $x = 40$ mm & $y = 30$ mm, $k = \frac{40}{27(10^3)}$

$$dI_x = y^2 dA = y^2\left(\frac{4}{3}y - \frac{4}{2700}y^3\right)dy$$

$$I_x = \int_0^{30}\left(\frac{4}{3}y^3 - \frac{4}{2700}y^5\right)dy$$

$$= \left[\frac{y^4}{3} - \frac{y^6}{4050}\right]_0^{30} = \underline{9(10^4)\ mm^4}$$

A/18



$$I_{x'} = I_{x_0} + A(a-\bar{y})^2 = I_x - A\bar{y}^2 + A(a-\bar{y})^2$$

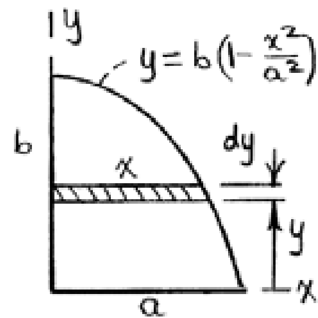
$$= I_x + A(a^2 - 2a\bar{y})$$

$$= \frac{1}{4}\left(\frac{1}{4}\pi a^4\right) + \frac{\pi a^2}{4}\left(a^2 - 2a\,\frac{4a}{3\pi}\right)$$

$$= \frac{\pi a^4}{16} + \frac{\pi a^4}{4}\left(1 - \frac{8}{3\pi}\right) = \underline{0.315\,a^4}$$

A/19

(a) Horizontal strip



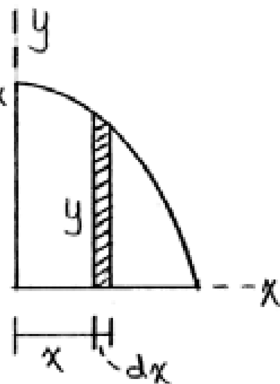
$$I_x = \int y^2 dA = \int_0^b y^2 x\, dy$$

$$= \int_0^b y^2 a \sqrt{1 - y/b}\, dy$$

$$= \frac{a}{\sqrt{b}} \int_0^b y^2 \sqrt{b-y}\, dy$$

$$= \frac{a}{\sqrt{b}} \frac{2}{105(-1)} (8b^2 + 12by + 15y^2) \sqrt{(b-y)^3} \Big|_0^b = \frac{16ab^3}{105}$$

(b) Vertical strip



$$I_x = \int_0^a \frac{1}{3} y^2 (y\, dx) = \frac{1}{3} \int_0^a b^3 (1 - \frac{x^2}{a^2})^3 dx$$

$$= \frac{b^3}{3} \frac{1}{a^6} \int_0^a (a^6 - 3a^4 x^2 + 3a^2 x^4 - x^6)\, dx$$

$$= \frac{b^3}{3a^6} \left[a^6 x - a^4 x^3 + \frac{3a^2 x^5}{5} - \frac{x^7}{7} \right]_0^a$$

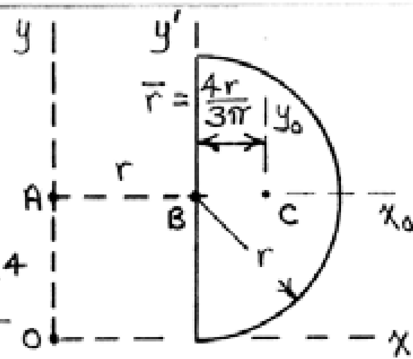
$$= \frac{16ab^3}{105}$$

A/20

$I_{x_0} = \frac{1}{2}\left(\frac{\pi r^4}{4}\right)$

$I_x = I_{x_0} + Ar^2$

$= \frac{\pi r^4}{8} + \frac{\pi r^2}{2} r^2 = \underline{\frac{5}{8}\pi r^4}$



$I_y = I_{y_0} + A(r+\bar{r})^2 = I_{y'} - A\bar{r}^2 + A(r+\bar{r})^2$

$= I_{y'} + Ar^2 + 2Ar\bar{r} = \frac{1}{8}\pi r^4 + \frac{\pi r^4}{2} + 2\frac{\pi r^2}{2} r \frac{4r}{3\pi}$

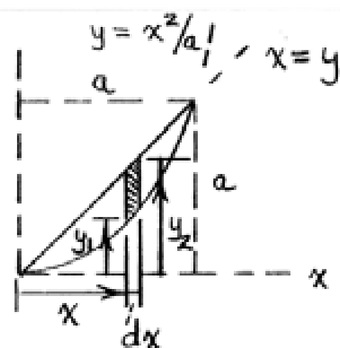
$= \pi r^4\left(\frac{1}{8} + \frac{1}{2} + \frac{4}{3\pi}\right) = \pi r^4\left(\frac{5}{8} + \frac{4}{3\pi}\right)$

$I_z = I_x + I_y = \frac{5}{8}\pi r^4 + \frac{5\pi r^4}{8} + \frac{4\pi r^4}{3\pi} = \pi r^4\left(\frac{5}{8} + \frac{4}{3\pi}\right)$

$I_z = Ak_z^2$, so $k_z = \sqrt{I_z/A}$

or $k_o = \sqrt{\frac{\pi r^4}{\pi r^2/2}\left(\frac{5}{4} + \frac{4}{3\pi}\right)} = \underline{1.830\,r}$

A/21

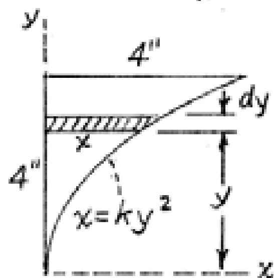


$$dA = (y_2 - y_1)\,dx$$
$$= \left(x - \frac{x^2}{a}\right) dx$$

$$I_x = \int \frac{1}{3} y_2^{\,3}\,dx - \int \frac{1}{3} y_1^{\,3}\,dx = \frac{1}{3}\int_0^a \left(x^3 - \frac{x^6}{a^3}\right) dx$$
$$= \frac{1}{3}\left[\frac{x^4}{4} - \frac{x^7}{7a^3}\right]_0^a = \frac{a^4}{3}\left(\frac{1}{4} - \frac{1}{7}\right) = \underline{a^4/28}$$

$$I_y = \int x^2\,dA = \int x^2\left(x - \frac{x^2}{a}\right) dx = \int_0^a \left(x^3 - \frac{x^4}{a}\right) dx$$
$$= \left[\frac{x^4}{4} - \frac{x^5}{5a}\right]_0^a = \frac{a^4}{4} - \frac{a^4}{5} = \underline{a^4/20}$$

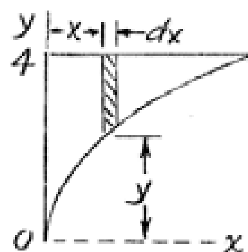
A/22 $dI_x = y^2 dA = y^2 x\,dy = y^2(y^2/4)\,dy = (y^4/4)\,dy$



$$I_x = \frac{1}{4}\int_0^4 y^4\,dy = \frac{1}{4}\frac{y^5}{5}\Big]_0^4 = \frac{256}{5}$$

$$= \underline{51.2 \text{ in.}^4}$$

$k = 1/4,\ x = y^2/4$



$$dI_x = \frac{1}{3}(dx)\left(4^3 - y^3\right)$$

$$= \frac{1}{3}\left(4^3 - [2\sqrt{x}]^3\right)dx$$

$$I_x = \frac{1}{3}\int_0^4 (4^3 - 8x^{3/2})\,dx$$

$$= \frac{1}{3}\left[4^3 x - 8\,\frac{2}{5}\,x^{5/2}\right]_0^4$$

$$= \frac{256}{3}\left(1 - \frac{2}{5}\right) = \frac{256}{5} = \underline{51.2 \text{ in.}^4}$$

A/23 (a) $k_o^2 = k_c^2 + \overline{oc}^2$ where $k_c^2 = I_c/A = \frac{40(10^4)}{1600}\ mm^2$
$= 250\ mm^2$

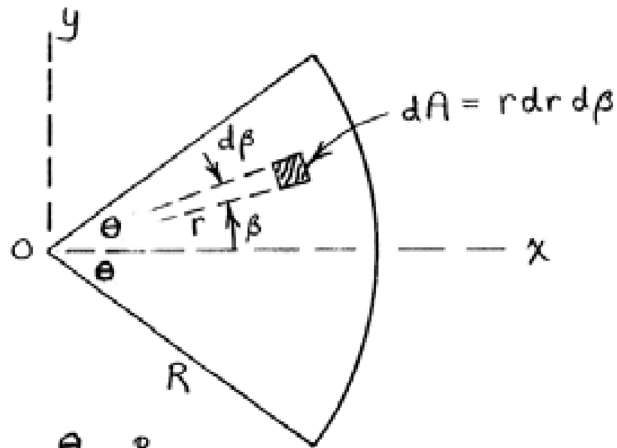
$k_o^2 = 250 + (30\sqrt{2})^2 = 2050\ mm^2$

$k_o = \sqrt{2050} = \underline{45.3\ mm}$

(b) $I_{x_o} = k_{x_o}^2 A$ & $I_{x_o} + I_{y_o} = I_c$ & $I_{y_o} = I_{x_o}$ so $I_{x_o} = \frac{1}{2} I_c$

so $k_{x_o}^2 = I_c/2A$, $k_{x_o} = \sqrt{\frac{40(10^4)}{2(1600)}} = \underline{11.18\ mm}$

A/24

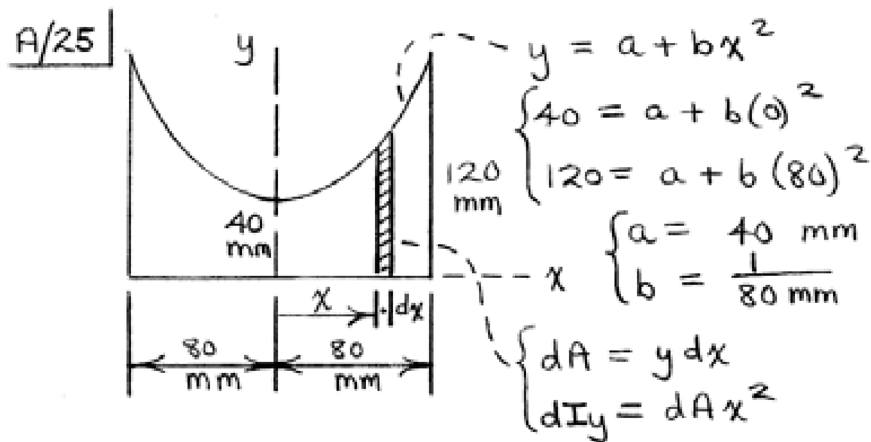


$$I_x = \int y^2 dA = \int_{-\theta}^{\theta} \int_0^R (r \sin\beta)^2 \, r \, dr \, d\beta$$

$$= \frac{R^4}{4}\left(\frac{\beta}{2} - \frac{\sin 2\beta}{4}\right)_{-\theta}^{\theta} = \underline{\frac{R^4}{4}\left(\theta - \frac{1}{2}\sin 2\theta\right)}$$

$$I_y = \int x^2 dA = \int_{-\theta}^{\theta} \int_0^R (r \cos\beta)^2 \, r \, dr \, d\beta$$

$$= \frac{R^4}{4}\left(\frac{\beta}{2} + \frac{\sin 2\beta}{4}\right)_{-\theta}^{\theta} = \underline{\frac{R^4}{4}\left(\theta + \frac{1}{2}\sin 2\theta\right)}$$



$$A = \int dA = \int y\,dx = 2\int_0^{80}\left(40 + \frac{1}{80}x^2\right)dx$$

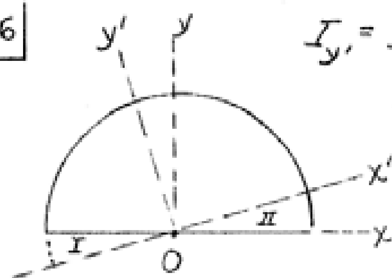
$$= 2\left[40x + \frac{x^3}{240}\right]_0^{80} = 10\,670 \text{ mm}^2$$

$$I_y = \int dI_y = \int x^2 y\,dx = 2\int_0^{80}\left[40x^2 + \frac{x^4}{80}\right]dx$$

$$= 2\left[\frac{40}{3}x^3 + \frac{x^5}{400}\right]_0^{80} = 30.0(10^6) \text{ mm}^4$$

$$k_y = \sqrt{I_y/A} = \sqrt{\frac{30.0(10^6)}{10\,670}} = \underline{53.1 \text{ mm}}$$

A/26



$I_{y'} = I_{\text{half circle}} - I_{I} + I_{II}$

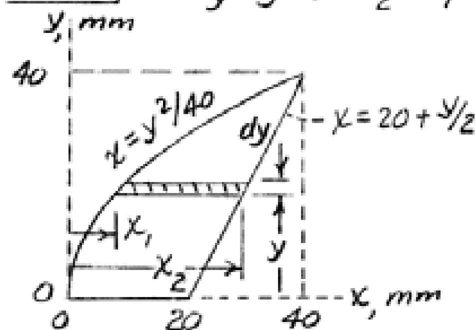
But $I_{I} = I_{II}$ by symmetry

Similarly for $I_{x'}$

So $\underline{I_{x'} = I_{y'} = I_x = I_y}$

A/27

$$dI_y = \frac{1}{3}dy(x_2^3 - x_1^3) = \frac{1}{3}\left[(20+\frac{y}{2})^3 - (\frac{y^2}{40})^3\right]dy$$



$$\int_0^{40}(20+\frac{y}{2})^3\,dy$$

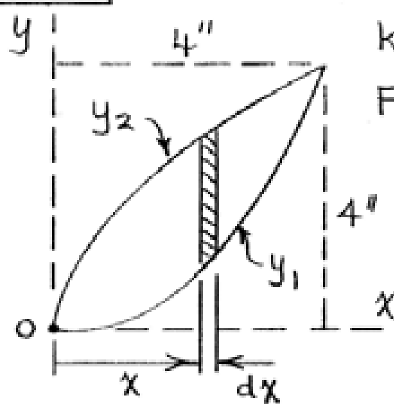
$$= \frac{1}{2}(20+\frac{y}{2})^4\Big]_0^{40} = 120(10^4)\ mm^4$$

$$\int_0^{40}\frac{y^6}{40^3}dy = \frac{1}{40^3}\,\frac{y^7}{7}\Big]_0^{40}$$

$$= 36.57(10^4)\ mm^4$$

$$I_y = \frac{1}{3}(120 - 36.6)10^4 = \underline{27.8\,(10^4)\ mm^4}$$

A/28



From $y_1 = k_1 x^3$: $4 = k_1 4^3$

$k_1 = \frac{1}{16}$ in.$^{-2}$ & $y_1 = \frac{x^3}{16}$

From $y_2 = k_2\sqrt{x}$: $4 = k_2\sqrt{4}$

$k_2 = 2$ in.$^{1/2}$ & $y_2 = 2\sqrt{x}$

$$dA = (y_2 - y_1)\,dx = \left(2\sqrt{x} - \frac{x^3}{16}\right)dx$$

$$I_y = \int x^2 dA = \int_0^4 x^2\left(2x^{\frac{1}{2}} - \frac{x^3}{16}\right)dx$$

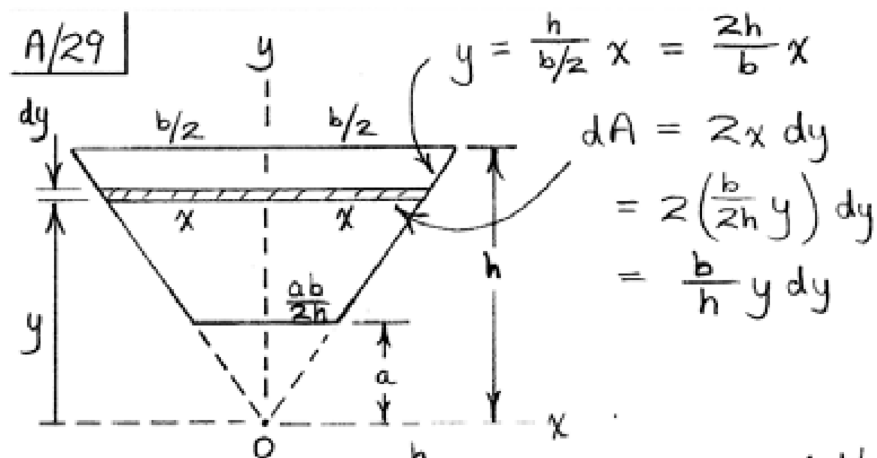
$$= \left[\frac{4}{7}x^{7/2} - \frac{x^6}{96}\right]_0^4 = \underline{30.5 \text{ in.}^4}$$

$$I_x = \int\left[\frac{1}{3}y_2^3\,dx - \frac{1}{3}y_1^3\,dx\right] = \frac{1}{3}\int_0^4\left[(2\sqrt{x})^3 - \left(\frac{x^3}{16}\right)^3\right]dx$$

$$= \frac{1}{3}\left[8\cdot\frac{2}{5}x^{5/2} - \frac{x^{10}}{10\cdot 16^3}\right]_0^4 = \underline{25.6 \text{ in.}^4}$$

$$I_o = I_x + I_y = \underline{56.1 \text{ in.}^4}$$

A/29



$$dA = 2x\,dy = 2\left(\frac{b}{2h}y\right)dy = \frac{b}{h}y\,dy$$

$$I_x = \int y^2\,dA = \int_a^h y^2\,\frac{b}{h}y\,dy = \frac{b}{h}\frac{y^4}{4}\Bigg|_a^h$$

$$= \underline{\frac{b}{4h}\left(h^4 - a^4\right)}$$

$$I_y = \int dI_y = \int \frac{1}{12}(2x)^3\,dy = \frac{2}{3}\int_a^h\left(\frac{b}{2h}y\right)^3 dy$$

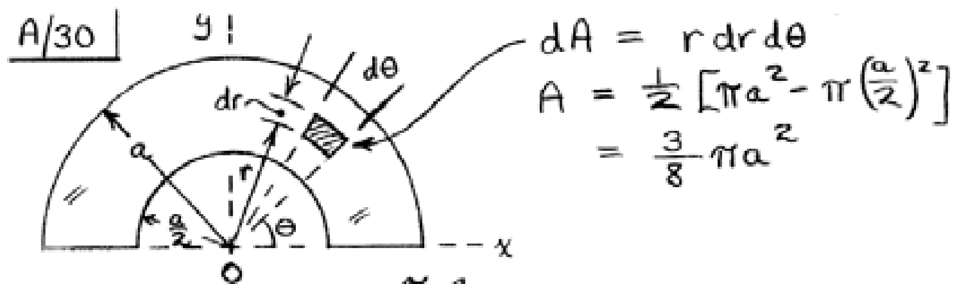
$$= \frac{1}{12}\frac{b^3}{h^3}\frac{y^4}{4}\Bigg|_a^h = \underline{\frac{1}{48}\frac{b^3}{h^3}\left(h^4 - a^4\right)}$$

$$I_z = I_x + I_y = \underline{\frac{b}{4h}\left(1 + \frac{b^2}{12h^2}\right)\left(h^4 - a^4\right)}$$

$$A = \frac{bh}{2} - \frac{1}{2}\left(\frac{ab}{h}\right)(a) = \frac{b}{2}\left(h - \frac{a^2}{h}\right)$$

$$k_O = \sqrt{\frac{I_z}{A}} = \underline{\sqrt{\frac{1}{2}\left(1 + \frac{b^2}{12h^2}\right)\left(h^2 + a^2\right)}}$$

A/30



$$I_x = \int y^2 dA = \int_0^{\pi}\int_{a/2}^{a} (r\sin\theta)^2\, r\,dr\,d\theta$$

$$= \int_0^{\pi} \frac{15}{64} a^4 \sin^2\theta\, d\theta = \frac{15}{128}\pi a^4$$

$$I_y = \int x^2 dA = 2\int_0^{\pi/2}\int_{a/2}^{a} (r\cos\theta)^2\, r\,dr\,d\theta$$

$$= 2\int_{a/2}^{a} \frac{15}{64} a^4 \cos^2\theta\, d\theta = \frac{15}{128}\pi a^4$$

$$\underline{k_x} = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{\frac{15}{128}\pi a^4}{\frac{3}{8}\pi a^2}} = \underline{\frac{\sqrt{5}}{4}a = k_y}$$

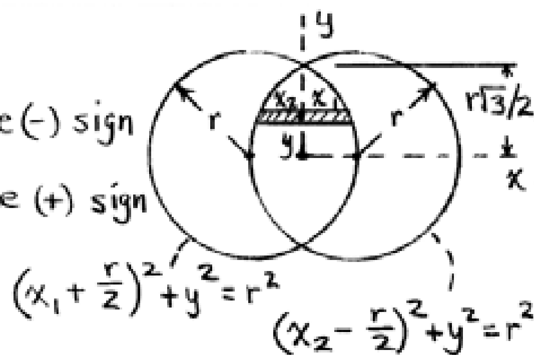
$$k_z^2 = k_x^2 + k_y^2 = 2\left(\frac{5}{16}a^2\right)$$

$$\underline{k_z = \frac{\sqrt{10}}{4}a}$$

▶ A/31

$x_2 = \frac{r}{2} \pm \sqrt{r^2-y^2}$ use (−) sign

$x_1 = -\frac{r}{2} \pm \sqrt{r^2-y^2}$ use (+) sign



$(x_1+\frac{r}{2})^2+y^2=r^2$

$(x_2-\frac{r}{2})^2+y^2=r^2$

$$(x_1-x_2) = -\frac{r}{2}+\sqrt{r^2-y^2}-\frac{r}{2}+\sqrt{r^2-y^2} = 2\sqrt{r^2-y^2}-r$$

$$dA = (2\sqrt{r^2-y^2}-r)\,dy$$

$$I_x = \int y^2 dA = 2\int_0^{r\sqrt{3}/2} y^2(2\sqrt{r^2-y^2}-r)\,dy$$

$$= 4\left\{-\frac{y}{4}\sqrt{(r^2-y^2)^3}+\frac{r^2}{8}\left(y\sqrt{r^2-y^2}+r^2\sin^{-1}\frac{y}{r}\right)\right\}-\frac{2r}{3}y^3\Bigg|_0^{\frac{r\sqrt{3}}{2}}$$

$$= 4\left\{-\frac{r\sqrt{3}}{8}\frac{r^3}{8}+\frac{r^2}{8}\left(\frac{r\sqrt{3}}{2}\frac{r}{2}+r^2\frac{\pi}{3}\right)\right\}-\frac{2\sqrt{3}}{8}r^4-0$$

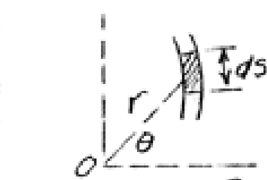
$$= \frac{r^4}{2}\left\{-\frac{\sqrt{3}}{8}+\frac{\sqrt{3}}{4}+\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right\} = \frac{r^4}{2}\left\{\frac{\pi}{3}-\frac{3\sqrt{3}}{8}\right\}$$

$$= 0.1988\,r^4$$

► A/32 $\overline{ds}^2 = \overline{dr}^2 + \overline{r\,d\theta}^2, \quad ds = \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2}\, d\theta$

$$= \sqrt{\left(\frac{R}{2\pi}\right)^2 + \left(\frac{R}{2\pi}\right)^2 \theta^2}\, d\theta$$

$$= \frac{R}{2\pi}\sqrt{1+\theta^2}\, d\theta$$



$r = k\theta, \quad k = \frac{R}{2\pi}$

$r = \frac{R}{2\pi}\theta$

$$\text{Area} = \int b\, ds = \frac{Rb}{2\pi}\int_0^{2\pi} \sqrt{1+\theta^2}\, d\theta$$

$$= \frac{Rb}{2\pi}\frac{1}{2}\left[\theta\sqrt{1+\theta^2} + \ln\left(\theta + \sqrt{1+\theta^2}\right)\right]_0^{2\pi}$$

$$= \frac{Rb}{4\pi}\left[2\pi\sqrt{1+4\pi^2} + \ln\left(2\pi + \sqrt{1+4\pi^2}\right)\right] = 3.383\, Rb$$

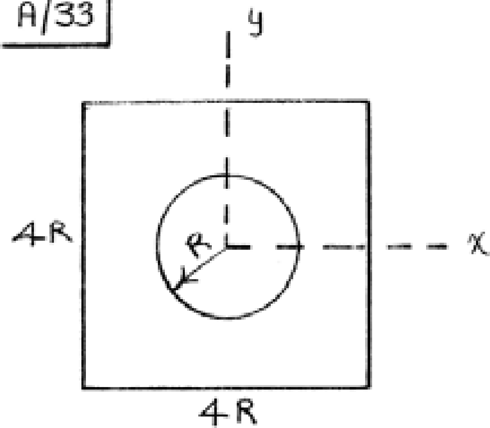
$$I_o = \int r^2 b\, ds = \int \frac{R^2}{4\pi^2} b\, \theta^2 \left(\frac{R}{2\pi}\sqrt{1+\theta^2}\, d\theta\right) = \int_0^{2\pi} \frac{R^3 b}{8\pi^3}\, \theta^2 \sqrt{1+\theta^2}\, d\theta$$

$$I_o = \frac{R^3 b}{8\pi^3}\left[\frac{\theta}{4}\sqrt{(1+\theta^2)^3} - \frac{\theta}{8}\sqrt{1+\theta^2} - \frac{1}{8}\ln\left(\theta + \sqrt{1+\theta^2}\right)\right]_0^{2\pi}$$

$$= \frac{R^3 b}{8\pi^3}\left[\frac{\pi}{2}\sqrt{(1+4\pi^2)^3} - \frac{\pi}{4}\sqrt{1+4\pi^2} - \frac{1}{8}\ln\left(2\pi + \sqrt{1+4\pi^2}\right)\right] = \underline{1.609 R^3 b}$$

$$k_o = \sqrt{I_o/A} = R\sqrt{\frac{1.6094}{3.383}} = \underline{0.690\, R}$$

A/33



Without hole, $I_x = \frac{1}{12}(4R)(4R)^3 = \frac{64}{3}R^4$

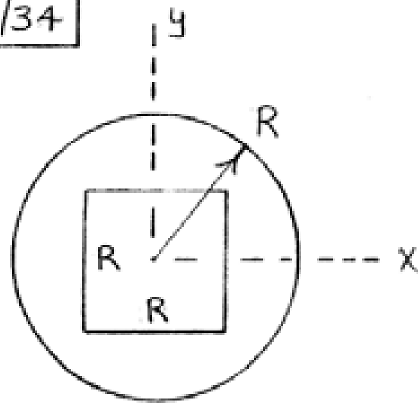
$(21.3R^4)$

With hole, $I_x = \frac{64}{3}R^4 - \frac{1}{4}(\pi R^2)R^2$

$= \underline{20.6R^4}$

(a 3.68% reduction)

A/34



Without square hole:

$$I_z = 2I_x = 2\left(\tfrac{1}{4}\pi R^2 \cdot R^2\right) = \underline{1.571 R^4}$$

With hole:

$$I_z = 1.571 R^4 - 2\left(\tfrac{1}{12} R \cdot R^3\right) = \underline{1.404 R^4}$$

(a reduction of 10.61%)

A/35

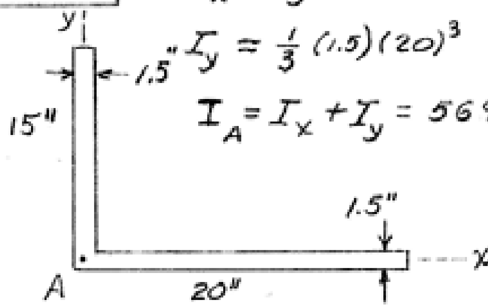
$$I_z = \frac{1}{2}\left[\frac{\pi a^4}{2} - \frac{\pi\left(\frac{a}{2}\right)^4}{2}\right] = \frac{15}{64}\pi a^4$$

$$k_z = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{\frac{15}{64}\pi a^4}{\frac{3}{8}\pi a^2}} = \underline{\frac{\sqrt{10}}{4}a}$$

From $k_x^2 + k_y^2 = k_z^2$ and the fact that $k_x = k_y$ for the present case,

$$2k_x^2 = \left(\frac{\sqrt{10}}{4}a\right)^2, \quad k_x = k_y = \underline{\frac{\sqrt{5}}{4}a}$$

A/36



$$I_x \simeq \frac{1}{3}(1.5)(15)^3 + 0 = 1690 \text{ in.}^4$$

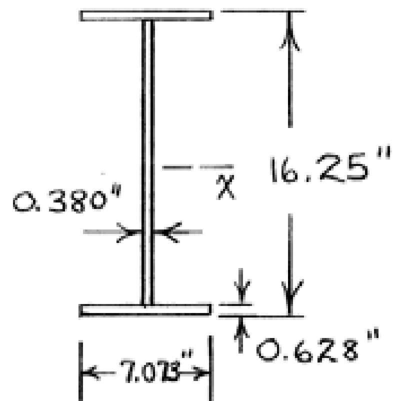
$$I_y \simeq \frac{1}{3}(1.5)(20)^3 + 0 = 4000 \text{ in.}^4$$

$$I_A = I_x + I_y = 5690 \text{ in.}^4$$

$$k_A = \sqrt{I_A/A} = \sqrt{\frac{5690}{1.5(15+20)}}$$

$$= \underline{10.4 \text{ in.}}$$

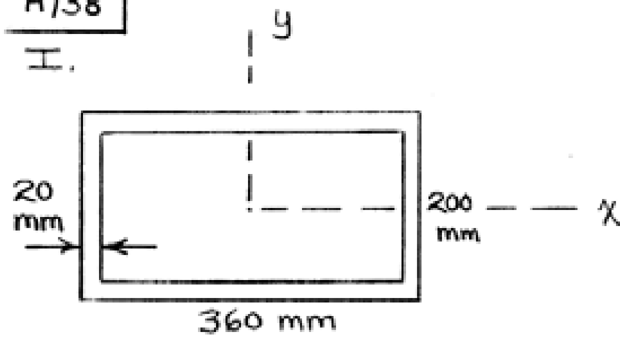
A/37



$$I_x = \frac{1}{12}(0.380)\left[16.25 - 2(0.628)\right]^3$$
$$+2\left\{\frac{1}{12}(7.073)(0.628)^3 + 7.073(0.628)\left[\frac{16.25}{2} - \frac{0.628}{2}\right]^2\right\}$$
$$= \underline{649 \text{ in.}^4}$$

A/38

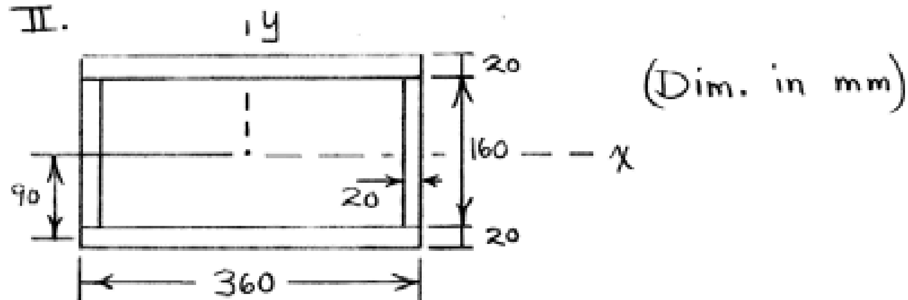
I.



$$I_x = \frac{1}{12}(360)(200)^3 - \frac{1}{12}(320)(160)^3$$

$$= \underline{130.8\,(10^6)\ \text{mm}^4}$$

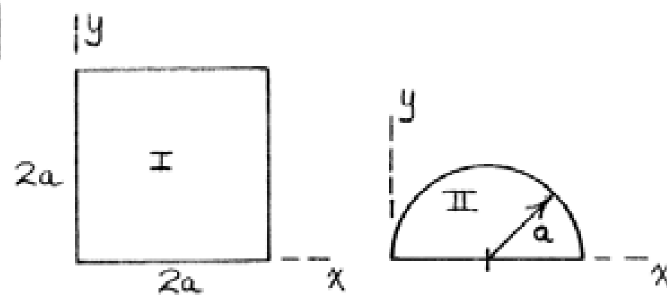
II.



$$I_x = 2\left[\frac{1}{12}(360)(20^3) + 360(20)(90)^2\right]$$

$$+ 2\left[\frac{1}{12}(20)(160)^3\right] = \underline{130.8\,(10^6)\ \text{mm}^4}$$

A/39



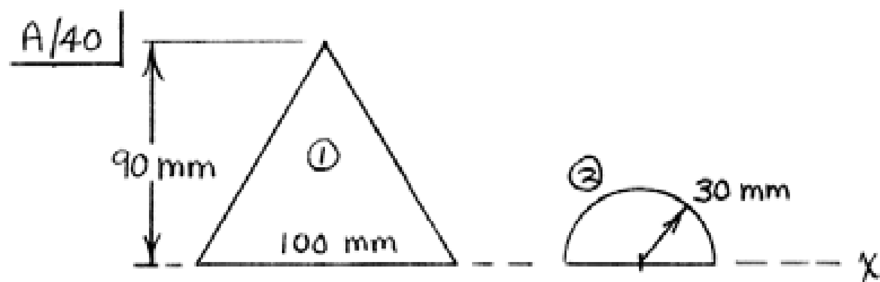
I. Square $I_x = I_y = \frac{1}{3}(4a^2)(2a)^2 = \frac{16}{3}a^4$

II. Semicircle $I_x = \frac{1}{8}\pi a^4$

$$I_y = \frac{1}{8}\pi a^4 + \frac{1}{2}\pi a^2(a^2) = \frac{5}{8}\pi a^4$$

Combined: $I_x = \frac{16}{3}a^4 - \frac{\pi}{8}a^4 = \underline{4.94a^4}$

$$I_y = \frac{16}{3}a^4 - \frac{5}{8}\pi a^4 = \underline{3.37a^4}$$



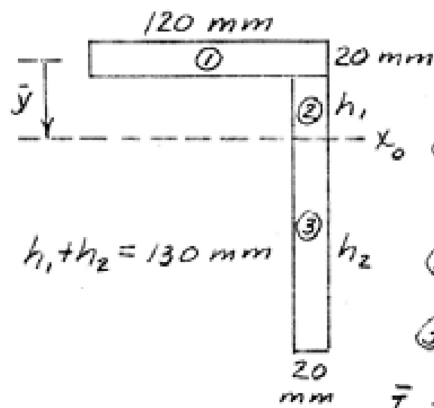
$$I_{x_1} = \frac{1}{12}(100)(90^3) = 6.08(10^6)\ \text{mm}^4$$

$$I_{x_2} = -\frac{\pi(30^4)}{8} = -0.318(10^6)\ \text{mm}^4$$

So $I_x = (6.08 - 0.318)10^6 = \underline{5.76(10^6)\ \text{mm}^4}$

A/41

$$\bar{y} = \Sigma A y / \Sigma A = \frac{0 + (130)(20)(65+10)}{250(20)} = 39.0 \text{ mm}$$



so $h_1 = 29.0$ mm

$h_2 = 101.0$ mm

① $\bar{I}_x = \frac{1}{12}(120)(20^3) + (120)(20)(39.0)^2$

$= 3.73(10^6)$ mm^4

② $\bar{I}_x = \frac{1}{3}(20)(29.0)^3 = 0.163(10^6)$ mm^4

③ $\bar{I}_x = \frac{1}{3}(20)(101.0)^3 = 6.87(10^6)$ mm^4

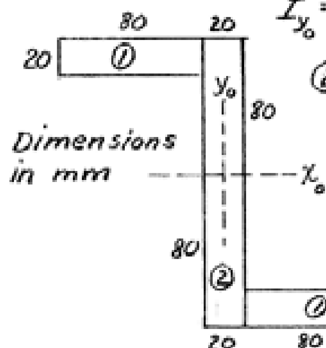
$\bar{I}_x = (3.73 + 0.163 + 6.87)10^6$

$= \underline{10.76(10^6) \text{ mm}^4}$

A/42

① $I_{x_0} = \frac{1}{12}(80)(20)^3 + (80)(20)(70)^2 = 7.89(10^6)\ mm^4$

$I_{y_0} = \frac{1}{12}(20)(80)^3 + (20)(80)(50)^2 = 4.85(10^6)\ mm^4$



② $I_{x_0} = \frac{1}{12}(20)(160)^3 = 6.83(10^6)\ mm^4$

$I_{y_0} = \frac{1}{12}(160)(20)^3 = 0.1067(10^6)\ mm^4$

Total $\bar{I}_x = [2(7.89) + 6.83](10^6)$

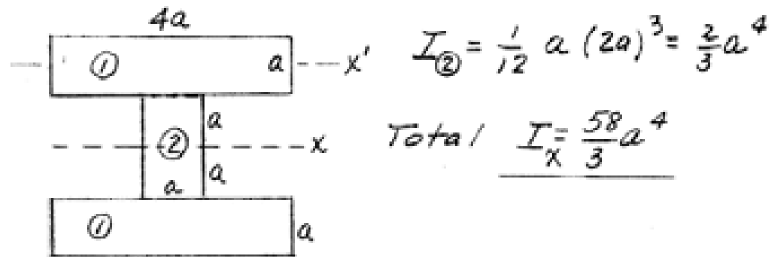
$= \underline{22.6(10^6)\ mm^4}$

$\bar{I}_y = [2(4.85) + 0.1067](10^6)$

$= \underline{9.81(10^6)\ mm^4}$

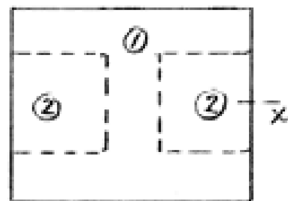
A/43

Sol. I $\quad I_{①} = 2\left[\frac{1}{12} 4a(a^3) + 4a^2\left(\frac{3a}{2}\right)^2\right] = \frac{56}{3}a^4$



$I_{②} = \frac{1}{12}\, a\,(2a)^3 = \frac{2}{3}a^4$

Total $\quad \underline{I_x = \frac{58}{3}a^4}$



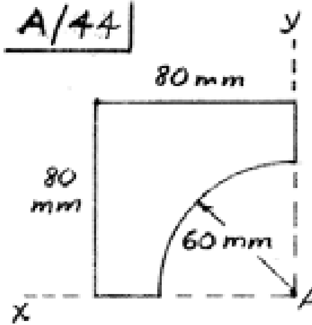
Sol. II

$I_{①} = \frac{1}{12}(4a)(4a)^3 = \frac{64}{3}a^4$

$I_{②} = -\frac{1}{12}(3a)(2a)^3 = -2a^4$

$\text{Total} = \left(\frac{64}{3} - \frac{6}{3}\right)a^4 = \underline{\frac{58}{3}a^4}$

A/44



Square:

$I_x = I_y = \frac{1}{3}(80)(80)^3 = 13.65(10^6)\ mm^4$

Quarter-circular area:

$I_x = I_y = \frac{-1}{4}\left(\frac{1}{4}\pi[60]^4\right) = -2.54(10^6)\ mm^4$

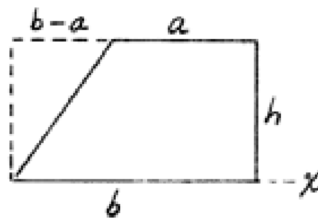
Area $A = (80)^2 - \frac{1}{4}\pi(60)^2 = 3573\ mm^2$

$I_z = I_x + I_y = 2(13.65 - 2.54)10^6$

$= 22.22(10^6)\ mm^4$

$k_A^2 = I_z/A = 22.22(10^6)/3573 = 6219\ mm^2$; $k_A = \sqrt{6219} = \underline{78.9\ mm}$

A/45 Distort to a rectangle and a triangle without altering y-distribution of area



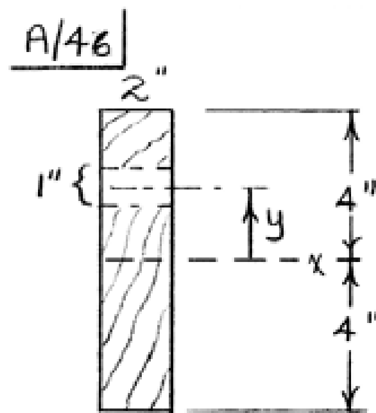
Rectangle $I_x = \frac{1}{3} b h^3$

Triangle (Sample Problem A2)

$I_x = \frac{-(b-a)h^3}{4}$

For trapezoid, $I_x = \frac{bh^3}{3} - \frac{b-a}{4} h^3 = \underline{\frac{1}{12}(b+3a)h^3}$

A/46



Without hole,

$$I_x = \frac{1}{12} bh^3 = \frac{1}{12}(2)(8)^3$$
$$= 85.3 \text{ in.}^4$$

With hole, $I_x' = I_x - (\bar{I}_{hole} + Ay^2)$

$$I_x' = 85.3 - \left[\frac{1}{12}(2)(1)^3 + (2)(1)y^2\right]$$
$$= 85.2 - 2y^2 \quad (y \text{ in in., } I_x' \text{ in in.}^4)$$

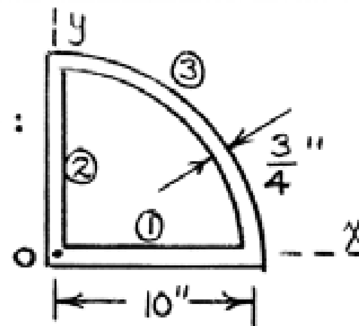
Percent reduction $n = \dfrac{I_x - I_x'}{I_x}(100\%)$

$$n = \frac{85.3 - (85.2 - 2y^2)}{85.3}(100\%) = \underline{0.1953 + 2.34y^2}$$
(in percent)

For $y = 2$ in.,

$$n = 0.1953 + 2.34(2)^2 = \underline{9.57\%}$$

A/47



Parts 1 and 2 separately:

$I_o = \frac{1}{3}\left(10 \times \frac{3}{4}\right)10^2 = \frac{10^3}{4}$ in.^{4}

(polar); Area $= 10\left(\frac{3}{4}\right)$ in.^{2}

Part 3: $I_o = Ar^2 = \frac{\pi(10)}{2}\left(\frac{3}{4}\right)10^2 = \frac{3\pi}{8}10^3$ in.^{4}

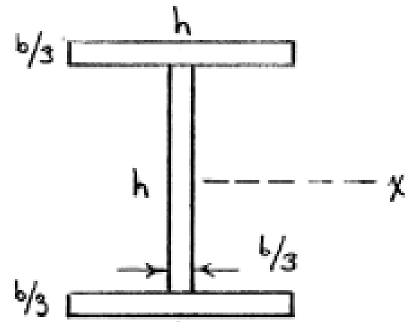
Area $= 10\left(\frac{3\pi}{8}\right)$ in.^{2}

Combined: $I_o = 2\left(\frac{10^3}{4}\right) + \frac{3\pi}{8}(10^3)$

Area $= 2(10)\left(\frac{3}{4}\right) + 10\left(\frac{3\pi}{8}\right)$

$k_o = \sqrt{\frac{I_o}{A}} = \sqrt{\frac{\frac{1}{2} + \frac{3\pi}{8}}{\frac{3}{2} + \frac{3\pi}{8}}}\ 10 = \underline{7.92 \text{ in.}}$

A/48



For area (a),

$$I_x = \frac{1}{12} b h^3$$

For area (b),

$$I_x = \frac{1}{12}\frac{b}{3}h^3 + 2\left[\frac{1}{12}h\left(\frac{b}{3}\right)^3 + h\frac{b}{3}\left(\frac{h}{2}+\frac{b}{6}\right)^2\right]$$

$$= \frac{hb}{9}\left(\frac{7}{4}h^2 + \frac{2}{9}b^2 + hb\right)$$

If $h = 200$ mm and $b = 60$ mm, we have

(a) $I_x = \frac{1}{12}(60)(200)^3 = 40(10^6)\ \text{mm}^4$

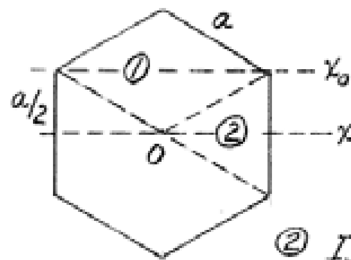
(b) $I_x = \frac{200(60)}{9}\left(\frac{7}{4}(200)^2 + \frac{2}{9}(60)^2 + 200(60)\right)$

$= 110.4(10^6)\ \text{mm}^4$

Percent increase $n = \frac{110.4 - 40}{40}(100\%) = 176.0\%$

A/49

① $I_{x_0} = 2\left(\frac{1}{12}bh^3\right) = \frac{1}{6}(2a\sqrt{3}/2)(a/2)^3 = \frac{\sqrt{3}}{48}a^4$



$I_x = I_{x_0} + Ad^2$

$= \frac{\sqrt{3}}{48}a^4 + (2a\sqrt{3}/2)(a/2)(a/2)^2$

$= \frac{7\sqrt{3}}{48}a^4$

② $I_x = 2\left(\frac{1}{12}bd^3\right) = \frac{1}{6}(a\sqrt{3}/2)(a/2)^3 = \frac{\sqrt{3}}{96}a^4$

Total $I_x = 2\left(\frac{7\sqrt{3}}{48}a^4 + \frac{\sqrt{3}}{96}a^4\right) = \underline{\frac{5\sqrt{3}}{16}a^4}$

A/50

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A}$$

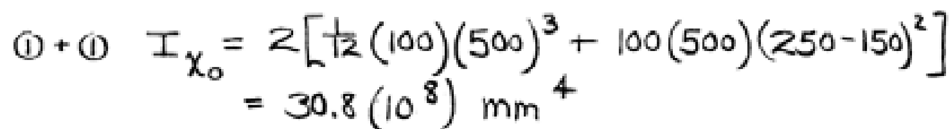
$$= \frac{2[(100)(500)(250)] + 500(100)(-50)}{2(100)(500) + 100(500)}$$

$$= 150 \text{ mm}$$

$$A = 2(100)(500) + 100(500)$$

$$= 15(10^4) \text{ mm}^2$$

(Dim. in mm)



① + ① $I_{x_0} = 2\left[\frac{1}{12}(100)(500)^3 + 100(500)(250-150)^2\right]$

$$= 30.8(10^8) \text{ mm}^4$$

$$I_{y_0} = 2\left[\frac{1}{12}(500)(100)^3 + 100(500)(150+50)^2\right] = 40.8(10^8) \text{ mm}^4$$

② $I_{x_0} = \frac{1}{12}(500)(100)^3 + 100(500)(50+150)^2 = 20.4(10^8) \text{ mm}^4$

$$I_{y_0} = \frac{1}{12}(100)(500)^3 = 10.42(10^8) \text{ mm}^4$$

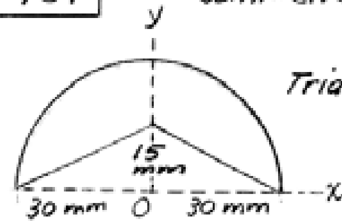
Totals ① + ① + ②: $I_{x_0} = 51.2(10^8) \text{ mm}^4$

$I_{y_0} = 51.2(10^8) \text{ mm}^4$

$$I_C = I_{x_0} + I_{y_0} = 102.5(10^8) \text{ mm}^4$$

$$k_C = \sqrt{I_C/A} = \sqrt{\frac{102.5(10^8)}{15(10^4)}} = \underline{261 \text{ mm}}$$

A/51



Semi-circle: $I_z = \frac{1}{4}\pi r^4 = \frac{1}{4}\pi(30)^4$
$= 0.6362(10^6)\ mm^4$

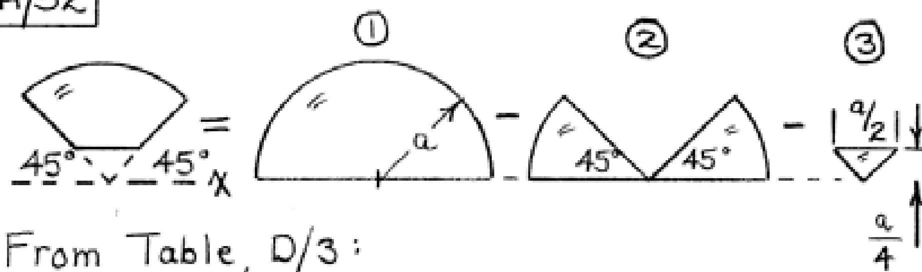
Triangle: $I_x = \frac{-1}{12}bh^3 = \frac{-1}{12}(60)(15)^3$
$= -0.01688(10^6)\ mm^4$

$I_y = -\frac{2}{12}(15)(30)^3$
$= -0.06750(10^6)\ mm^4$

$I_z = I_x + I_y = -(0.01688 + 0.06750)10^6$
$= -0.0844(10^6)\ mm^4$

Total $I_z = (0.6362 - 0.0844)(10^6) = \underline{0.552(10^6)\ mm^4}$

A/52



From Table D/3:

$$I_{x_1} = \frac{\pi a^4}{8}$$

$$I_{x_2} = 2\frac{a^4}{8}\left[\frac{\pi}{4} - \frac{1}{2}\sin\left(2\cdot\frac{\pi}{4}\right)\right] = a^4\left(\frac{\pi}{16} - \frac{1}{8}\right)$$

$$I_{x_3} = \frac{a}{2}\left(\frac{a}{4}\right)^3 / 4 = \frac{a^4}{512}$$

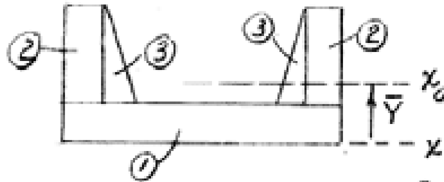
So $I_x = I_{x_1} - I_{x_2} - I_{x_3}$

$$= \frac{\pi a^4}{8} - a^4\left(\frac{\pi}{16} - \frac{1}{8}\right) - \frac{a^4}{512}$$

$$= a^4\left(\frac{\pi}{16} + \frac{63}{512}\right) = \underline{0.319 a^4}$$

A/53

Part	A in.^{2}	$\bar{y}$ in.	$\bar{y}A$ in.^{3}	$\bar{I}_x$ in.^{4}	d in.	Ad^2 in.^{4}
1	8.40	0.35	2.94	0.343	0.696	4.07
2	4.29	2.35	10.08	3.894	1.304	7.30
3	0.33	1.80	0.59	0.200	0.754	0.19
Totals	13.02		13.61	4.437		11.56



① $A_1 = 0.70 \times 12 = 8.40$ in.^{2}

② $A_2 = 0.65 \times 3.30 \times 2 = 4.29$ in.^{2}

③ $A_3 = 2\left(\frac{1}{2}\right)(0.10)(3.30) = 0.33$ in.^{2}

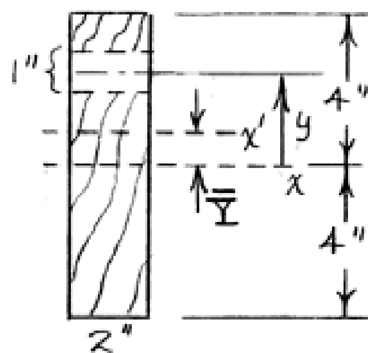
$$\bar{Y} = \frac{13.61}{13.02} = 1.046 \text{ in.}$$

$$\bar{I}_x = I_{x_0} = \Sigma \bar{I}_x + \Sigma A d^2$$

$$= 4.437 + 11.56$$

$$= \underline{16.00 \text{ in.}^4}$$

▸A/54 Without hole, $x' = x$ and $I_x = I_{x'} = \frac{1}{12}bh^3 = \frac{1}{12}(2)(8)^3 = 85.3 \text{ in.}^4$



Centroid location:

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{8(2)(0) - 2(1)y}{8(2) - 2(1)}$$

$$= -0.1429y \quad \left(\text{or } -\frac{1}{7}y\right)$$

$$I'_{x'} = \frac{1}{3}(2)(4+\bar{Y})(4+\bar{Y})^2 + \frac{1}{3}(2)(4-\bar{Y})(4-\bar{Y})^2 - \left[\frac{1}{12}(2)(1)^3 + 2(1)(y-\bar{Y})^2\right]$$

$$= 85.2 + 16\bar{Y}^2 - 2(y-\bar{Y})^2$$

With $\bar{Y} = -\frac{1}{7}y$: $I'_{x'} = 85.2 + 16\left(-\frac{1}{7}y\right)^2 - 2\left(y + \frac{1}{7}y\right)^2$

or $I'_{x'} = 85.2 - \frac{112}{49}y^2 = 85.2 - 2.29y^2$

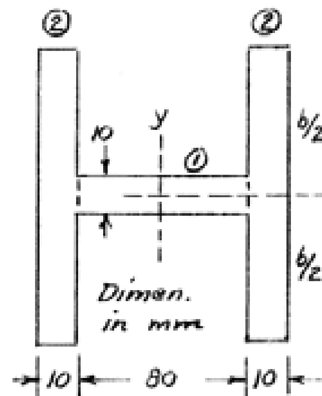
Percent reduction $n = \frac{I_{x'} - I'_{x'}}{I_{x'}}(100\%)$

$$= (100\%)\frac{85.3 - (85.2 - 2.29y^2)}{85.3} = 0.1953 + 2.68y^2$$

(in percent)

For $y = 2$ in., $n = 10.91\%$

▶A/55



① $I_x = \frac{1}{12}(80)(10)^3 = 0.00667(10^6)\ mm^4$

$I_y = \frac{1}{12}(10)(80)^3 = 0.427(10^6)\ mm^4$

② $I_x = 2\left[\frac{1}{12}(10)b^3\right] = 1.667b^3$

$I_y = 2\left[\frac{1}{12}b(10)^3\right] + (10b)(45)^2\right]$

$= 0.0407(10^6)b$

Total I_x = Total I_y

$(0.00667)(10^6) + 1.667b^3$

$= (0.427 + 0.0407)(10^6)$

or $b^3 - 0.0244(10^6)b - 0.252(10^6) = 0$

Solve by cubic formula; $\left[\frac{0.252(10^6)}{2}\right]^2 < \left[\frac{0.0244(10^6)}{3}\right]^3$ so 3 real roots

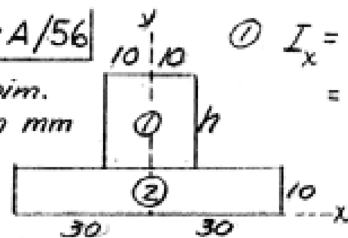
$\cos u = \frac{q}{p\sqrt{p}}$ where $q = \frac{252(10^3)}{2} = 126(10^3)$, $p = \frac{24.4(10^3)}{3} = 8.13(10^3)$

$\cos u = 126(10^3)/[8.13(10^3)90.2] = 0.1718$, $u = 80.11°$

$b_1 = 2\sqrt{p}\cos u/3 = 2(90.2)(0.8933) = 161.1$ mm or $\underline{b = 161.1 \text{ mm}}$

$b_2 = 2\sqrt{p}\cos\left(\frac{u}{3} + 120°\right) = (-)$, $b_3 = 2\sqrt{p}\cos\left(\frac{u}{3} + 240°\right) = -(\ )$

▶A/56



① $I_x = \frac{1}{12}(20)h^3 + 20h\left(\frac{h}{2}+10\right)^2$

$= 20\left(h^3/3 + 10h^2 + 100h\right)$ mm^4

$I_y = \frac{1}{12}h(20)^3 = \frac{2000h}{3}$ mm^4

② $I_x = \frac{1}{3}(60)(10)^3 = 20000$ mm^4

$I_y = \frac{1}{12}(10)(60)^3 = 180\,000$ mm^4

Thus for equal I_x & I_y totals

$$20\left(h^3/3 + 10h^2 + 100h\right) + 20000 = \frac{2000h}{3} + 180\,000$$

or $h^3 + 30h^2 + 200h = 24000$

Substitute $h = u - 10$ & get $u^3 = 100u + 24000$ & solve by Appen. B/4-4: Let $p = 100/3$, $q = 12000$ Case II $q^2 - p^3 = (+)$

$(q^2 - p^3)^{1/2} = \sqrt{144(10^6) - 10^6/27} = 11.99846\,(10^3)$

$u_1 = (12000 + 11998)^{1/3} + (12000 - 11998.46)^{1/3} = 28.84 + 1.16 = 30.00$

$h = 30.00 - 10 = \underline{20.0 \text{ mm}}$

A/57 $A = (30)(60) = 1800\ mm^2$ for each

$\bar{I}_{xy} = 0$ for each, so $I_{xy} = 0 + A\,d_x d_y$

(a) $I_{xy} = (40)(50)(1800) = 360\,(10^4)\ mm^4$

(b) $I_{xy} = (40)(-50)(1800) = -360\,(10^4)\ mm^4$

(c) $I_{xy} = (-40)(-50)(1800) = 360\,(10^4)\ mm^4$

(d) $I_{xy} = (-40)(50)(1800) = \underline{-360\,(10^4)\ mm^4}$

A/58

$$I_{xy} = -(3)(3)\left[(-3.5)(3.5) + (-3.5)(-3.5) + (3.5)(-3.5)\right]$$

$$= \underline{110.2 \text{ in.}^4}$$

A/59

$$I_x = \frac{1}{12}(400)(200)^3 - 3\left[\frac{1}{4}\pi(30^4) + \pi(30)^2(50)^2\right]$$
$$= \underline{2.44(10^8)\ \text{mm}^4}$$

$$I_y = \frac{1}{12}(200)(400)^3 - 3\left[\frac{1}{4}\pi(30^4) + \pi(30)^2(100)^2\right]$$
$$= \underline{9.80(10^8)\ \text{mm}^4}$$

$$I_{xy} = -\pi(30)^2\left[(100)(50) + (-100)(50) + (-100)(-50)\right]$$
$$= \underline{-14.14(10^6)\ \text{mm}^4}$$

A/60

(a) $I_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (60)(40)(80)(50)$

$= \underline{9.60(10^6)\ mm^4}$

(b) $I_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (-60)(40)(\pi \cdot 25^2)$

$= \underline{-4.71\ (10^6)\ mm^4}$

(c) $I_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (-60)(-40)(80)(50)$

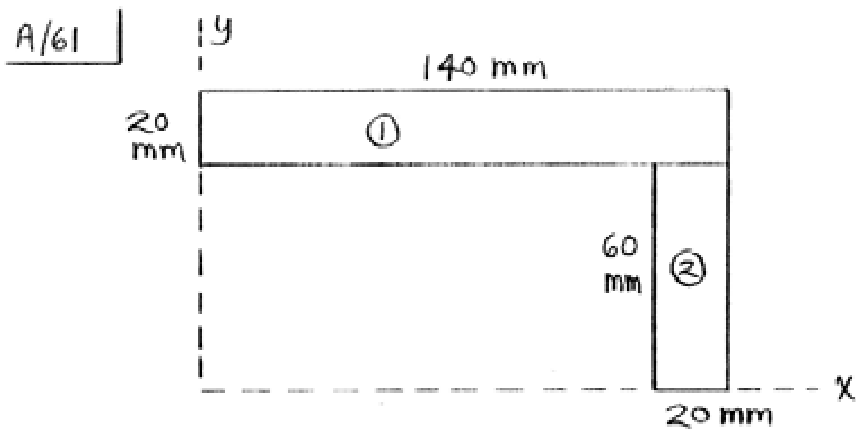
$= \underline{9.60\ (10^6)\ mm^4}$

(d) $I_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (60)\left(-40 - \frac{4(25)}{3\pi}\right)$

$\times\ (\pi \cdot 25^2)/2$

$= \underline{-2.98\,(10^6)\ mm^4}$

A/61

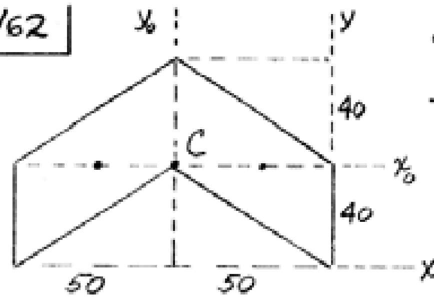


$$I_{xy_1} = 20(140)(70)(70) = 13.72(10^6)\ \text{mm}^4$$

$$I_{xy_2} = 60(20)(130)(30) = 4.68(10^6)\ \text{mm}^4$$

$$\text{Total}: \underline{I_{xy} = 18.40(10^6)\ \text{mm}^4}$$

A/62



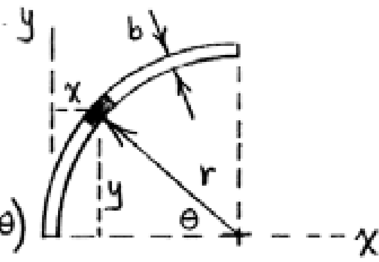
Centroid is at C

$I_{xy} = I_{x_0y_0} + d_x d_y A$

$= 0 + (40)(-50)(40 \times 100)$

$= \underline{-8(10^6)\ \text{mm}^4}$

A/63



$$I_{xy} = \int xy\, dA$$

$$= \int_0^{\pi/2} (r - r\cos\theta)\, r\sin\theta\, (b r\, d\theta)$$

$$= br^3 \int_0^{\pi/2} (\sin\theta - \sin\theta\cos\theta)\, d\theta$$

$$= br^3 \left[-\cos\theta + \frac{1}{4}\cos 2\theta \right]_0^{\pi/2}$$

$$= br^3 \left[0 - \frac{1}{4} + 1 - \frac{1}{4} \right] = \underline{br^3/2}$$

A/64

$$I_{xy} = \int xy\,dA = \int_0^{\pi/2}\int_0^r (r\cos\theta)(r\sin\theta)\,r\,dr\,d\theta$$

$$= \int_0^{\pi/2} \frac{\sin 2\theta}{2}\frac{r^4}{4}\,d\theta = \frac{r^4}{16}\left(-\cos 2\theta\right)_0^{\pi/2}$$

$$= \frac{r^4}{16}\left(1-[-1]\right) = \underline{r^4/8}$$

$$\bar{I}_{xy} = I_{xy} - d_x d_y A = \frac{r^4}{8} - \frac{4r}{3\pi}\left(\frac{4r}{3\pi}\right)\frac{\pi r^2}{4} = \frac{r^4}{8}\left(1-\frac{32}{9\pi}\right)$$

$$= \underline{-0.01647\,r^4}$$

A/65

(1) By direct integration

$(x-r)^2+y^2=r^2$

For elemental strip,

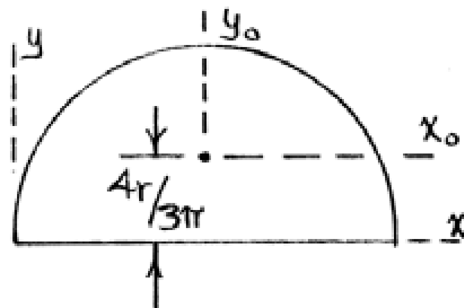
$$dI_{xy} = x\,\frac{y}{2}\,dA = \frac{xy}{2}\,y\,dx$$

$$= \frac{x}{2}\left[r^2-(x-r)^2\right]dx$$

$$I_{xy} = \frac{1}{2}\int_0^{2r}(xr^2-x^3+2rx^2-r^2x)\,dx$$

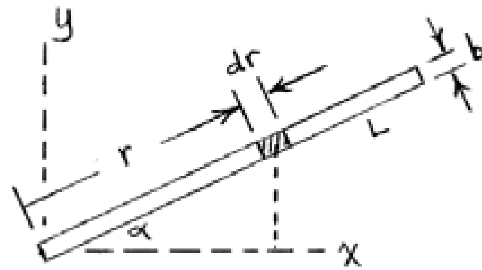
$$= \frac{1}{2}\left[\frac{x^2r^2}{2}-\frac{x^4}{4}+\frac{2rx^3}{3}-\frac{r^2x^2}{2}\right]_0^{2r} = \underline{\frac{2}{3}r^4}$$

(2) By axis transfer



$$I_{xy} = I_{x_0y_0} + Ad_xd_y = 0 + \frac{\pi r^2}{2}(r)\left(\frac{4r}{3\pi}\right) = \underline{\frac{2}{3}r^4}$$

A/66 Strip 1:



$$I_{xy_1} = \int xy\, dA = \int_0^L (r\cos\alpha)(r\sin\alpha)\, b\, dr$$

$$= b\cos\alpha\sin\alpha \left.\frac{r^3}{3}\right|_0^L = \frac{bL^3}{3}\cos\alpha\sin\alpha$$

or $I_{xy_1} = \frac{bL^3}{6}\sin 2\alpha$

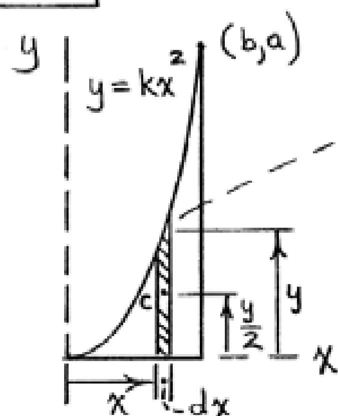
By inspection, $I_{xy_2} = -I_{xy_1} = -\frac{bL^3}{6}\sin 2\alpha$

$$I_{xy_3} = \frac{bL^3}{6}\sin\left[2(90° - \alpha)\right] = \frac{bL^3}{6}\sin 2\alpha$$

$$I_{xy_4} = -I_{xy_3} = -\frac{bL^3}{6}\sin 2\alpha$$

$\sum_{i=1}^{4} I_{xy_i} = 0$, which must be the case due to symmetry about the y-axis.

A/67



$y = kx^2$: $a = kb^2$, $k = a/b^2$

$\Rightarrow y = \frac{a}{b^2}x^2$

$dA = y\,dx = \frac{a}{b^2}x^2\,dx$

$d\bar{I}_{xy} = 0$

$dI_{xy} = d\bar{I}_{xy} + dA\,(x)\left(\frac{y}{2}\right)$

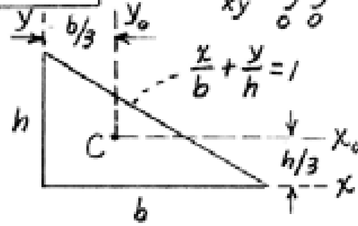
$= 0 + y\,dx\,(x)\left(\frac{y}{2}\right)$

$= \frac{1}{2}\frac{a^2}{b^4}x^5\,dx$

$$I_{xy} = \int dI_{xy} = \int_0^b \frac{1}{2}\frac{a^2}{b^4}x^5\,dx$$

$$= \frac{1}{2}\frac{a^2}{b^4}\frac{x^6}{6}\Big|_0^b = \underline{\frac{1}{12}a^2b^2}$$

A/68



$$I_{xy}=\int_0^h\int_0^x xy\,dx\,dy=\int_0^h y\left(\frac{x^2}{2}\right)\Big]_0^{b(1-y/h)}dy$$

$$=\int_0^h \frac{b^2}{2}\left(y-\frac{2y^2}{h}+\frac{y^3}{h^2}\right)dy$$

$$=\frac{b^2}{2}\left[\frac{h^2}{2}-\frac{2h^2}{3}+\frac{h^2}{4}\right]=\underline{\frac{b^2h^2}{24}}$$

$$I_{x_0y_0}=\bar{I}_{xy}=I_{xy}-d_xd_yA=\frac{b^2h^2}{24}-\frac{bh}{2}\left(\frac{h}{3}\right)\left(\frac{b}{3}\right)=\underline{-\frac{b^2h^2}{72}}$$

A/69

$\frac{x}{y} = \frac{b}{h}$

First: $I_{xy} = \int_0^b \int_0^y xy\,dy\,dx$

$= \int_0^b \frac{x}{2} y^2 \Big]_0^{\frac{hx}{b}} dx = \int_0^b \frac{h^2}{2b^2} x^3 dx$

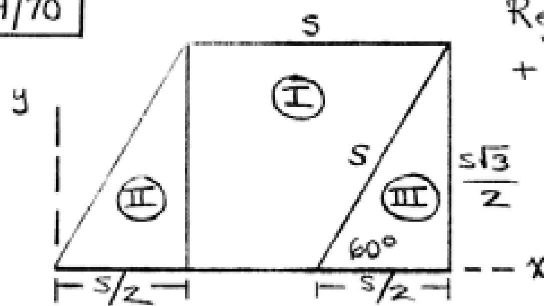
$= \frac{h^2}{2b^2} \frac{b^4}{4} = \underline{\frac{b^2h^2}{8}}$

Second: $dI_{xy} = dI_{x_0 y_0} + d_x d_y (dA)$

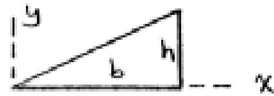
$= 0 + \frac{y}{2} x (y\,dx) = \frac{h^2}{2b^2} x^3 dx$

$I_{xy} = \frac{h^2}{2b^2} \int_0^b x^3 dx = \frac{h^2}{2b^2} \frac{b^4}{4} = \underline{\frac{b^2h^2}{8}}$

A/70



Regard as I (rectangle) + II − III (triangles)



Use result $I_{xy} = \frac{b^2h^2}{8}$ from Prob. A/69.

$$\bar{I}_{xy} = I_{xy} - Ad_x d_y = \frac{b^2h^2}{8} - \frac{bh}{2}\left(\frac{2b}{3}\right)\left(\frac{h}{3}\right) = \frac{1}{72} b^2h^2$$

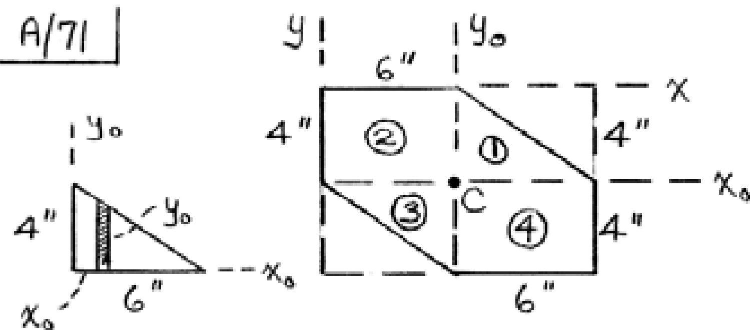
$$I_{I} = \bar{I}_{xy} + Ad_x d_y = 0 + s\,\frac{s\sqrt{3}}{2}\left(s\right)\left(\frac{s\sqrt{3}}{4}\right) = \frac{3}{8}s^4$$

$$I_{II} = \frac{1}{8}\left(\frac{s}{2}\right)^2\left(\frac{s\sqrt{3}}{2}\right)^2 = \frac{3}{128}s^4$$

$$I_{III} = \bar{I}_{xy} + Ad_x d_y = \frac{1}{72}\left(\frac{s}{2}\right)^2\left(\frac{s\sqrt{3}}{2}\right)^2$$
$$+ \frac{1}{2}\left(\frac{s}{2}\right)\left(\frac{s\sqrt{3}}{2}\right)\left[s + \frac{2}{3}\frac{s}{2}\right]\left[\frac{1}{3}\frac{s\sqrt{3}}{2}\right]$$
$$= \frac{11}{128}s^4$$

$$I_{xy} = I_{I} + I_{II} - I_{III} = s^4\left[\frac{3}{8} + \frac{3}{128} - \frac{11}{128}\right]$$
$$= \underline{\frac{5}{16}s^4}$$

A/71



Part 1: $I_{x_0y_0} = \int_0^6 x_0 \frac{y_0}{2}(y_0 dx_0)$, $y_0 = 4 - \frac{2}{3}x_0$

$= \frac{1}{2}\int_0^6 x_0\left(16 - \frac{16}{3}x_0 + \frac{4}{9}x_0^2\right)dx_0$

$= \frac{1}{2}\left[8x_0^2 - \frac{16}{9}x_0^3 + \frac{1}{9}x_0^4\right]_0^6 = 24 \text{ in.}^4$

Part 3 : $I_{x_0y_0} = 24 \text{ in.}^4$

Part 2 : $I_{x_0y_0} = 4(6)(-3)(+2) = -144 \text{ in.}^4$

Part 4 : $I_{x_0y_0} = -144 \text{ in.}^4$

Combined : $I_{x_0y_0} = 2(24) + 2(-144) = -240 \text{ in.}^4$

Combined area $= 2(4)(6) + 2(\frac{1}{2})(4)(6) = 72 \text{ in.}^4$

So $I_{xy} = I_{x_0y_0} + Ad_xd_y = -240 + 72(+6)(-4)$

$= \underline{-1968 \text{ in.}^4}$

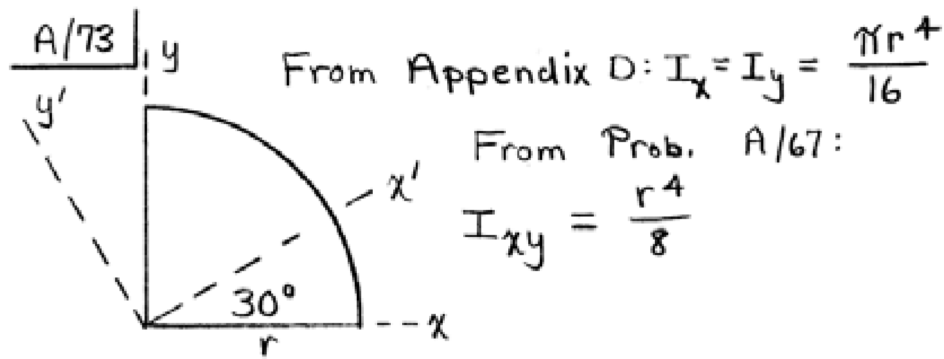
A/72 $I_{xy} = \bar{I}_{xy} + d_x d_y A$, $I_{x'y'} = \bar{I}_{xy} + d_{x'} d_{y'} A$

$I_{xy} - I_{x'y'} = (d_x d_y - d_{x'} d_{y'})A$

where $d_x = 20$ mm, $d_y = 30$ mm, $d_{x'} = -40$ mm, $d_{y'} = 80$ mm

$(80 - [-420])10^5 = [(20)(30) - (-40)(80)]A$

$$A = \frac{500(10^5)}{3800} = \underline{1.316(10^4)\ \text{mm}^2}$$



From Appendix D: $I_x = I_y = \dfrac{\pi r^4}{16}$

From Prob. A/67:

$$I_{xy} = \frac{r^4}{8}$$

Eq. A/9: $I_{x'} = \dfrac{\pi r^4}{16} + 0 - \dfrac{r^4}{8}\sin 60° = \underline{\dfrac{r^4}{16}\left[\pi - \sqrt{3}\right]}$

$$I_{y'} = \frac{\pi r^4}{16} - 0 + \frac{r^4}{8}\sin 60° = \underline{\frac{r^4}{16}\left[\pi + \sqrt{3}\right]}$$

Eq. A/9a:

$$I_{x'y'} = 0 + \frac{r^4}{8}\cos 60° = \underline{\frac{r^4}{16}}$$

A/74 $I_x = \frac{1}{3} b(b^3) = \frac{1}{3} b^4$; $I_y = \frac{1}{3} b^4$

$I_{xy} = 0 + \frac{b}{2}\frac{b}{2} b^2 = \frac{1}{4} b^4$

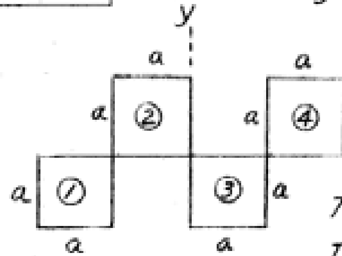
with $\theta = 30°$, Eqs. A/9 & A/9a give

$I_{x'} = \frac{b^4}{3} + 0 - \frac{1}{4} b^4 \sin 60° = (\frac{1}{3} - \frac{\sqrt{3}}{8}) b^4 = \underline{0.1168 b^4}$

$I_{y'} = \frac{b^4}{3} + 0 + \frac{1}{4} b^4 \sin 60° = (\frac{1}{3} + \frac{\sqrt{3}}{8}) b^4 = \underline{0.5498 b^4}$

$I_{x'y'} = 0 + \frac{b^4}{4}\frac{1}{2} = \frac{b^4}{8} = \underline{0.1250 b^4}$

A/75 ① $I_x = a^4/3$, $I_y = a^4/12 + a^2(3a/2)^2 = 7a^4/3$; $I_{xy} = +3a^4/4$



② $I_x = a^4/3$, $I_y = a^4/3$, $I_{xy} = -a^4/4$

③ same as ②

④ same as ①

Thus for composite area

$I_x = 4a^4/3$, $I_y = 16a^4/3$, $I_{xy} = +a^4$

From Eq. A/11,

$$I_{max} = \frac{I_x + I_y}{2} + \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2} = \left(\frac{10}{3} + \sqrt{5}\right)a^4 = \underline{5.57a^4}$$

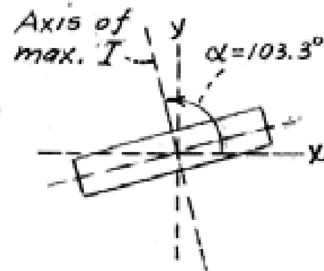
$$I_{min} = \frac{I_x + I_y}{2} - \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2} = \left(\frac{10}{3} - \sqrt{5}\right)a^4 = \underline{1.097a^4}$$

From Eq. A/10

$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2a^4}{(16/3 - 4/3)a^4} = +\frac{1}{2}$$

$2\alpha = 26.6°$ or $206.6°$

$\alpha = 13.3°$ or $\underline{103.3°}$



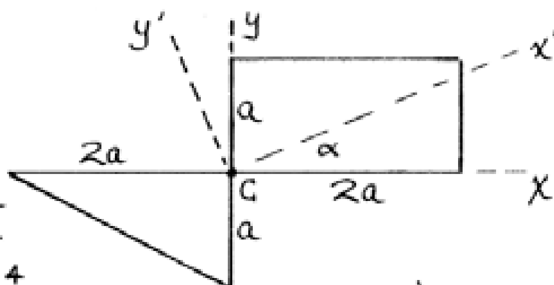
A/76 | Multipy Eqs. A/11 together and get

$$I_{max} I_{min} = \left(\frac{I_x + I_y}{2}\right)^2 - \left(\frac{\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}}{2}\right)^2$$

$$= \frac{1}{4}\left(I_x^2 + 2I_xI_y + I_y^2 - I_x^2 + 2I_xI_y - I_y^2 - 4I_{xy}^2\right)$$

$$= I_xI_y - I_{xy}^2, \text{ so } \underline{I_{xy} = \sqrt{I_xI_y - I_{max}I_{min}}}$$

A/77



For the triangle,

$I_x = \frac{1}{12}(2a)(a^3) = \frac{a^4}{6}$

$I_y = \frac{1}{12}(a)(2a)^3 = \frac{2a^4}{3}$

$I_{xy} = \frac{1}{24}(2a)^2 a^2 = \frac{a^4}{6}$ (from Prob. A/68)

For the rectangle, $I_x = \frac{1}{3}(2a^2)a^2 = \frac{2a^4}{3}$

$I_y = \frac{1}{3}(2a^2)(2a)^2 = \frac{8a^4}{3}$, $I_{xy} = 2a^2\left(\frac{a}{2}\right)(a) = a^4$

Totals: $I_x = \frac{5a^4}{6}$, $I_y = \frac{10a^4}{3}$, $I_{xy} = \frac{7a^4}{6}$

Eqs. A/11:

$$I_{max} = \frac{\frac{5}{6}+\frac{10}{3}}{2}a^4 + \frac{1}{2}\sqrt{\left(\frac{5}{6}-\frac{10}{3}\right)^2 a^8 + 4\left(\frac{7}{6}\right)^2 a^8} = \underline{3.79a^4}$$

$$I_{min} = \frac{\frac{5}{6}+\frac{10}{3}}{2}a^4 - \frac{1}{2}\sqrt{\left(\frac{5}{6}-\frac{10}{3}\right)^2 a^8 + 4\left(\frac{7}{6}\right)^2 a^8} = \underline{0.373a^4}$$

Eq. A/10:

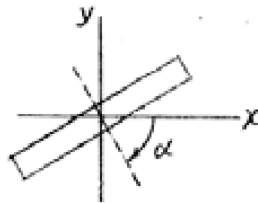
$$\tan 2\alpha = \frac{2\left(\frac{7}{6}a^4\right)}{\frac{10}{3}a^4 - \frac{5}{6}a^4} = \frac{14}{15},\quad 2\alpha = 43.0^\circ \text{ or } 223^\circ$$

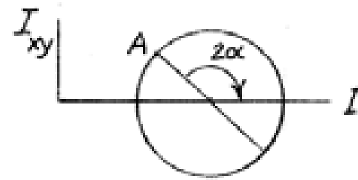
$\alpha = 21.5^\circ$ or 111.5°

$\underline{\alpha = 111.5^\circ}$ for axis of I_{max}

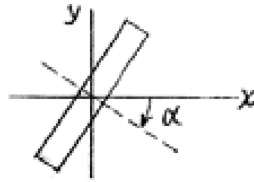
A/78

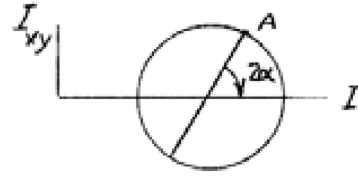
(a) $I_x < I_y$, I_{xy} (+)



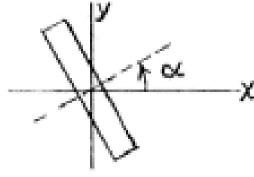


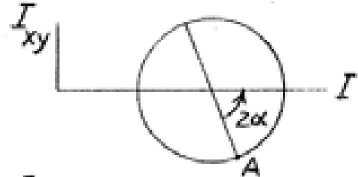
(b) $I_x > I_y$, I_{xy} (+)



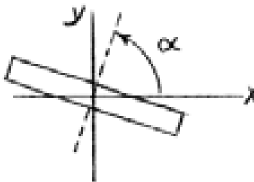


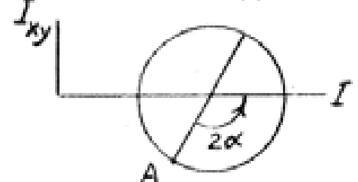
(c) $I_x > I_y$, I_{xy} (−)





(d) $I_x < I_y$, I_{xy} (−)





A/79

① $I_x = I_y = \frac{1}{4}(\frac{1}{4}\pi b^4) = \frac{1}{16}\pi b^4$

$dI_{xy} = dI_{x_0 y_0} + dA(y)(-|\frac{x}{2}|)$

$= 0 \quad + |x|dy(-|\frac{xy}{2}|)$

But $(-x)^2 = b^2 - y^2$ so $dI_{xy} = -\frac{1}{2}(b^2 y - y^3)dy$

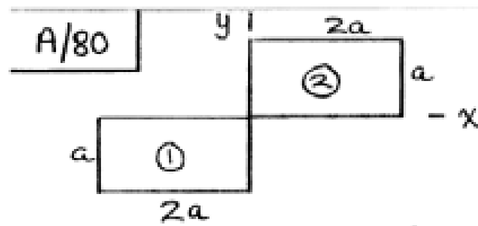
$I_{xy} = -\frac{1}{2}\int_0^b (b^2 y - y^3)dy = -b^4/8$

② From Sample Prob. A/2, $I_x = \frac{1}{4}b^4$, $I_y = \frac{1}{12}b^4$, $I_{xy} = \frac{1}{8}b^4$ (Prob. A/69)

For total, $I_x = (\frac{\pi}{16} + \frac{1}{4})b^4$, $I_y = (\frac{\pi}{16} + \frac{1}{12})b^4$, $I_{xy} = 0$

$I_x = \underline{0.446\, b^4}$, $I_y = \underline{0.280\, b^4}$

x = axis of max. moment of inertia } since $I_{xy} = 0$
y = " " min. " " "



① $I_x = \frac{1}{3}(2a)(a^3) = \frac{2}{3}a^4$, $I_y = \frac{1}{3}(a)(2a)^3 = \frac{8}{3}a^4$

$I_{xy} = (2a^2)(a)(a/2) = a^4$

② $I_x = \frac{2}{3}a^4$, $I_y = \frac{8}{3}a^4$, $I_{xy} = a^4$

Eq. A/11 : $I_{min} = \frac{I_x + I_y}{2} - \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$

$= \frac{10}{3}a^4 - \frac{1}{2}\sqrt{(-4a^4)^2 + 16a^8} = \underline{0.505a^4}$

$I_{max} = \frac{I_x + I_y}{2} + \frac{1}{2}\sqrt{(I_x^2 - I_y^2) + 4I_{xy}^2}$

$= \frac{10}{3}a^4 + \frac{1}{2}\sqrt{(-4a^4)^2 + 16a^8} = \underline{6.16a^4}$

Eq. A/10: $\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{4a^4}{\left(\frac{16}{3} - \frac{4}{3}\right)a^4}$

$2\alpha = 45°$ or $225°$

$\alpha = 22.5°$ for I_{min}

or $\alpha = \underline{112.5°}$ for I_{max}

A/81 | From figure, I_{xy} is (−)

Add Eqs. A/11 & get $I_{max} + I_{min} = I_x + I_y$

so $I_x + I_y = (12+2)10^6 = 14(10^6)$ mm^4

From the 1st of Eqs. A/11,

$$(I_x - I_y)^2 = [2I_{max} - (I_x + I_y)]^2 - 4I_{xy}^2$$

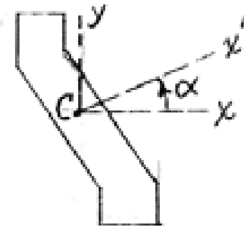
$$= [2(12) - 14]^2 10^{12} - 4(-4)^2 10^{12} = 36(10^{12}) \text{ mm}^8$$

$I_x - I_y = 6(10^6)$ mm^4
$I_x + I_y = 14(10^6)$ mm^4 } add & get $I_x = 10(10^6)$ mm^4

& $I_y = 4(10^6)$ mm^4

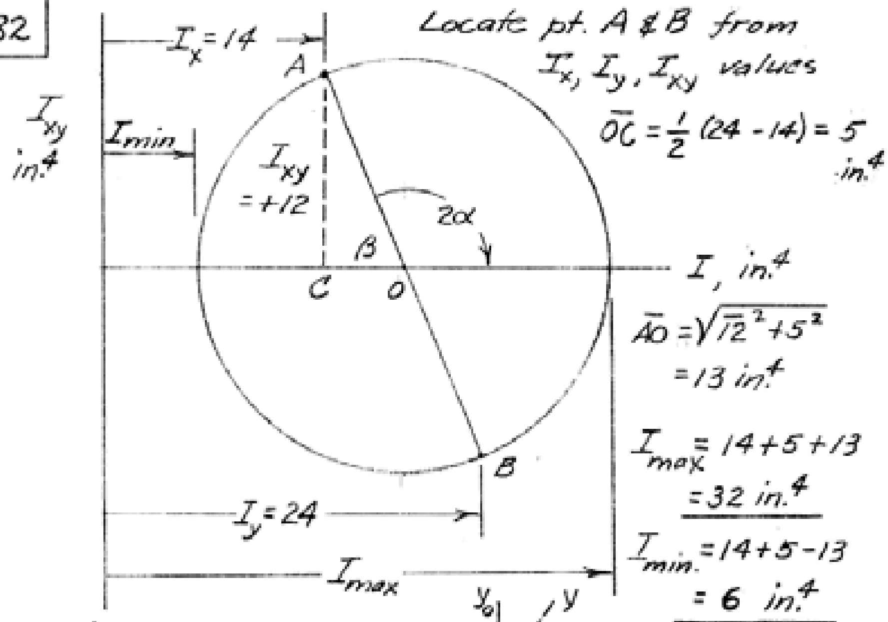
From Eq. A/10,

$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2(-4)(10^6)}{-6(10^6)} = 4/3$$



$2\alpha = 53.13°$, $\alpha = 26.6°$

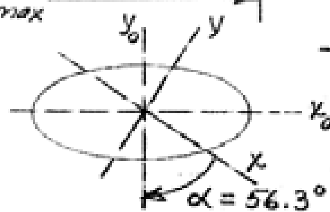
A/82



$\beta = \sin^{-1}\frac{12}{13} = 67.38°,$

$2\alpha = 180 - 67.38 = 112.6°$

$\alpha = 56.3°$ Clockwise



A/83

$I_x = \frac{1}{12}bh^3 = \frac{1}{12}4(8^3) = 170.7 \text{ in.}^4$

$I_y = \frac{1}{12}8(4^3) = 42.7 \text{ in.}^4$

$I_{xy} = \frac{b^2h^2}{24} = \frac{1}{24}(4^2)(8^2) = 42.7 \text{ in.}^4$

y, 8″, x′, α, O, 4″, x

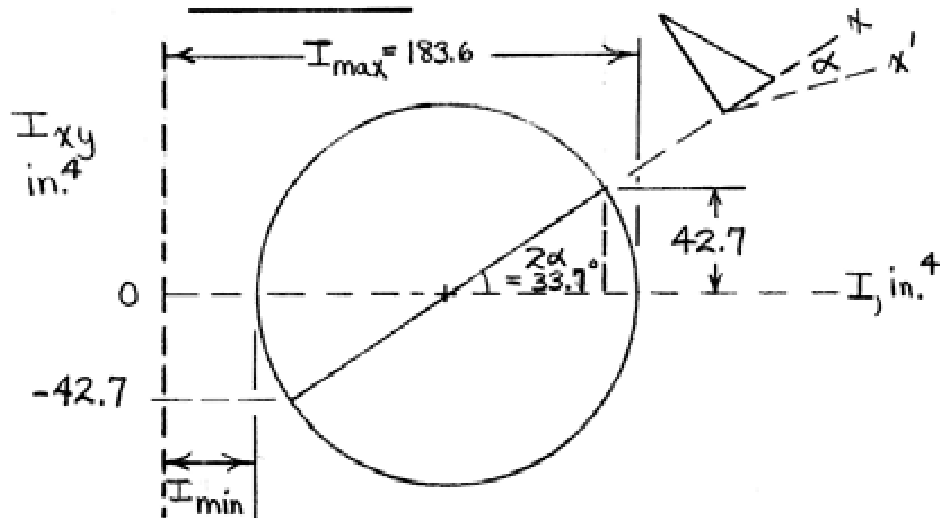
Eq. A/11:

$$I_{max} = \frac{170.7 + 42.7}{2} + \frac{1}{2}\sqrt{(170.7 - 42.7)^2 + 4(42.7)^2}$$

$$= 106.7 + 76.9 = \underline{183.6 \text{ in.}^4}$$

Eq. A/10: $\tan 2\theta_{cr} = \tan 2\alpha = \frac{2(42.7)}{42.7 - 170.7} = -0.667$

$\underline{\alpha = -16.85°}$



A/84

① $I_x = \frac{1}{3}(10)(80)^3 = 1.707(10^6)\ \text{mm}^4$, $I_y = \frac{1}{3}(80)(10)^3 = 0.0267(10^6)\ \text{mm}^4$

② $I_x = \frac{1}{3}(50)(10)^3 = 0.0167(10^6)\ \text{mm}^4$,

$I_y = \frac{1}{12}(10)(50)^3 + 10(50)(35)^2 = 0.7167(10^6)\ \text{mm}^4$

① $I_{xy} = 0 + 80(10)(40)(5) = 0.1600(10^6)\ \text{mm}^4$

② $I_{xy} = 0 + 50(10)(35)(5) = 0.0875(10^6)\ \text{mm}^4$

Totals: $I_x = 1.723(10^6)\ \text{mm}^4$

$I_y = 0.743(10^6)\ \text{mm}^4$

$I_{xy} = 0.248(10^6)\ \text{mm}^4$

y, y′, x′, α, ①, ②, 80 mm, A, 10 mm, 50 mm, 10 mm, x

From Eqs. A/11

$$I_{max} = \frac{I_x + I_y}{2} + \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}, \quad I_{min} = \frac{I_x + I_y}{2} - \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$$

$$I_{max} = \left[\frac{1.723 + 0.743}{2} + \frac{1}{2}\sqrt{(1.723 - 0.743)^2 + 4(0.248)^2}\right]10^6 = \underline{1.782(10^6)\ \text{mm}^4}$$

$$I_{min} = \left[\frac{1.723 + 0.743}{2} - \frac{1}{2}\sqrt{(1.723 - 0.743)^2 + 4(0.248)^2}\right]10^6 = \underline{0.684(10^6)\ \text{mm}^4}$$

From Eq. A/10

$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2(0.248)}{0.743 - 1.723} = -0.5051$$

$2\alpha = -26.8°$

$\underline{\alpha = -13.40°}$

***A/85**

$I_x = \frac{1}{3}(1)(3^3) + \frac{1}{3}(1)(1^3) = 28/3 = 9.333 \text{ in.}^4$

$I_y = \frac{1}{3}(2)(1^3) + \frac{1}{3}(1)(2^3) = 10/3 = 3.333 \text{ in.}^4$

$I_{xy} = (3)(1)(1.5)(0.5) + (1^2)(0.5)(1.5) = 3 \text{ in.}^4$

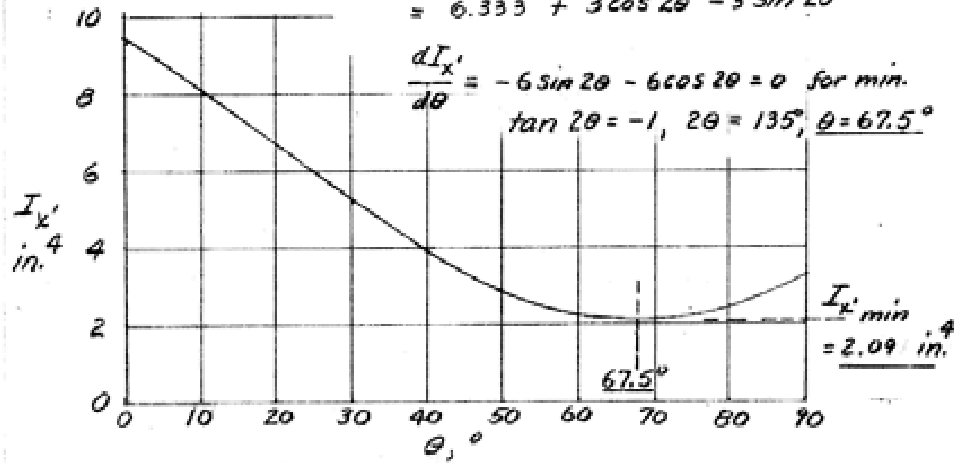
Eq. A/9

$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$

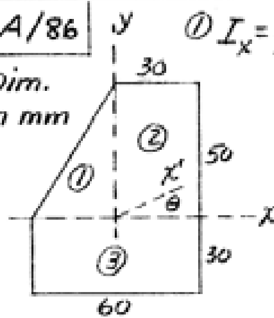
$= 6.333 + 3\cos 2\theta - 3\sin 2\theta$

$\frac{dI_{x'}}{d\theta} = -6\sin 2\theta - 6\cos 2\theta = 0$ for min.

$\tan 2\theta = -1, \; 2\theta = 135°, \; \underline{\theta = 67.5°}$



*A/86



① $I_x = \frac{1}{12}(30)(50)^3 = \frac{125}{4}(10^4)$ mm^4

$I_y = \frac{1}{12}(50)(30)^3 = \frac{45}{4}(10^4)$ mm^4

$I_{xy} = -\frac{(30)^2(50)^2}{24} = -\frac{75}{8}(10^4)$ mm^4 (Prob. A/49)

② $I_x = \frac{1}{3}(30)(50)^3 = 125(10^4)$ mm^4

$I_y = \frac{1}{3}(50)(30)^3 = 45(10^4)$ mm^4

$I_{xy} = (30)(50)\frac{50}{2}\frac{30}{2} = \frac{225}{4}(10^4)$ mm^4

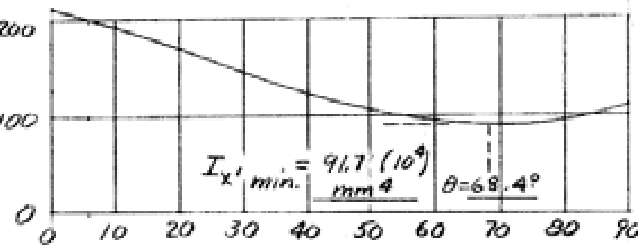
③ $I_x = \frac{1}{3}(60)(30)^3 = 54(10^4)$ mm^4, $I_y = \frac{1}{12}(30)(60)^3 = 54(10^4)$ mm^4, $I_{xy} = 0$

Totals $I_x = 210.25(10^4)$ mm^4, $I_y = 110.25(10^4)$ mm^4, $I_{xy} = 46.875(10^4)$ mm^4

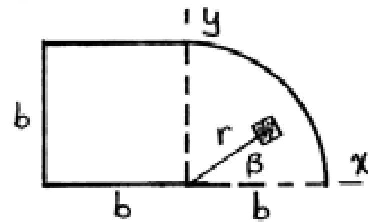
Eq. A/9

$I_{x'} = [160.25 + 50\cos 2\theta - 46.875\sin 2\theta]10^4$

Compute & plot



*A/87



$I_x = \frac{1}{3}b^4 + \frac{1}{16}\pi b^4 = 0.530\,b^4$

$I_y = \frac{1}{3}b^4 + \frac{1}{16}\pi b^4 = 0.530\,b^4$

Quarter circle : $I_{xy} = \int_0^{\pi/2}\int_0^{b} (r\cos\beta)(r\sin\beta)\, r\,dr\,d\beta$

$= \frac{r^4}{4}\Big|_0^b \times \left(-\frac{1}{4}\cos 2\beta\right)\Big|_0^{\pi/2} = \frac{b^4}{4}\left(\frac{1}{4}+\frac{1}{4}\right) = \frac{b^4}{8}$

Square : $I_{xy} = b^2\left(-\frac{b}{2}\right)\left(\frac{b}{2}\right) = -\frac{b^4}{4} = -0.25\,b^4$

Combined : $I_{xy} = \frac{b^4}{8} - \frac{b^4}{4} = -\frac{b^4}{8} = -0.125\,b^4$

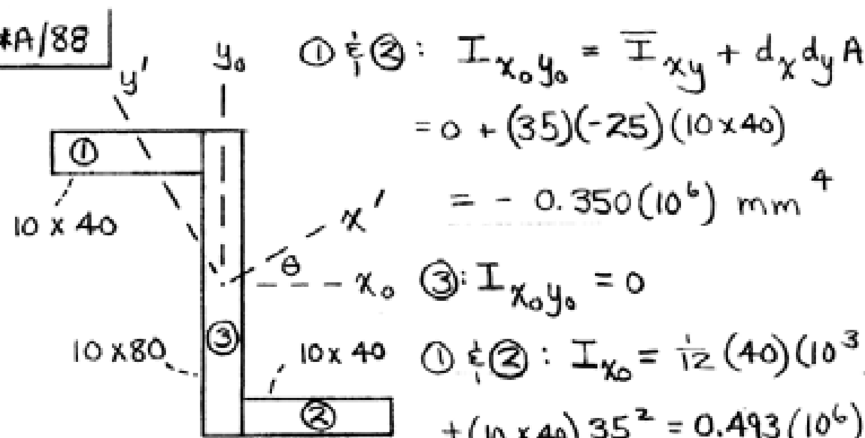
Eq. A/9 : $I_{x'} = \frac{2(0.530\,b^4)}{2} + 0 - (-0.125b^4)\sin 2\theta$

$= (0.530 + 0.125\sin 2\theta)\,b^4$

For critical angle $\theta = \alpha$, Eq. A/10 gives

$\tan 2\alpha = \frac{2(0.530\,b^4)}{0}, \quad 2\alpha = \frac{\pi}{2}, \quad \alpha = \frac{\pi}{4}$

*A/88



① ¢ ②: $I_{x_0 y_0} = \bar{I}_{xy} + d_x d_y A$

$= 0 + (35)(-25)(10 \times 40)$

$= -0.350(10^6)\ \text{mm}^4$

③: $I_{x_0 y_0} = 0$

① ¢ ②: $I_{x_0} = \frac{1}{12}(40)(10^3)$

$+ (10 \times 40)35^2 = 0.493(10^6)\ \text{mm}^4$

$I_{y_0} = \frac{1}{12}(10)(40)^3 + (10 \times 40)25^2 = 0.303(10^6)\ \text{mm}^4$

③: $I_{x_0} = \frac{1}{12}(10)(80)^3 = 0.427(10^6)\ \text{mm}^4$

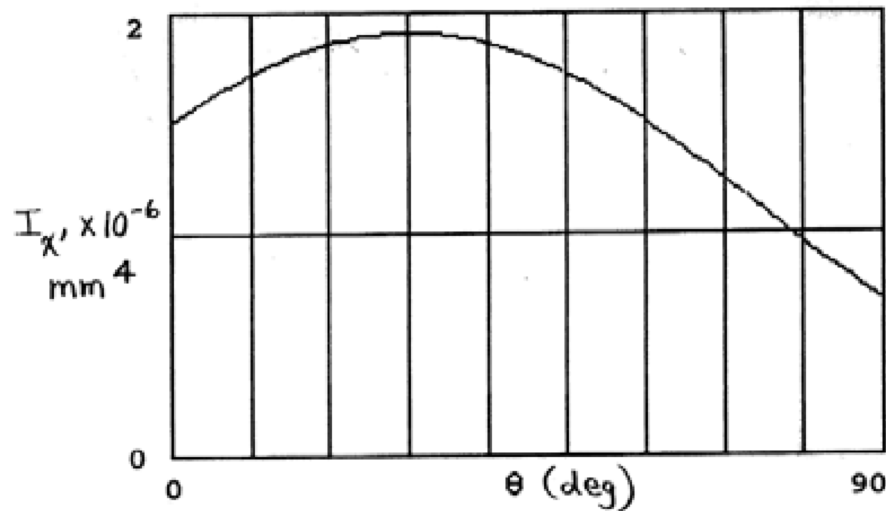
$I_{y_0} = \frac{1}{12}(80)(10)^3 = 0.00667(10^6)\ \text{mm}^4$

Totals: $\begin{cases} I_{x_0} = 1.413(10^6)\ \text{mm}^4 \\ I_{y_0} = 0.613(10^6)\ \text{mm}^4 \\ I_{x_0 y_0} = -0.700(10^6)\ \text{mm}^4 \end{cases}$

From Eq. A/9:

$$I_{x'} = \left[\frac{1.413 + 0.613}{2} + \frac{1.413 - 0.613}{2}\cos 2\theta + 0.700 \sin 2\theta\right] 10^6$$

$$= [1.013 + 0.4 \cos 2\theta + 0.7 \sin 2\theta]\, 10^6$$



Eq. (A/11): $I_{max} = \frac{I_x + I_y}{2} + \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$

$= \frac{1.413 + 0.613}{2} 10^6 + \frac{1}{2}\sqrt{(1.413 - 0.613)^2 (10^6)^2 + 4(-0.7 \times 10^6)^2}$

$= \underline{1.820(10^6) \text{ mm}^4}$

Eq. (A/10): $\tan 2\alpha = \frac{2 I_{xy}}{I_y - I_x}$

$= \frac{2(-0.7)}{0.613 - 1.413}$

$\Rightarrow \alpha = \underline{30.1^\circ}, 120.1^\circ$

(Values from Eqs. A/10 & A/11 agree with plot.)

*A/89 Quarter circle: $I_x = I_y = \frac{1}{16}\pi r^4$

Use horiz strip & get $dI_{xy} = 0 + (x\,dy)\,y\,\frac{x}{2} = \frac{b^2y - y^3}{2}\,dy$

& $I_{xy} = \frac{1}{2}\int_0^b (b^2y - y^3)\,dy = b^4/8$

y
b
x'
θ
x
b/2 b/2

Triangle: $I_x = \frac{1}{12}\frac{b}{2}b^3 = b^4/24$; $I_y = \frac{1}{12}\,b\left(\frac{b}{2}\right)^3 = \frac{b^4}{96}$

$I_{xy} = \frac{1}{24}\left(\frac{b}{2}\right)^2 b^2 = \frac{b^4}{96}$ (Prob A/49)

Composite: $I_x = \left(\frac{\pi}{16} - \frac{1}{24}\right)b^4$, $I_y = \left(\frac{\pi}{16} - \frac{1}{96}\right)b^4$, $I_{xy} = \left(\frac{1}{8} - \frac{1}{96}\right)b^4$

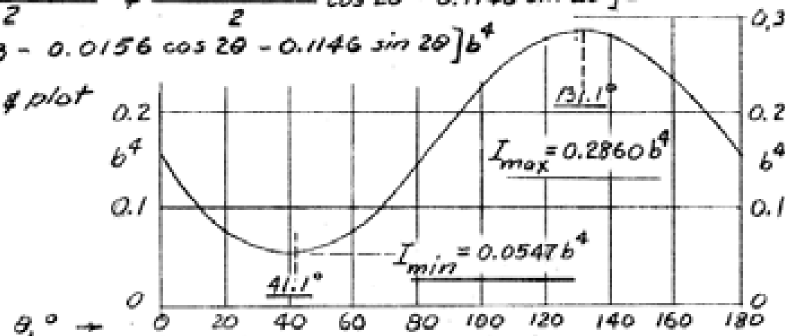
$= 0.1547\,b^4$, $= 0.1859\,b^4$, $= 0.1146\,b^4$

From Eq. A/9

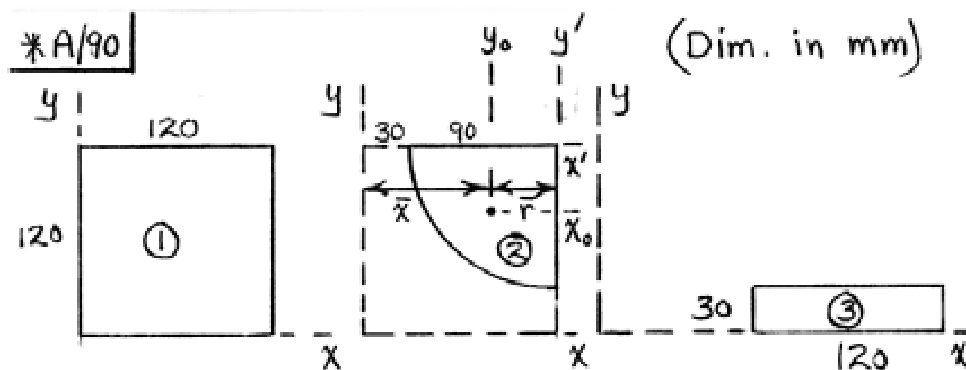
$I_{x'} = \left[\frac{0.1547 + 0.1859}{2} + \frac{0.1547 - 0.1859}{2}\cos 2\theta - 0.1146 \sin 2\theta\right] b^4$

$= \left[0.1703 - 0.0156 \cos 2\theta - 0.1146 \sin 2\theta\right] b^4$

Calculate & plot



∗A/90 (Dim. in mm)



① $I_x = I_y = \frac{1}{3}\,120^4 = 69.1(10^6)\ mm^4$, $A = 120^2 = 1.44(10^4)\,mm^2$

$I_{xy} = 120^2(60)(60) = 51.8\ mm^4$

② $\bar{r} = \frac{4r}{3\pi} = \frac{4(90)}{3\pi} = 38.2\,mm$, $\bar{x} = 120 - \bar{r} = 81.8\ mm$

$A = \frac{1}{4}\pi(90^2) = 0.636(10^4)\ mm^2$

$I_{x'} = I_{y'} = \frac{1}{4}\left(\frac{1}{4}\pi \times 90^4\right) = 12.88(10^6)\ mm^4$

$I_x = I_y = I_{y_0} + A\bar{x}^2 = I_{y'} - A\bar{r}^2 + A\bar{x}^2$

$= [12.88 + 0.00636(81.8^2 - 38.2^2)]\,10^6 = 46.2(10^6)\,mm^4$

$I_{xy} = I_{x_0y_0} + A\bar{x}\bar{y} = I_{x'y'} - A\bar{r}\bar{r} + A\bar{x}\bar{y}$

where $I_{x'y'} = \frac{r^4}{8} = \frac{90^4}{8} = 8.20(10^6)\,mm^4$ {See Sol. to Prob. A/65

$\therefore I_{xy} = [8.20 + 0.00636(81.8^2 - 38.2^2)]\,10^6 = 41.5(10^6)\,mm^4$

③ $I_x = \frac{1}{3}(120)(30)^3 = 1.08(10^6)\ mm^4$

$I_y = \frac{1}{12}(30)(120)^3 + 30(120)(180^2) = 121.0(10^6)\ mm^4$

$I_{xy} = 30(120)(180)(15) = 9.72(10^6)\ mm^4$

Combined: $\begin{cases} I_x = (69.1 - 46.2 + 1.08)10^6 = 24.0(10^6)\ mm^4 \\ I_y = (69.1 - 46.2 + 121.0)10^6 = 143.9(10^6)\ mm^4 \\ I_{xy} = (51.8 - 41.5 + 9.72)10^6 = 20.1(10^6)\ mm^4 \end{cases}$

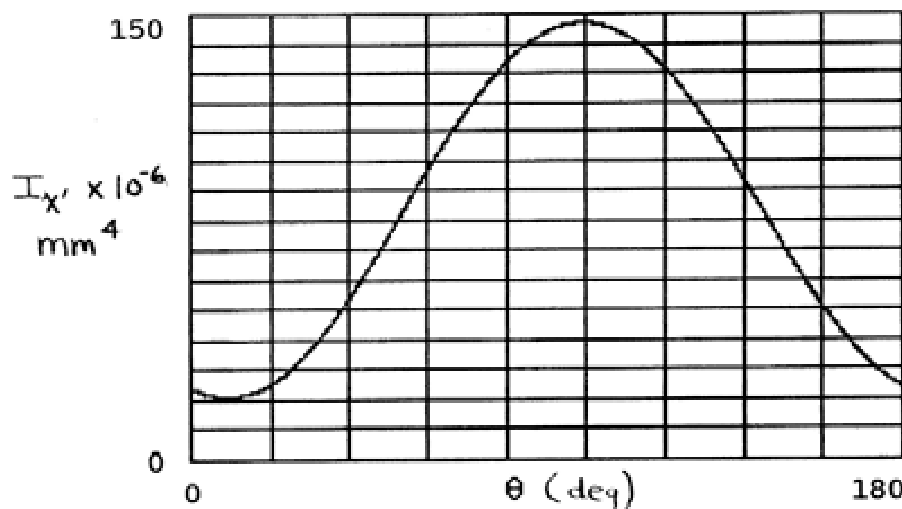
A/9: $I_{x'} = \left\{\frac{24.0 + 143.9}{2} + \frac{24.0 - 143.9}{2}\cos 2\theta - 20.1 \sin 2\theta\right\} \times 10^6$

$I_{x'}(10^{-6}) = 84.0 - 59.9\cos 2\theta - 20.1 \sin 2\theta \quad mm^4$

A/10: $\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2(20.1)}{143.9 - 24.0} = 0.335$

$2\alpha = 18.51°, \quad \alpha = 9.26° \text{ (minimum } I)$

$\alpha = 9.26 + 90 = 99.3° \text{ (maximum } I)$



$I_{min} = 20.8(10^6)\ mm^4$ @ $\theta = 9.26°$

$I_{max} = 147.2(10^6)\ mm^4$ @ $\theta = 99.3°$